Markovian Elections

John Duggan
and
Jean Guillaume Forand

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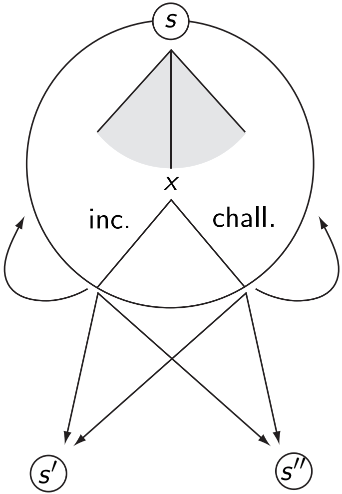
Outline

Motivation

- Our goal is a model of dynamic elections in the presence of a state variable that evolves endogenously over time.
- It should be:
  - General: it should be amenable to a range of structure on preferences, policies, states, etc.
  - Viable: equilibria should exist widely to allow non-constructive characterizations.
  - Useful: it should be possible to solve special cases of the model to generate novel insights.

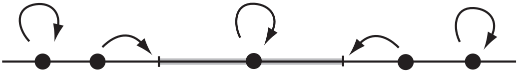
Motivation (cont.)

- More precisely, we model:
  - Sequence of elections over an infinite horizon.
  - State variable evolves over time.
  - Incumbent chooses policy.
  - Challenger is drawn.
  - An election is held.
  - Repeat.



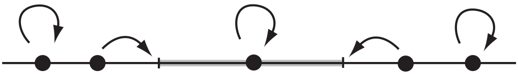
Electoral accountability

	adverse selection
single state	simple partitional equil.



Electoral accountability (cont.)

	adverse selection	complete information
single state	simple partitional equil.	simple partitional equil.



Electoral accountability (cont.)

	adverse selection	complete information
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multi state	big mess	

Electoral accountability (cont.)

	adverse selection	complete information
single state	simple partitional equil.	simple partitional equil.
multi state	big mess	this paper

Commitment

- To generate equilibria with a partitional form we assume state-by-state commitment.
- Each period begins with some state s.
- If the office holder chooses x, she is "bound" to x if s is realized again next period.
- This commitment lasts until the state changes to $s' \neq s$, at which time she is "free."
- Could be supported by history-dependent punishments for breaking commitments (penalizing flip-floppers).

Outline

Components

N	voters	finite
M	politicians	countably infinite
T	types	finite
S	states	countable
Y	policy space	general

Timing

- State s and incumbent's type t are publicly observed.
- Office holder chooses policy x in feasible set $Y_t(s)$.
- Office holder receives stage utility $w_t(s,x)$, and voter and out of office politicians receive $u_t(s,x)$.
- Challenger with unobserved type t' drawn from untried politicians with probability $q_t(t'|s,x)$.

Timing (cont.)

- If the office holder does not seek reelection, the challenger takes office ($e = 0$).
- Otherwise, an election takes place, and if a decisive coalition $C \in \mathcal{D}(s)$ of voters vote to reelect, the incumbent wins ($e = 1$); otherwise, the challenger wins ($e = 0$).
- State s' is drawn from $p_t(s'|s, x, e)$.
- Move to next period (discounting by δ_t) and repeat.

Assumptions

- Finite T and countable S.
  - Types can represent preferences or ability, states can represent economic variables, distributions of preferences, etc.
- Sets $Y_t(s)$ are closed subsets of compact metric space Y.
  - Can be finite, or convex subset of Euclidean space, or space of tax functions.
- Collection $\mathcal{D}(s)$ is monotonic.
  - Captures majority rule, quota rules, electoral college, non-democratic systems

Assumptions (cont.)

- Stage utilities $u_t(s, x)$ and $w_t(s, x)$ are bounded and continuous.
  - **Policy motivation:** $w_t(s, x) = u_t(s, x)$.
  - **Mixed motivation:** $w_t(s, x) = u_t(s, x) + b$.
- State transition $p_t(s'|s, x, e)$ and challenger distribution $q_t(t'|s, x)$ are continuous.
  - Policy can influence evolution of future economic states.
- No convexity conditions are imposed.

Strategies

- Policy strategies: $\pi_t(\cdot|s)$
  - Type symmetry, stationarity wrt s
- Commitment:
  - Mixing only occurs when transitioning from another state.
  - Once x is chosen in s, the office holder is **bound** to x for successive realizations of s.
  - When the state leaves s, the office holder is **free**.

Strategies (cont.)

- Voting strategy: $\rho(s, t, x) \in [0, 1]$
  - Stationarity wrt s, t, x
  - Mixing occurs when the office holder is initially bound to x in s, then electoral decision carries over for successive realizations of s — but no commitment.
  - Reduced form of more detailed voting game, where mixing is generated by indifferent voters.
- Let $\sigma = (\pi, \rho)$ be a **simple Markov strategy profile**.

Remarks

- The model subsumes the single-state model with adverse selection or with complete information, substituting state-by-state commitment for private information about types or history-dependent punishments.
- We also capture competition between two infinitely-lived parties that alternate in power; they may be purely policy motivated or receive office benefit.
- As a special case, we can parameterize commitment: assume a fixed $\gamma \leq 1$ such that given policy choice x in state s, if s is subsequently realized, the office holder is bound to x with probability γ.

Continuation values

- We consider expected discounted utilities for type τ voter conditional on three kinds of events:

$V_\tau^B(s,t,x)$	electing a type t incumbent committed to x in s (and continuing to do so) before next state is realized
$V_\tau^C(s,t,x)$	electing a challenger after a type t incumbent has chosen x in s

Continuation values (cont.)

- We consider expected discounted utilities for type t office holders conditional on two events:

$W_t^B(s,x)$	choosing x in state s and being reelected (and continuing to choose x in s and being reelected in s)
$W_t^C(s,x)$	choosing x in state s and being replaced by a challenger

Recursive conditions

Outline

Reelection sets

- Given strategy profile σ, define for all s, all t, and all τ,

$$\begin{aligned} P_\tau(s,t) &= \{x \in Y_t(s) : V_\tau^B(s,t,x) > V_\tau^C(s,t,x)\} \\ R_\tau(s,t) &= \{x \in Y_t(s) : V_\tau^B(s,t,x) \geq V_\tau^C(s,t,x)\}. \end{aligned}$$

- And for all coalitions C of types,

$$P_C(s,t) = \bigcap_{\tau \in C} P_\tau(s,t) \quad \text{and} \quad R_C(s,t) = \bigcap_{\tau \in C} R_\tau(s,t).$$

Reelection sets (cont.)

- Finally, define the **strict** and **weak reelection sets** as

$$\begin{aligned} P(s,t) &= \bigcup_{C \in \mathcal{D}(s)} P_C(s,t) \\ R(s,t) &= \bigcup_{C \in \mathcal{D}(s)} R_C(s,t), \end{aligned}$$

respectively.

Equilibrium concept

- Strategy profile σ is a **simple Markov electoral equilibrium** if two conditions hold:
- **Optimal policies:** $\pi_t(\cdot|s)$ puts probability one on solutions to

$$\max_{x \in Y_t(s)} \rho(s,t,x) W_t^B(s,x) + (1-\rho(s,t,x)) W_t^C(s,x).$$

- **Optimal voting:**

$$\rho(s,t,x) = \begin{cases} 1 & \text{if } x \in P(s,t) \\ 0 & \text{if } x \notin R(s,t). \end{cases}$$

Outline

Overview of results

- Existence and continuity
- Representative voters
- Dynamic core convergence

Existence...

Theorem

There exists a simple Markov electoral equilibrium.

- Existence does not follow from results in extant literature.
- Requires precise selection of reelection probabilities following different policy choices.
- Proof is built on existence proof for general bargaining environments.

Idea of proof

. . . and continuity

- Parameterize utilities and transitions by the elements γ of a metric space Γ,

$$u_t(s, x, \gamma),\ w_t(s, x, \gamma),\ p_t(s'|s, x, e, \gamma),\ q_t(t'|s, x, \gamma),$$

  and assume joint continuity.
- Define $\mathcal{E}(\gamma)$ to consist of policy strategy vectors $\pi = (\pi_t(\cdot|s))$ corresponding to simple Markov electoral equilibria.

Theorem

The correspondence $\mathcal{E}\colon \Gamma \rightrightarrows \Delta(X)^{S\times T}$ has closed graph.

Representative voters

- Elections are simplified if there is one voter type whose preferences determine the outcome.
- Given a simple Markov electoral equilibrium σ, a type κ is **representative** at s if for all t,

$$P(s,t) \;=\; P_{\kappa}(s,t) \quad \text{and} \quad R(s,t) \;=\; R_{\kappa}(s,t).$$

Representative voters (cont.)

- We can show representativeness of the median voter under additional assumptions.

  D1 $Y \subseteq \Re$,

  D2 u_τ is quadratic in x and independent of state for all τ, i.e.,

$$u_\tau(x) \quad = \quad -|x - \hat{x}_\tau|^2,$$

  D3 $\delta_\tau = \delta$ for all τ.

- The electoral rule $\mathcal{D}(s)$ is **strong** if $C \notin \mathcal{D}(s)$ implies $N \setminus C \in \mathcal{D}(s)$
- Then let $\kappa(s)$ be the median voter at s.

Representative voter (cont.)

Theorem

Let σ be a simple Markov electoral equilibrium. And assume (D1)–(D3), fix s, and assume $\mathcal{D}(s)$ is strong. Then the median voter $\kappa(s)$ is representative at s.

- The result extends to multiple dimensions under (D1)–(D3) if the core at s is nonempty,

Representative voters (cont.)

- For each s, t, and x, there are probability measures $\mu(\cdot|s,t,x)$ and $\nu(\cdot|s,t,x)$ on X such that

$$\begin{aligned} V_\tau^B(s,t,x) &= \frac{1}{1-\delta}\int_{x'} u_\tau(x')\mu(dx'|s,t,x) \\ V_\tau^C(s,t,x) &= \frac{1}{1-\delta}\int_{x'} u_\tau(x')\nu(dx'|s,t,x). \end{aligned}$$

- An election presents voters with the choice, effectively, between two lotteries.

Dynamic core convergence

- In the standard Downsian model, candidates take policy positions at the ideal point of the median voter in equilibrium.
- That is, the candidates offer the policy that the median voter would choose herself.
- To formulate this question, we abstract away from politicians: assume a representative voter $\kappa(s)$ for each state, feasible policies $Y(s)$, and state transition $p(s'|s, x)$.
- Complications: there may be different medians in different states, and a median voter's optimal policy choice is endogenous.

Dynamic core convergence (cont.)

- Define an associated **dynamic representative voting game**: in state s,
  - voter $\kappa(s)$ chooses x from $Y(s)$
  - a new state is drawn from $p(\cdot|s,x)$
  - period payoffs are $u_{\kappa(s)}(s',x)$
  - discount factors are $\delta_{\kappa(s)}$.
- If the representative voter is κ (fixed), this is a dynamic programming problem with optimal value function V_κ^*.

Dynamic core convergence (cont.)

- We provide core convergence results under the following assumptions:
  - E1 $Y_t(s)$ is independent of t,
  - E2 $p_t(s'|s, x, e)$ is independent of t and e,
  - E3 mixed motives, i.e., $w_t(s, x) = u_t(s, x) + b$,
  - E4 for all t, $\delta_t > 0$,
  - E5 for all s, there is a representative voter type $\kappa(s)$,
  - E5′ there is a representative voter κ fixed across states.

Weak core convergence

Theorem

Assume (E1)–(E5) with b large. Let $\tilde{\pi} = (\tilde{\pi}_s)$ be a stationary Markov perfect equilibrium (possibly in mixed strategies) of the dynamic representative voting game. Then there is a simple Markov electoral equilibrium $\sigma = (\pi, \rho)$ such that for all s and all t, $\pi_t(\cdot|s) = \tilde{\pi}_s$, i.e., politicians implement the equilibrium policy strategies $\tilde{\pi}$.

- Adding (E5′), we can use pure policy strategies.

Weak convergence (cont.)

- Given such policy strategies, the representative voter in s is indifferent between every incumbent and every challenger.
- Using high office benefit, specify mixed electoral outcomes to make a type $t \neq \kappa(s)$ office holder indifferent between all policies in the support of $\tilde{\pi}_s$.
- Furthermore, the probability of reelection is zero for policies outside the support of $\tilde{\pi}_s$, so no deviations are profitable.

Strong core convergence

Theorem

In addition to (E1)–(E5′), assume:

- $b = 0$, *i.e., policy motivation,*
- *for all* s, $\sum_{m=1}^{\infty} p^m(s|s,x) = 1$,
- *for all* s, $\min_{t,x} q_t(\kappa|s,x) > 0$.

As $\delta \to 1$, *let* σ^δ *be a simple Markov electoral equilibrium, and let* $V_\kappa^{F,\delta}(s,t)$ *be the value of a free type* t *office holder in state* s*. Then for all* s *and all* t,

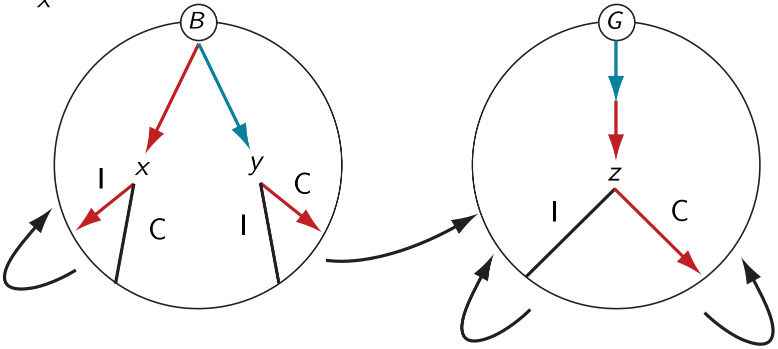
$$\frac{V_\kappa^{F,\delta}(s,t)}{V_\kappa^{*,\delta}(s)} \to 1.$$

Strong core convergence (cont.)

- When $b = 0$, the type κ voter and politician are perfectly aligned.
- The equilibrium strategies of the type κ voter and politician solve the Bellman equation for the unified player (so we can solve their optimization problem jointly).
- The type κ voter can always draw challengers until a type κ politician is elected, keeping her in office thereafter.
- When δ is close to one, the cost of this strategy becomes negligible.

Curse of ambition

ranking	b high
z	
y	
x	



Strong core convergence (cont.)

Theorem

Let σ be a simple Markov electoral equilibrium such that for all s and all t, policy strategies are pure and voting strategies are deferential, i.e.,

$$R(s,t) \neq \emptyset, \text{ and for all } x \in R(s,t),\ \rho(s,t,x) = 1.$$

In addition to (E1)–(E5′), assume:

- *b large,*
- *$p(s'|s,x)$ is independent of x,*
- *for all s, $\min_{t,x} q_t(\kappa|s,x) > 0$.*

Then for all s and all t, $V^F_\kappa(s,t) = V^*_\kappa(s)$.

Outline

Conclusion

- We analyze dynamic elections with an endogenously evolving state variable.
- The framework appears general, viable, and useful.
- Some topics that may be accessible with the model: growth and development, political transitions and instability, dynamics of income inequality...
- More to do!