Collective Dynamic Choice: The Necessity of Time Inconsistency

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Priorat, June 8, 2012

Introduction

- Groups often make intertemporal decisions collectively:
  - Political committees
  - Firms
  - Households
  - Multiple motives within an agent

Background - Impossibility

Impossibility of aggregating individual preferences with a rational representative agent

- Arrow (1951, 63): full domain
- Plott (1967), McKelvey (1976,79): Majority voting over multi-dimensional alternatives with diverse enough preferences results in cycles.
- Mongin (1995): Cannot aggregate subjective preferences/ nonatomic probabilities

Main Question

What if the alternatives are time sequences of consumption and agents discount sums of utilities of consumption?

Can a society aggregate preferences in a "rational" manner – **time consistent** and **transitive**?

Two Desiderata on Group Decision-Making

Time Consistency: ensures that decisions stand up over time without (costly) commitment devices

Transitivity: ensures that the process is well-defined and will not cycle endlessly, or be subject to agenda manipulations

Households – Heterogeneity of Time Preferences

Life expectancy (children born 2010-2015, UN stats):

US: 82 female, 78 male

France: 85 female, 79 male

China: 76 female, 72 male

Brazil: 77 female, 70 male…

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Combining these: Typical female married to male of 65 years expects to live 50% longer. (Browning, 2000)

➔ Different time preferences for consumption/savings plans within the same household (e.g., Schaner, 2010)

Another Perspective

View an individual as comprised of multiple personalities/motives.

Can such an individual act "rationally"?

Other Background Literature

- **Time inconsistencies**
  - **Evidence:** Hernstein (1961), Thaler (1981), Benzion, Rapoport, Yagil, (1989), Green, Myerson, McFadden (1997), Rubinstein (2003), della Vigna, Malmendier (2006), Benhabib, Bisin, Schotter (2009), Andreoni, Sprenger (2010), …
  - **Theory:** Strotz (1956), Laibson (1997), O'Donoghue, Rabin (1999), Ok, Masatlioglu (2003),…

Typical Time Inconsistency

Ainslie and Haslam (1992) :

Majority prefer 100$ certified check today to 200$ check cashable in two years; but prefer 200$ check cashable in eight years to 100$ check cashable in six years

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Inconsistency: $\mathbf{u(100) > \delta^2 u(200)}$

implies $\mathbf{\delta^6 u(100) > \delta^8 u(200)}$

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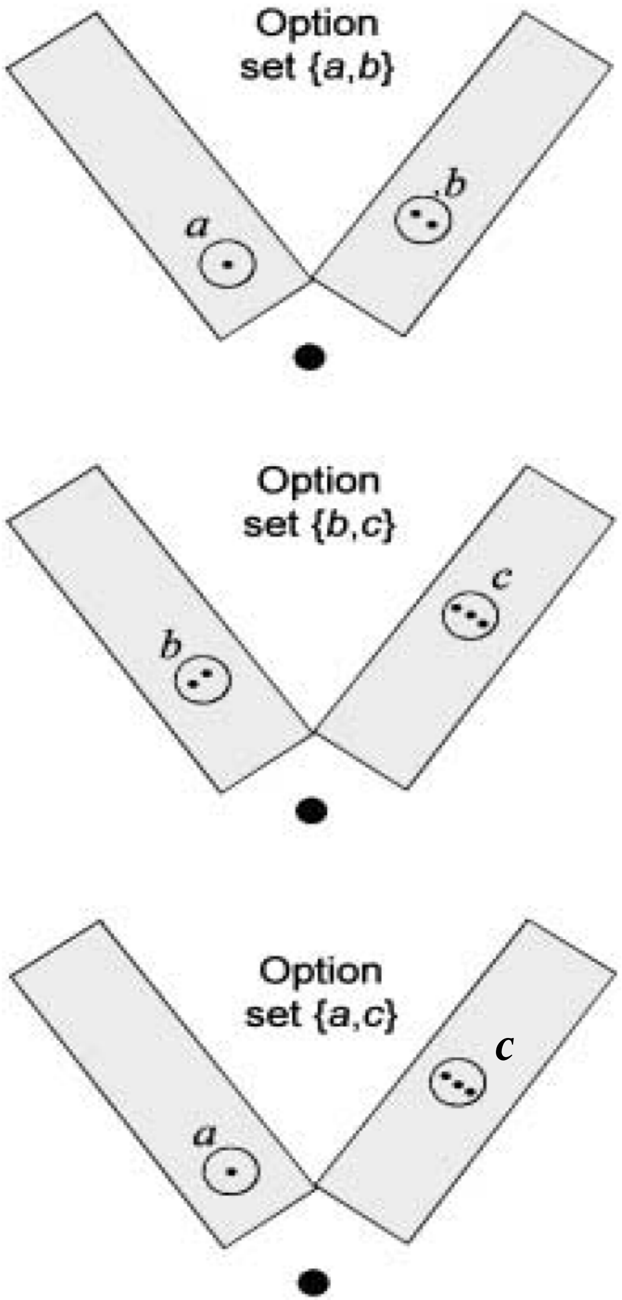
- **Intransitivities**
  - **Evidence:** Tversky (1969) humans, Shafir (1994) bees, Waite (2001) jays,…
  - **Theory:** Kahneman, Tversky (1974, 79), Fishburn (1991), van Zandt (1996),…

Typical Intransitivities

Intransitivity among grey jays, Waite (2001):

- 1, 2 or 3 raisins placed at various distances in a tube: effort/danger
- Some 'prefer' a to b to c but not a to c



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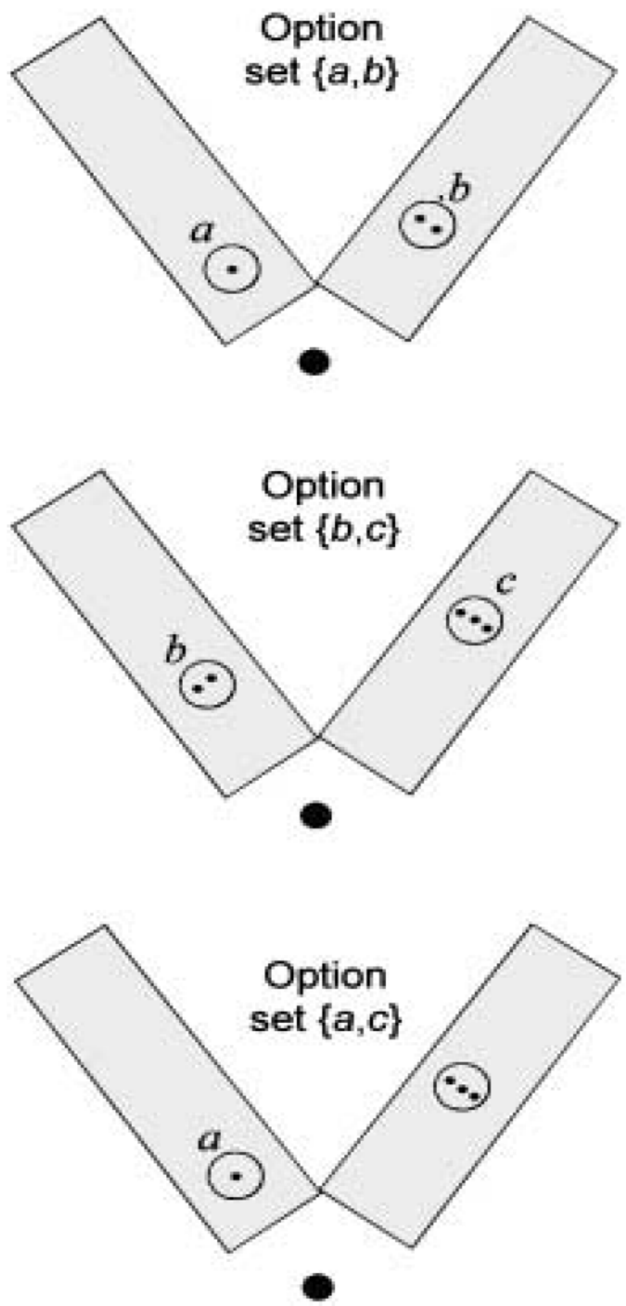
But U(a)>U(b)>U(c) implies U(a)>U(c)



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- **Models of multiple selves / aggregation of time preferences**
  - **Hot/cold/addiction:** Bernheim and Rangel (2004), Fudenberg, Levine (2006),…
  - **Planners:** Marglin (1963), Feldstein (1964), Green, Hojman (2009), Zuber (2010),…
  - **Current and future selves:** Thaler, Shefrin (1981), Gul, Pesendorfer (2001), Benabou, Tirole (2005), Ambrus, Rozen (2009),…

Setting

- Agents - $\{1,\ldots,n\}$

- $C=(c_1,c_2,\ldots.)$ time stream of consumption, c_t in $[0,M]$

- Agent i evaluates consumption as

$$U_i(C) = \sum_t \delta_i^{t-1} u_i(c_t)$$

- u_i is twice continuously differentiable, strictly increasing

- Agents evaluate a common stream of consumption

Setting

- $C=(c_1,c_2,\ldots.)$ common stream of consumption

- $U_1(C) = \sum_t \delta_1^{\,t-1}\, u_1(c_t)$

$$\vdots$$

$$U_n(C) = \sum_t \delta_n^{\,t-1}\, u_n(c_t)$$

- How to aggregate these?

Collective Decisions

- Society U = $(U_1, U_2, \ldots, U_n) = (\delta_1, u_1; \ldots; \delta_n, u_n)$

- Collective utility function (e.g., utilitarian):

  $V[U](C)$

- Collective preferences (e.g., majority vote):

  $C \quad R[U] \quad C'$

Examples

- **(Weighted) Utilitarian**

  $V[U](C) = \sum_i w_i \sum_t \delta_i^{t-1} u_i(c_t)$

- **Maximin**

  $V[U](C) = \mathrm{Min}_i \ (\sum_t \delta_i^{t-1} u_i(c_t))$

Outline

- Utilitarian aggregation
  - must be present-biased

- General aggregation of utilities
  - must be time inconsistent

- Voting over time streams
  - must be intransitive

Utilitarian Aggregation

- Utilitarian planner choosing efficient streams

$$V(C) = \sum w_i U_i(C)$$

Utilitarian Aggregation

- Utilitarian planner choosing efficient streams

$$\mathbf{V(C) = \sum w_i\, U_i(C)}$$

- What do we know? Marglin '63, Feldstein '64, Gollier, Zeckhauser '05, Zuber '10,…
  - Representative agent has time varying discount factor – so will be time inconsistent

- **Can we draw more general conclusions?**

Utilitarian Aggregation: Example 1

- Constantine has $\delta_1 = .5$
- Patience has $\delta_2 = .8$

$$C = (10,0,0,0\ldots) \quad \text{vs} \quad C' = (0,15,0,0\ldots)$$

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$C = (10,0,0,0\ldots)$ vs $C' = (0,15,0,0\ldots)$

V(C) = 10/2 + 10/2 = 10

V(C') = .5*15/2 + .8*15/2 = 9.75

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$V(C) = .5 * 10/2 + .8 * 10/2 = 6.5$

$V(C') = .5^2 * 15/2 + .8^2 * 15/2 = 6.675$

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$$V(C') = .5^2 * 15/2 + .8^2 * 15/2 = 6.675$$

Utilitarian Aggregation: Example 2

- A population with a uniform distribution of δ_i in $[0,1]$
- Utilitarian planner maximizes

$$\Sigma_t \int \delta^{t-1}\, u(c_t)\; d\delta \;=\; \Sigma_t\; u(c_t) \,/\, t$$

Hyperbolic discounting!

(cf Sozou (1998): uncertainty with exponential weights…)

Present-Bias, Preference Reversal

- There are c, c' such that

$V(c_1, 0\ldots) > V(0, \ldots, c'_{k+1}, 0\ldots)$ while

$V(0,\ldots,0,c_t,0\ldots) < V(0,\ldots,0,c'_{t+k},0\ldots)$ for all $t > 1$

There are some cases where one becomes more patient as decisions are moved towards the future

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There are some cases where one becomes more patient as decisions are moved towards the future

- *But* $V(0,\ldots,0,c_t,0\ldots) \leq V(0,\ldots,0,c'_{t+k},0\ldots)$ implies

  $V(0,\ldots,0,c_{t+1},0\ldots) \leq V(0,\ldots 0,c'_{t+k+1},0\ldots)$

But never the reverse: always become more patient, never more impatient, in the future

Utilitarian Aggregation

Proposition: *If V is utilitarian with positive weights on some agents who have different discount factors, then V is* ***present-biased.***

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Implications:

1. Utilitarian planners, be it within oneself or for a collective, will necessarily exhibit present-bias.
2. An econometrician measuring $\mathbf{E}(\delta_i^{t-1})$ would deduce present-bias when populations are heterogeneous.

Utilitarian Aggregation: Intuition

- $\delta_1 < \delta_2 \quad \ldots < \delta_n$
- $w_1 \delta_1^t + \ldots + w_n \delta_n^t$ tends to $w_n \delta_n^t$
- Relatively more influence from higher δ_i's as t grows regardless of explicit weighting
- → More patience as look further ahead in time
- Time inconsistent preferences when aggregating

Outline

- Utilitarian aggregation
  - must be present-biased

- General aggregation of utilities
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General Aggregation Rules

- Is there some other method (non-utilitarian) of aggregating utilities that will be time consistent?

- E.g., minmax, order statistic, measure of inequality of utilities, time varying weights on individuals, ….

Time Consistency

1. $V(C) > V(C')$ iff $V(c_1, C) > V(c_1, C')$

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$$C\,|_t\,C' = (c_1, \ldots, c_t, c'_{t+1}, c'_{t+2}, \ldots)$$

Time Consistency

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Let

$$C|_t C' = (c_1, \ldots, c_t, c'_{t+1}, c'_{t+2}, \ldots)$$

2. $V(C|_t C''|_{t'} C) > V(C'|_t C''|_{t'} C') \text{ iff}$

$$V(C|_t C'''|_{t'} C) > V(C'|_t C'''|_{t'} C')$$

Unanimity

If $U_i(C) > U_i(C')$ for all i,

then $V[U](C) > V[U](C')$

General Aggregation Rules

Consider U such that each agent has a different discount factor.

Theorem: *If a collective utility function is time consistent and satisfies unanimity, then it is dictatorial: there exists i such that, up to affine transformation,*

$$V[U](C) = \sum_t \delta_i^{t-1} u(c_t) \text{ for all } C.$$

General Aggregation Rules: Intuition

Koopmans (1960): Time consistency implies

$$V(C) = \sum_t \delta^{t-1} v(c_t)$$

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Time consistency has powerful implications for functional forms, and exponential discounting is essentially the only time consistent form

General Aggregation Rules: Intuition

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$$V(C) = \sum_t \delta^{t-1} v(c_t)$$

Unanimity: $v(c_t)=u(c_t)$ and $\delta = \delta_i$ for some i

- Suppose $v(c_t)=u(c_t)$
- Suppose $\delta \neq \delta_i$ for all i and that all δ_i are different
- $\{(1, \delta_i, \delta_i^2, \ldots)\}$ and $(1, \delta, \delta^2, \ldots)$ are linearly independent

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Unanimity: $v(c_t)=u(c_t)$ and $\delta = \delta_i$ for some i

- Suppose $v(c_t)=u(c_t)$
- Suppose $\delta \neq \delta_i$ for all i. From independence, find C, C' such that:

$$\sum_t \delta_i^{t-1} [u(c_t) - u(c'_t)] > 0, \quad i = 1, \ldots, n$$
$$\sum_t \delta^{t-1} [u(c_t) - u(c'_t)] < 0.$$

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- Suppose $v(c_t)=u(c_t)$
- Suppose $\delta \neq \delta_i$ for all i. From independence, find C, C' such that:

Agents prefer C to C' →

$$\sum_t \delta_i^{t-1} [u(c_t) - u(c'_t)] > 0, \quad i = 1, \ldots, n$$

Social planner prefers C' to C →

$$\sum_t \delta^{t-1} [u(c_t) - u(c'_t)] < 0.$$

General Aggregation Rules: Remarks

- Slightly more intricate proof for different instantaneous utilities

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General Aggregation Rules: Remarks

- Slightly more intricate proof for different instantaneous utilities
- Proof uses consumption variations on at least three dates
- May provide a bridge between empirical work showing time consistency on simple tasks and that illustrating time inconsistency on more intricate life decisions

Relaxing Time Consistency

- Suppose we allowed the social planner to have time-dependent discount rates, but maintained separability:

$$V(C) = \sum_t \delta(t)\, u(c_t)$$

Claim: *If a collective utility function of the form* $V(C) = \sum_t \delta(t)\, u(c_t)$ *satisfies unanimity, then it is a weighted utilitarian.*

In particular, it is present-biased if it weighs more than one individual.

Intuition

- If $(\delta(1), \delta(2), \ldots)$ is linearly independent of $\{(1, \delta_i, \delta_i^2, \ldots)\}$, there exists a pair of consumption streams such that the planner prefers one while all agents prefer the other, violating unanimity.

Intuition

- If $(\delta(1), \delta(2), \ldots)$ is linearly independent of $\{(1, \delta_i, \delta_i^2, \ldots)\}$, there exists a pair of consumption streams such that the planner prefers one while all agents prefer the other, violating unanimity.

- Therefore, $(\delta(1), \delta(2), \ldots)$ is a linear combination of $\{(1, \delta_i, \delta_i^2, \ldots)\}$, and is thus utilitarian, implying present-bias.

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- Utilitarian aggregation
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Voting

- Rank C and C' by voting
- Can we find a representative voter? Say with the median discount factor? (would look like a dictator…)
- Is the resulting ranking rational? (at a minimum transitive?)

Intuition: Cycles

There are "three dimensions" to decisions –

- **Short-term consumption**: impatient agents care most about
- **Overall consumption**: patient agents care most about
- **Distribution of consumption**: moderately patient care about

General Voting Rules

R[U] is a ***voting rule*** if it only depends on information regarding who prefers one alternative to another.

So includes weighted majority, supermajorities, other non-anonymous, non-neutral rules…

General Voting Rules

R[U] is a ***voting rule*** if it only depends on information regarding who prefers one alternative to another.

R[U] is ***locally non-dictatorial*** if it never picks in favor of one agent's ranking when all others have the opposite ranking.

Voting – Main Result

Let all agents have a strictly concave $u_i = u$

Theorem: *If a voting rule is locally non-dictatorial, then it is intransitive for some profiles of discount factors.*

Summary

- Utilitarian aggregation leads to a present-bias.
- Any non-dictatorial, unanimous collective utility function is time inconsistent.
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Summary

- Utilitarian aggregation leads to a present-bias.
- Any non-dictatorial, unanimous collective utility function is time inconsistent.
- Voting rules are necessarily intransitive
- See time inconsistency and intransitivity in the lab, 'planners' weigh utilitarian and egalitarian motives.

THE END

General Aggregation Rules: Intuition

Koopmans (1960): Time consistency implies

$$V(C) = \sum_t \delta^{t-1} v(c_t)$$

Unanimity: $v(c_t) = u_i(c_t)$ and $\delta = \delta_i$ for some i

- Suppose $v(c_t) = u(c_t) = u_i(c_t)$ for all i
- If we used average δ_i, what would go wrong?

Example: Problem with Averaging

$\delta_1 = 0 < \delta_{avg} = .5 < \delta_3 = 1,$

linear utility $u(c)=c$

Example: Problem with Averaging

$$\delta_1 = 0 < \delta_{avg} = .5 < \delta_3 = 1$$

$$C = (1, 1, 1, 0, 0, \ldots) \text{ vs}$$

$$C' = (1+\varepsilon, 1-6\varepsilon, 1+6\varepsilon, 0, 0, \ldots)$$

$$U_1(C) = 1 < U_1(C') = 1 + \varepsilon$$

$$U_3(C) = 3 < U_3(C') = 3 + \varepsilon$$

Example: Problem with Averaging

$$\delta_1 = 0 < \delta_{avg} = .5 < \delta_3 = 1$$

$$C = (1, 1, 1, 0, 0, \ldots) \text{ vs}$$

$$C' = (1+\varepsilon, 1-6\varepsilon, 1+6\varepsilon, 0, 0, \ldots)$$

$$U_1(C) = 1 < U_1(C') = 1 + \varepsilon$$

$$U_3(C) = 3 < U_3(C') = 3 + \varepsilon$$

$$U_{avg}(C) = 1.75 > U_{avg}(C') = 1.75 + \varepsilon - 3\,\varepsilon + 1.5\,\varepsilon$$

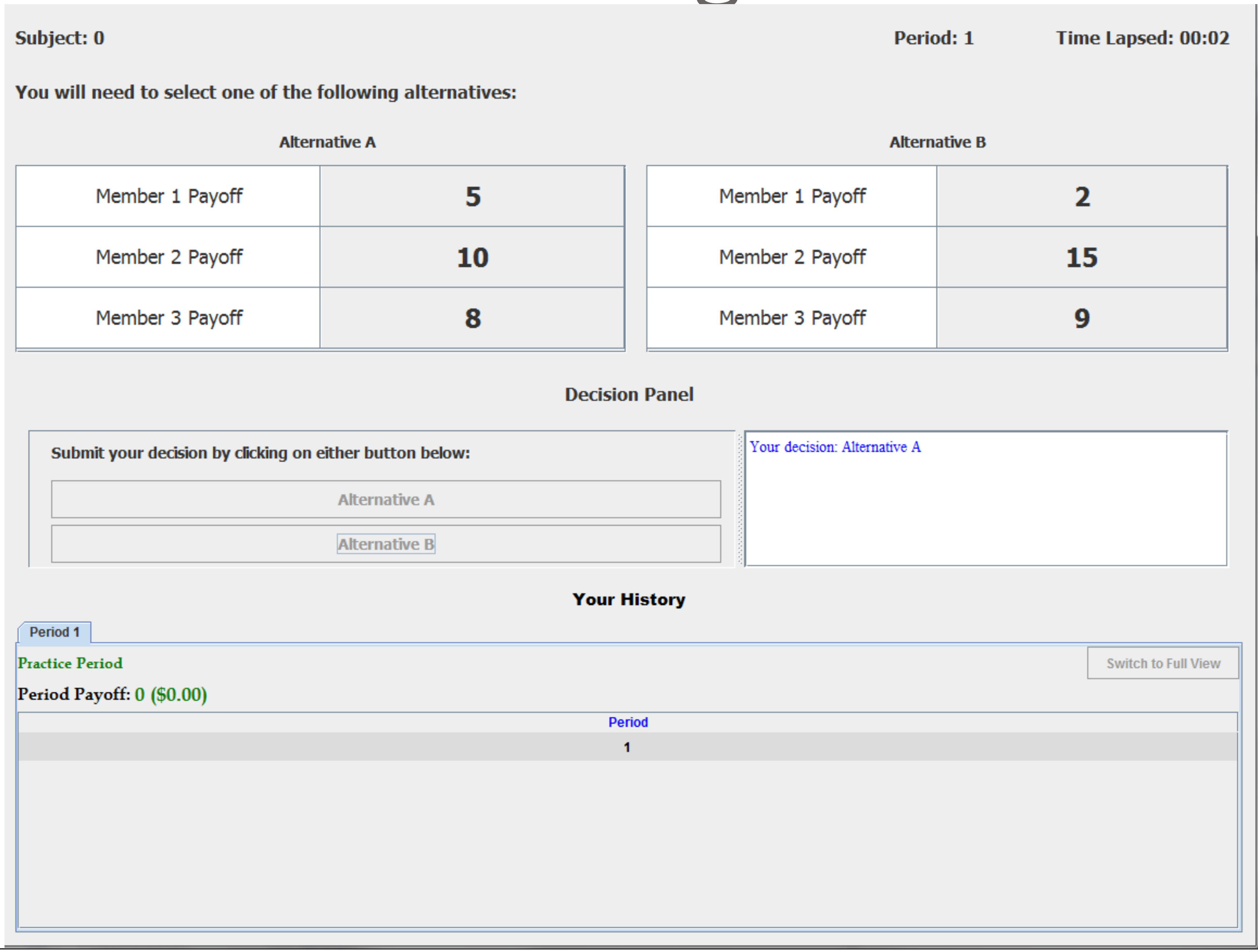
Experiments on a Shoestring

- Have groups of agents make decisions over streams of consumptions
  - Have some subjects act like planners and choose for a group: are they time consistent?
  - What heuristics do experimental social planners use?
  - Hold a vote over paths of consumption: is the group intransitive?

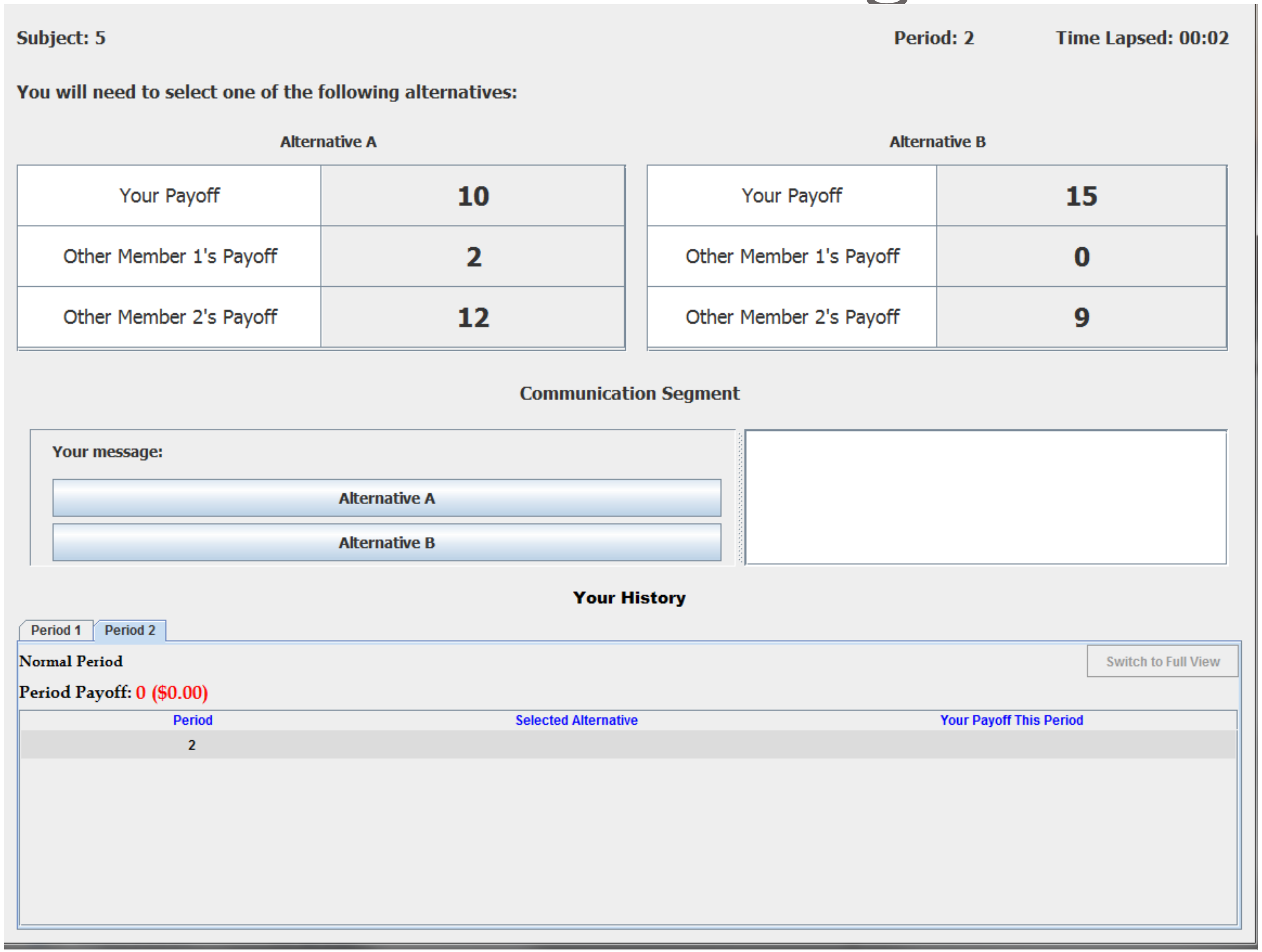
Experiments – Social Planners

- 96 planners in the lab
- Each makes about 35 allocation decisions between three individuals (presented as net present values), sometimes including herself
- Randomly rematched
- (For 60) Decisions affect the payoff of someone else in the room (chosen at random)

Decisions Affecting Others



Decisions also Affecting Self



A Note on Elicitation (and Using NPVs)

- Problems with delaying payments:
  - Arbitrage: bank account? What is the discount rate?
  - Confounding Uncertainty: subjects not sure of utility in future, credit constraints, etc.
- Also, want to isolate effect of aggregation
  - Want to ensure underlying preferences are time consistent: stack deck towards consistency
  - Want to control and vary underlying time preferences

New Method

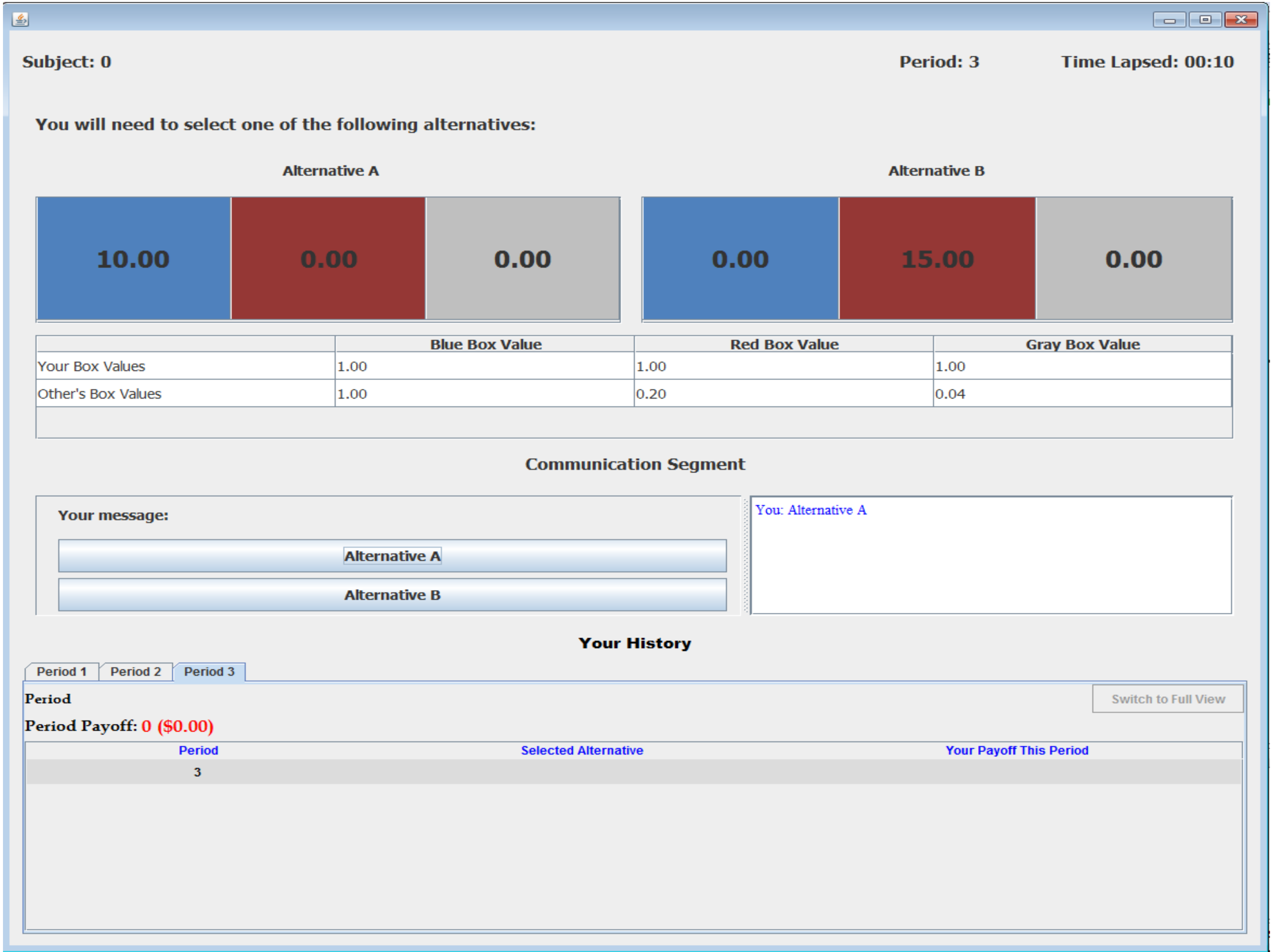
- Tokens of different color substitute for time
- Blue tokens worth 1, Red worth δ, Grey worth δ^2
- Vary δ to simulate time preferences

Alternative Elicitation

- To induce a discount factor $\delta_i = .9$
- Three types of tokens: Blue, Red, Grey
- Tokens are worth (1, .9, .81), respectively

Example of Choice: C= (105,0,0) vs C'=(0,160,0)

so payoffs ("NPVs") are 105 160x.9=144


Subject: 0
Period: 3
Time Lapsed: 00:10
You will need to select one of the following alternatives:
Alternative A
10.00
0.00
0.00
Alternative B
0.00
15.00
0.00
Blue Box Value
Red Box Value
Gray Box Value
Your Box Values
1.00
1.00
1.00
Other's Box Values
1.00
0.20
0.04
Communication Segment
Your message:
Alternative A
Alternative B
You: Alternative A
Your History
Period 1
Period 2
Period 3
Period
Switch to Full View
Period Payoff: 0 ($0.00)
Period
Selected Alternative
Your Payoff This Period
3

Alternative Elicitation

- We conducted an auxiliary set of experiments with the token method
- Results qualitatively the same
- Today: present results from experiments with NPVs

Example – Planner for Two

- Subject 3 is a planner: gets paid 80 regardless
- Subject 1 has $\delta_1 = .2$, Subject 2 has $\delta_2 = .9$

- 3 chooses between C and C' that both 1 and 2 consume:

Immediate: C= (105,0,0) vs C'=(0,160,0)

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NPVs (105, 105, 80) NPVs (32, 144, 80)

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Immediate: C= (105,0,0) vs C'=(0,160,0)

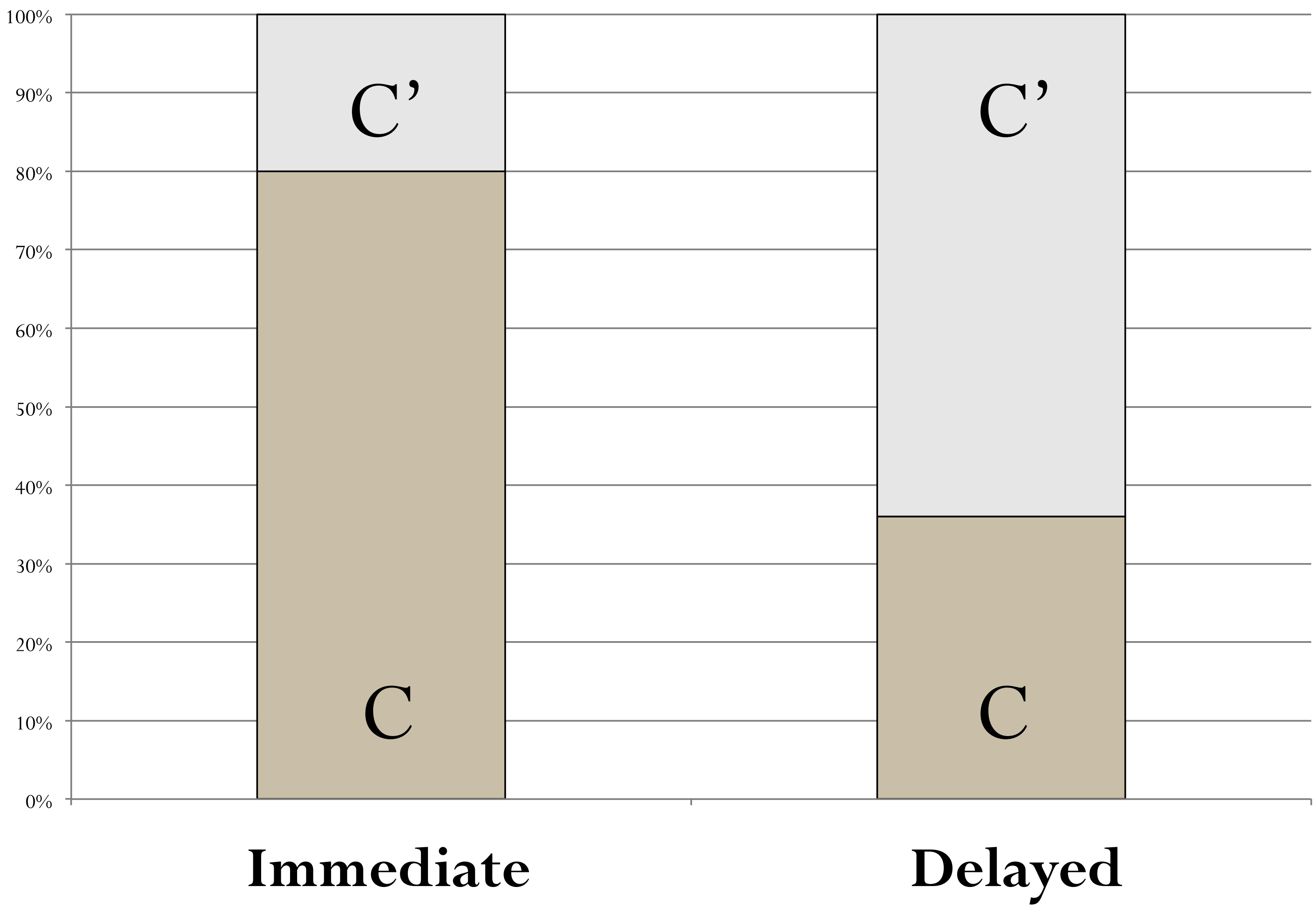
Delayed: C= (0,105,0) vs C'=(0,0,160)

Example – Planner for Two

Immediate: C= (105,0,0)	vs	C'=(0,160,0)
NPVs (105, 105, 80)		NPVs (32, 144, 80)
Total =290		Total = 256
Delayed: C= (0,105,0)	vs	C'=(0,0,160)
NPVs (21, 95, 80)		NPVs (6, 130, 80)
Total =116		Total = 136

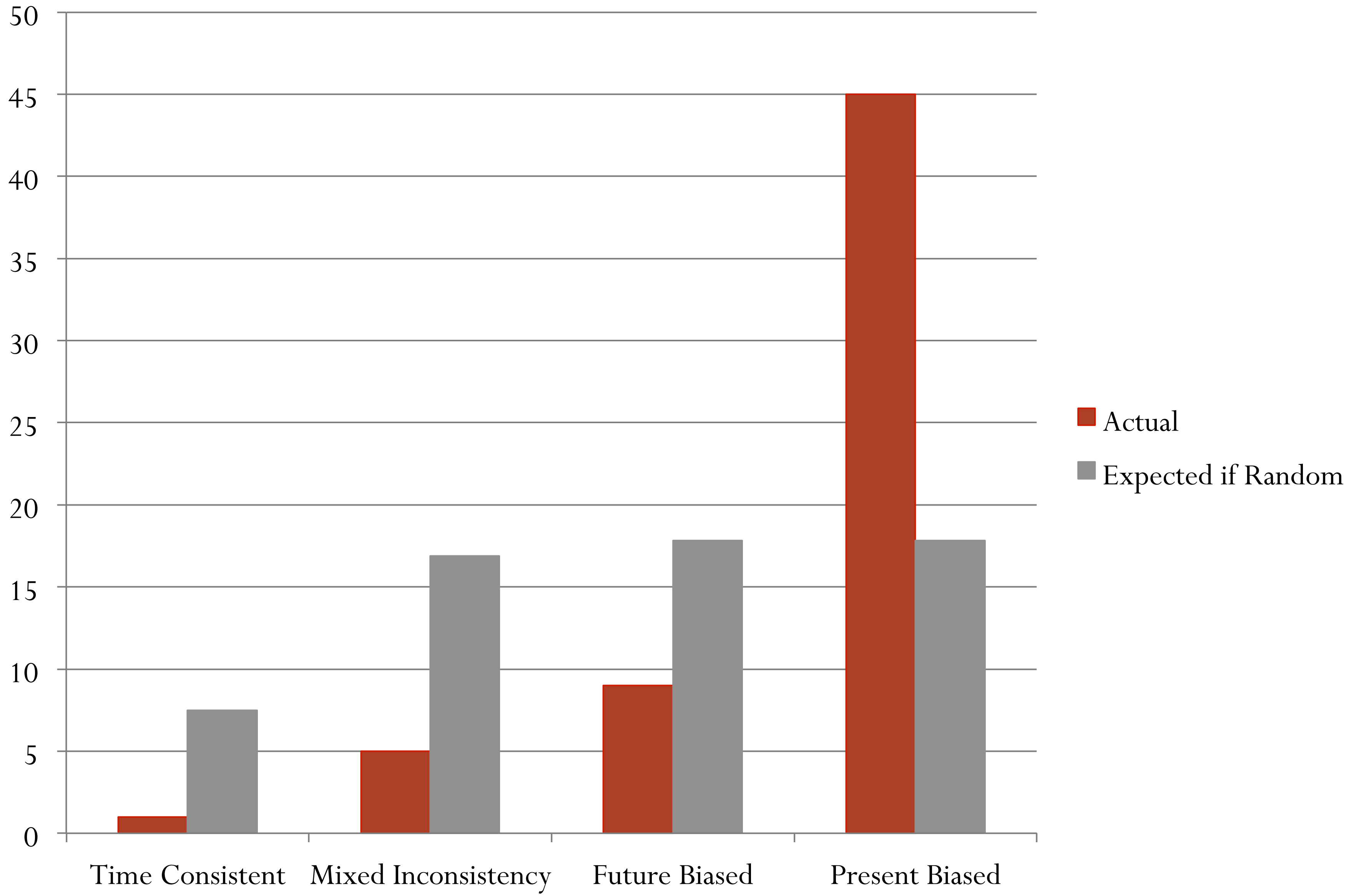
Present Biased/Time Inconsistent if pick C then C'

Results – Time Consistency

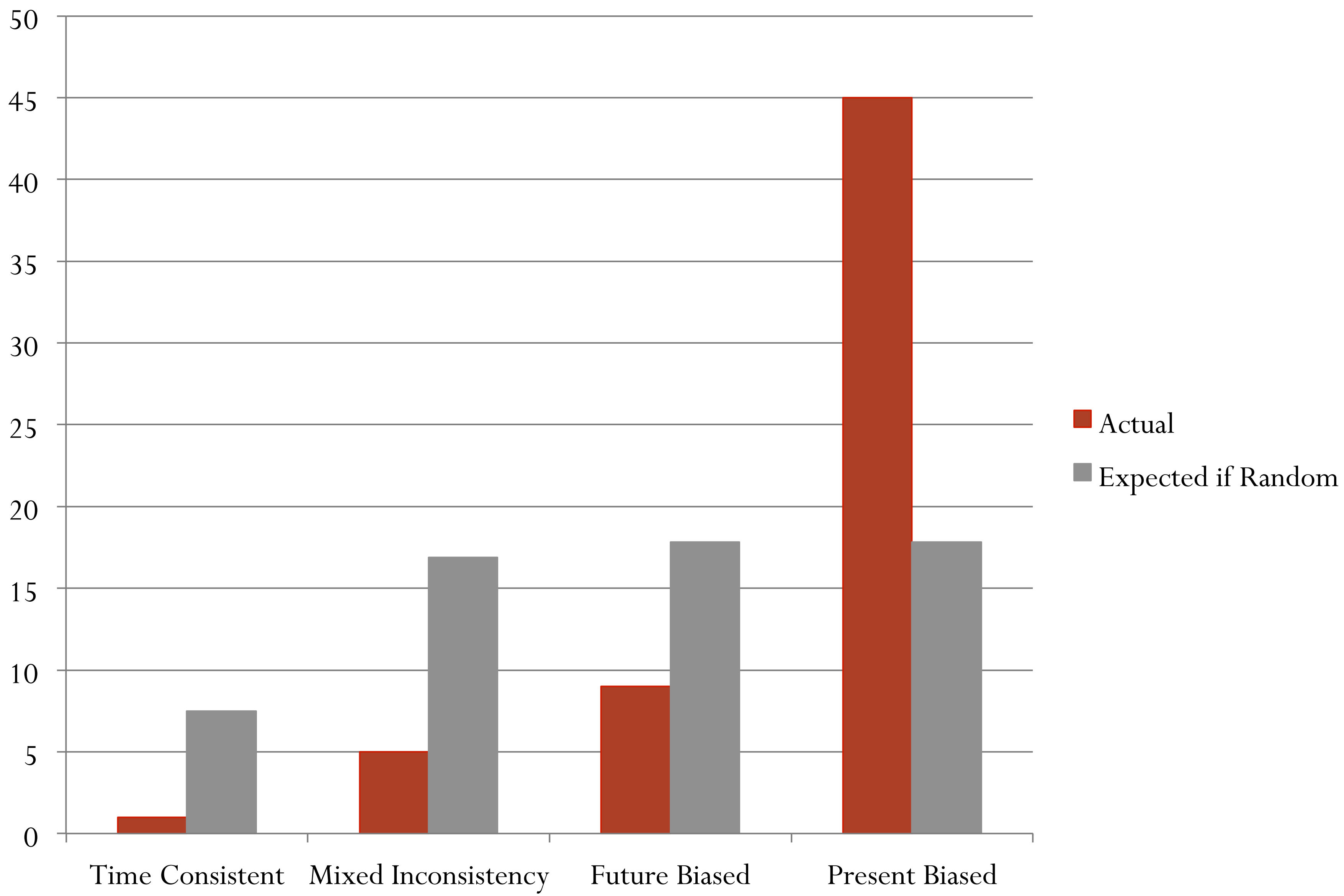


Individuals

- How many are always time consistent?
- How many are present biased?
- How many are future biased?
- How many exhibit mixed inconsistencies?



50
45
40
35
30
25
20
15
10
5
0
Time Consistent
Mixed Inconsistency
Future Biased
Present Biased
Actual
Expected if Random



[probability of *at least* 45 present biased individuals if randomly choosing is ~10↑−28]

Planner Types

- Can we distinguish between different 'types' of planners?

- Do they use different collective utility functions?
  - Utilitarian
  - Maximin
  - Equality…

Example – Planner for Two

- Subject 1 has $\delta_1 = .2$, Subject 2 has $\delta_2 = .9$

Decision 1: C= (105,0,0) vs C'=(0,160,0)

NPVs (105, 105, 80) NPVs (32, 144, 80)

290 (Util) 256

Decision 2: C= (0,105,0) vs C'=(0,0,160)

NPVs (21, 95, 80) NPVs (6, 130, 80)

116 136 (Util)

Note that a non-utilitarian planner might *not* show a reversal on these alternatives: e.g., maximin, inequality averse would pick C each time

Distinguishing Types

C = NPVs (105, 105, 80)

290 **(Util., Maxmin, Ineq)**

C' =NPVs (32, 144, 80)

256

C = NPVs (21, 95, 80)

116 **(Maxmin, Ineq)**

C' =NPVs (6, 130, 80)

136 **(Util.)**

C = NPVs (61, 55, 80)

116 **(Maxmin)**

C' =NPVs (46, 90, 80)

136 **(Util., Ineq)**

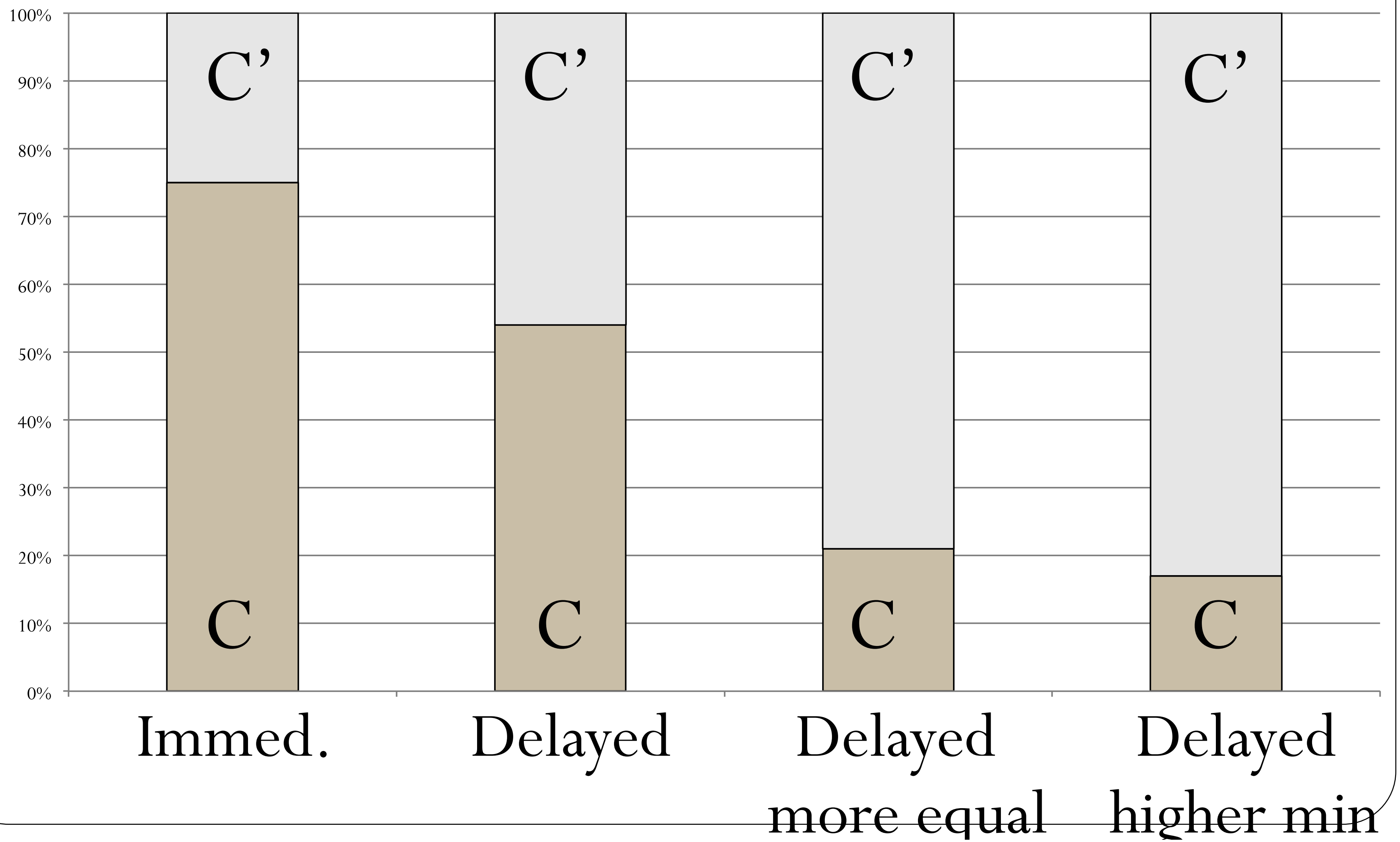
C = NPVs (21, 95, 80)

116

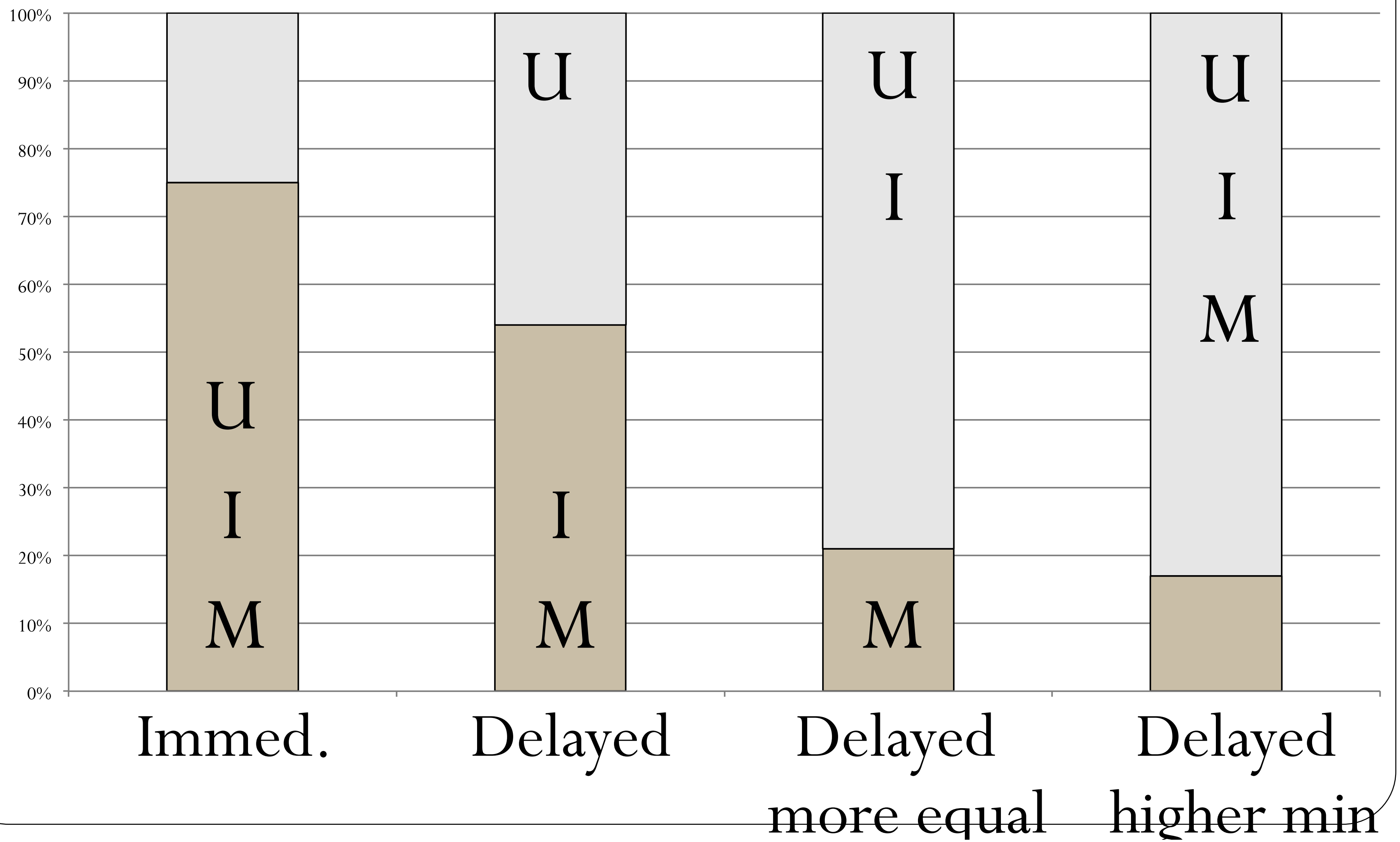
C' =NPVs (36, 100, 80)

136 **(Util, Ineq, Maxmin)**

Results – Inequality



Results – Inequality



Social Planners’ Objectives

- Looking at the set of all choices, the important parameters for choice appear to be:
  - The sum of utilities of each alternative;
  - The inequality (or variance) of the payoff distribution.

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$$a * Utilitarian - (1-a) * Standard\ Deviation$$

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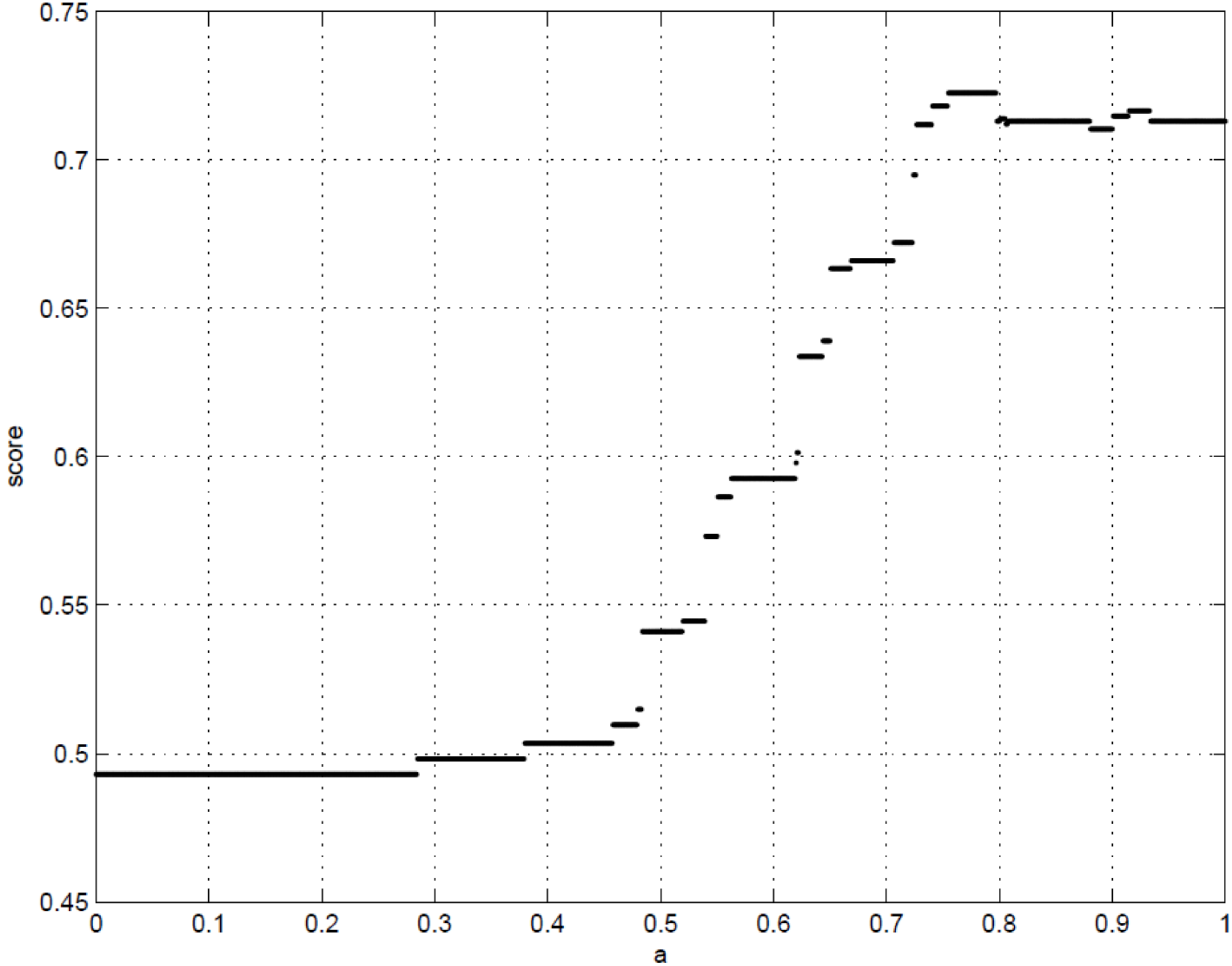
$$a * Utilitarian - (1\text{-}a) * Standard\ Deviation$$

- For each value of *a*, can define a *score*, the fraction of choices consistent with that choice of *a*.

Aggregate Data (individuals have the same a)



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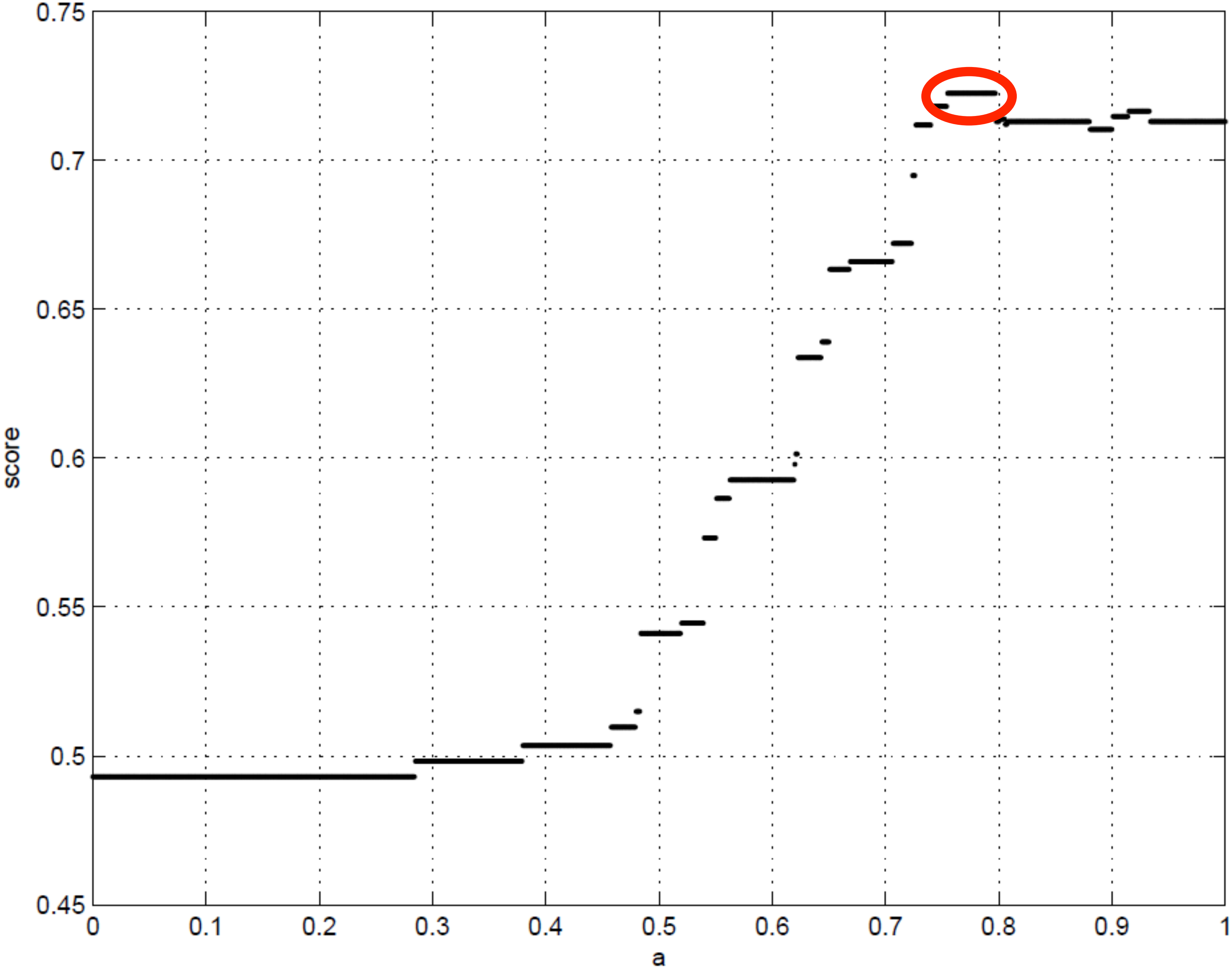
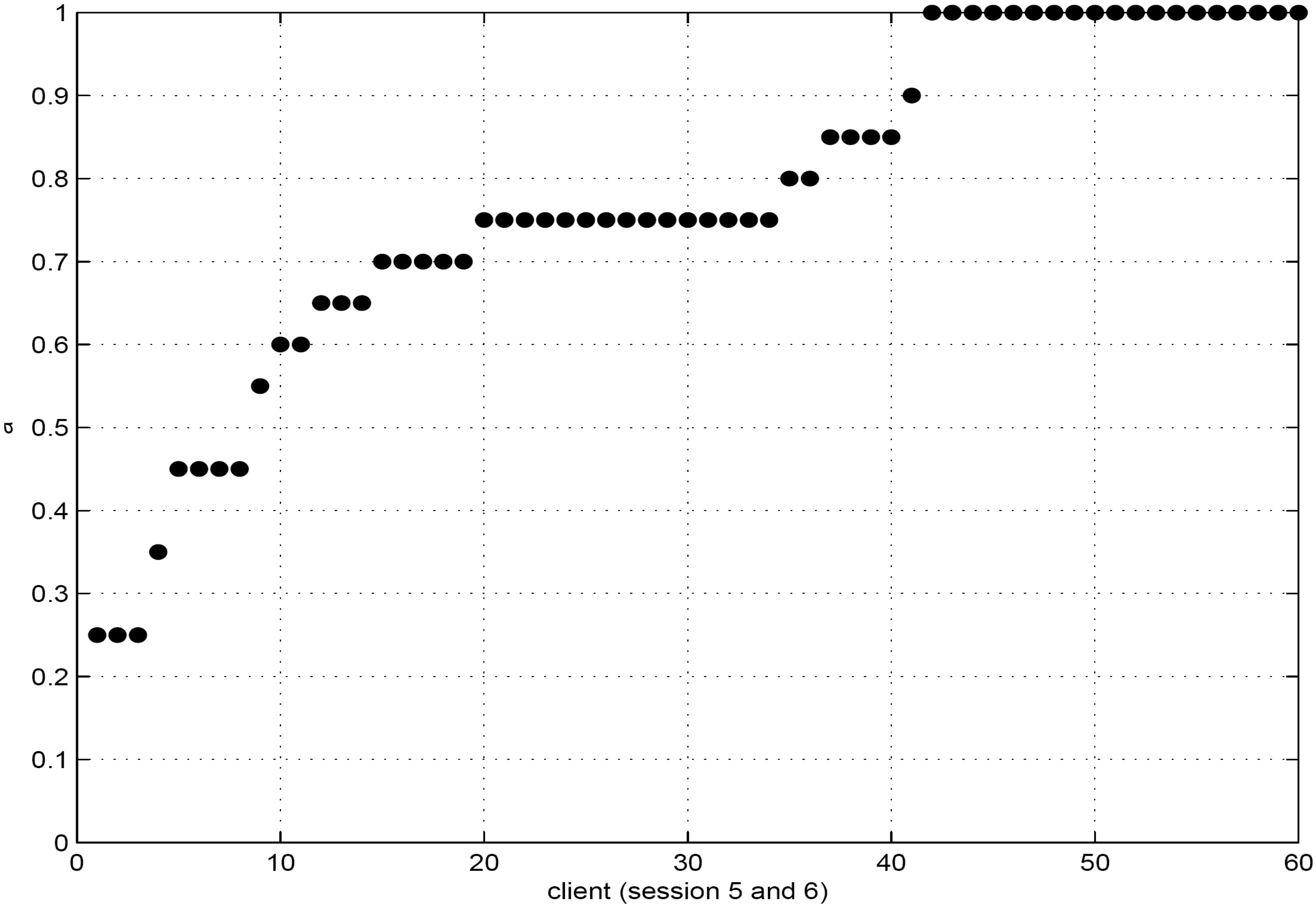
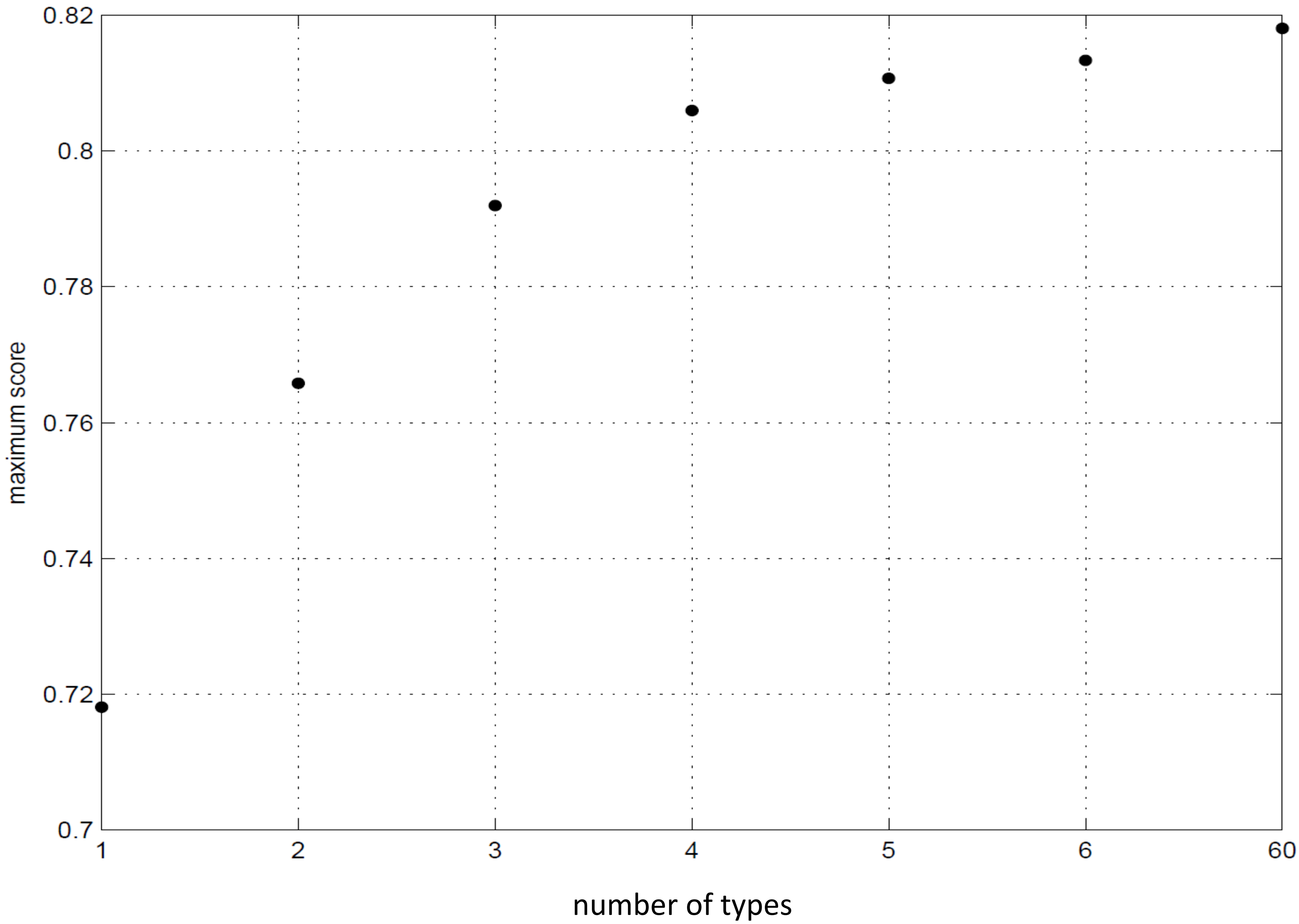


Individual best fitting a's (taking max if several):



Estimation by Types

Number of Types	Type 1	Type 2	Type 3	Type 4	Type 5	Type 6
1	0.75					
	(100%)					
Score	0.72					
2	0-0.25	0.95-1				
	(25%)	(75%)				
Score	0.67	0.80				
3	0-0.25	0.75	0.95-1			
	(21%)	(40%)	(39%)			
Score	0.66	0.79	0.86			
4	0-0.25	0.7	0.75	0.95-1		
	(17%)	(18%)	(27%)	(38%)		
Score	0.66	0.82	0.81	0.86		
5	0-0.25	0.55	0.7	0.75	0.95-1	
	(15%)	(7%)	(14%)	(26%)	(38%)	
Score	0.6449	0.7968	0.838	0.814	0.8653	
6	0-0.25	0.55	0.7	0.75	0.85	0.95-1
	(15%)	(7%)	(13%)	(25%)	(17%)	(23%)
Score	0.64	0.80	0.84	0.82	0.88	0.85


maximum score
0.82
0.8
0.78
0.76
0.74
0.72
0.7
1
2
3
4
5
6
60
number of types

Voting?

- Offer various pairs of streams to groups of three for voting
- Get cycles between 80 to 100 percent of the time predicted by selfish voting

Well-ordered Alternatives

- Suppose all i have $u_i = u$
- Differ only in discount factors δ_i

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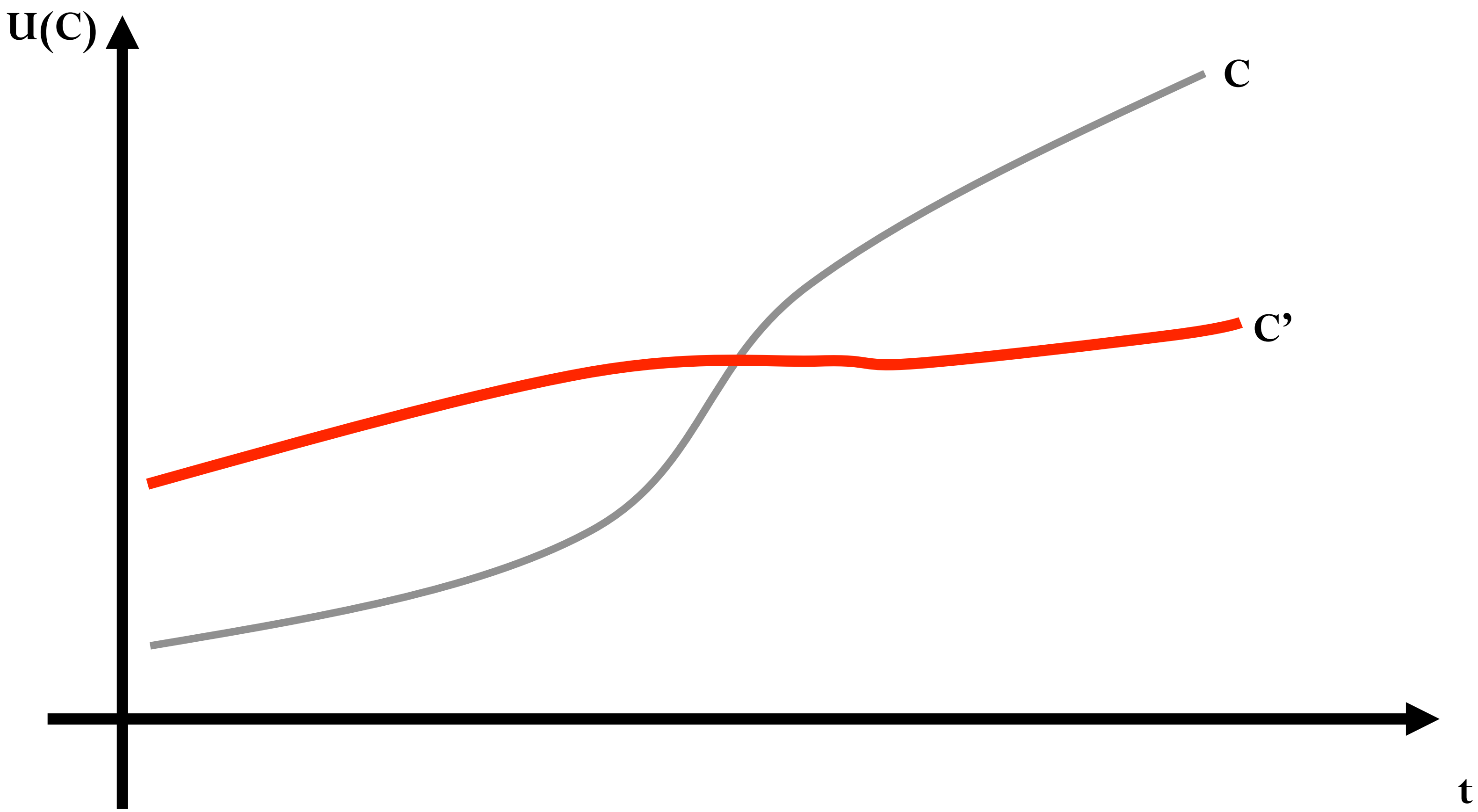
Well-ordered Alternatives



Well-ordered Alternatives

- Suppose all i have $u_i = u$
- Differ only in discount factors δ_i
- *C, C' are well-ordered if $u(c_t)$ -$u(c'_t)$ is monotone in t*
- **Implication:** if $\delta_i < \delta_k < \delta_j$ and i and j prefer C to C', then so does k.

Well-ordered Alternatives

- Suppose all i have $u_i = u$
- Differ only in discount factors δ_i

Proposition 3: *If a set of consumption streams is such that each pair is well ordered, then any neutral voting rule has a strict component that is transitive.*

Empirical Implications:

- Political Bodies/Households/Committees/Firms/Planners/Multiple Personalities/Representative Agents… will be intransitive and/or time inconsistent if they embody heterogeneity in time preferences

- Understanding time inconsistencies/intransitivities in individuals may mean that individuals are best modeled as multi-faceted

- Behavior depends on the aggregation method:
  - **voting is time consistent but intransitive**
  - **utilitarian/other weighting is transitive but time inconsistent**

Summary

- Aggregating preferences via nontrivial weighting leads to a present-bias.

- Aggregating preferences via voting results in intransitivities, even when restricting alternatives to pure consumption smoothing.

- Any non-dictatorial, unanimous collective utility function is time inconsistent.

Majority Voting: Intransitivities

Majority rule: *C R C'* iff a majority of agents do.

Proposition 2: *If the largest group of agents having identical discount factors is smaller than a majority, then majority rule is intransitive.*

In fact, for any interior *C*, can find a cycle:

C P C' P C" P C

[even if further restrict alternatives…]

Majority Voting: Technical Intuition

- Suppose we want agents *1,…,k* to prefer *C* to *C'* and agents *k+1, …,n* to prefer *C'* to *C*

- System of linear inequalities:

$$\sum_t \delta_i^{t-1} [u(c_t) - u(c'_t)] > 0, \quad i = 1,\ldots,k$$
$$\sum_t \delta_i^{t-1} [u(c_t) - u(c'_t)] < 0, \quad i = k+1,\ldots,n$$

- For different δ_i 's, linear independence of $\{(1, \delta_i, \ldots, \delta_i^{t-1}, \ldots)\}$ when the range of *u* is sufficiently rich

- In order to rule out intransitivities, need to rule out independence ↔ richness of range of instantaneous utility

General Voting Rules

$$p(U, C, C') = \{\ i \mid U_i(C) > U_i(C')\}$$

- $p(U, C, C')$ are agents who prefers C to C'

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$$p(U, C, C') = \{\ i \mid U_i(C) > U_i(C')\}$$

- $p(U, C, C')$ are agents who prefers C to C'
- R[U] is a ***voting rule if it only depends on information in p(U, C, C') and p(U, C', C)***
- So includes weighted majority, supermajorities, other non-anonymous, non-neutral rules…

Locally Non-dictatorial Voting

- R[U] is locally non-dictatorial if:

  $|p(U, C, C')| \geq n\text{-}1$ implies $C\ R[U]\ C'$

Locally Non-dictatorial Voting

- R[U] is locally non-dictatorial if:

  $$|p(U, C, C')| \geq n\text{-}1 \text{ implies } C\ R[U]\ C'$$

- No agent can drive collective preferences (locally for some C, C') when all other agents prefer C to C'

Restricting Consumption Sets

$C[x,g]$ = set of C's such that

- $c_1 + c_2/g + c_3/g^2 = x$
- $c_t = 0$ for $t > 3$

- So, only smoothing with growth or decay
- Only three periods
- Restricts ability to construct cycles

General Voting Rules: Intransitivities

Theorem: *Let all agents have a strictly concave $u_i = u$. If a voting rule is locally non-dictatorial, then it is intransitive.*

General Voting: Intuition

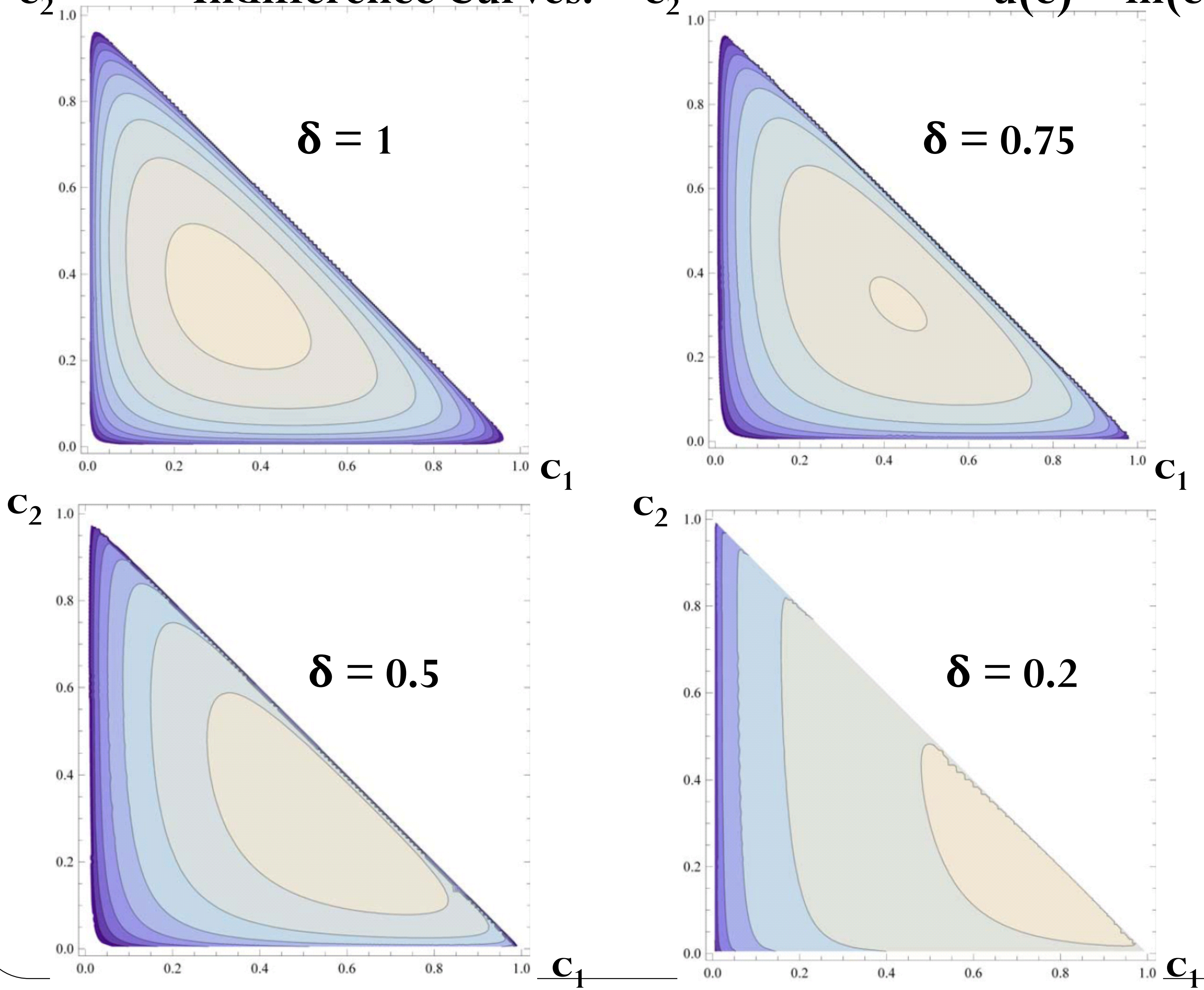
- Underlying tradeoff in decisions:
  - **Time preference**: push consumption forward
  - **Strict concavity**: equalize consumption over time

- Possibility of moving some consumption forward in exchange for other parts back, produces cycles

Ideas Behind Proof

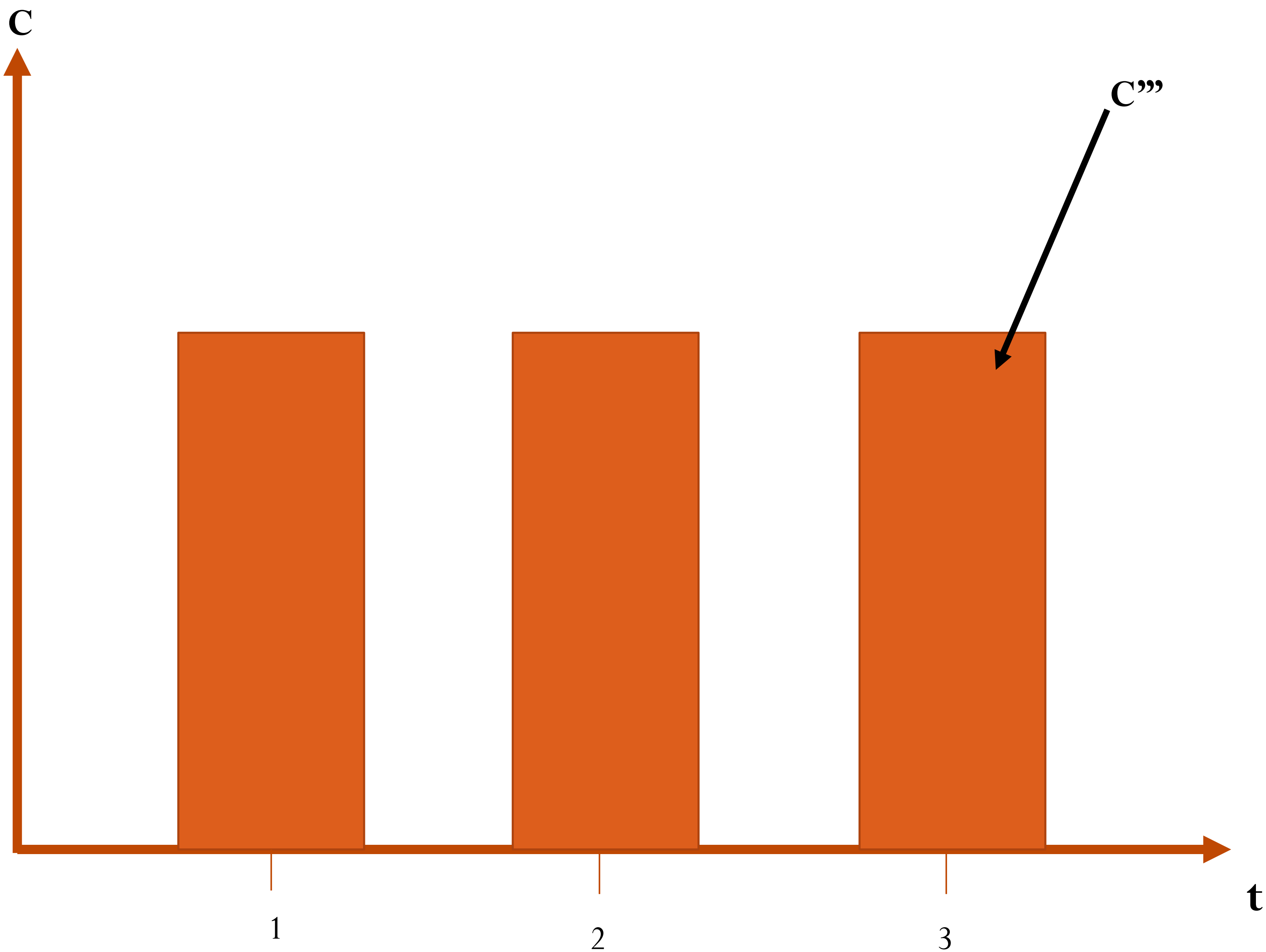
- Show there exist triples of alternatives with any possible preference ordering
- Not an environment with "single crossing"…
- Work with 3 groups: most patient, moderately patient, least patient
- Indifference curves cross multiple times…

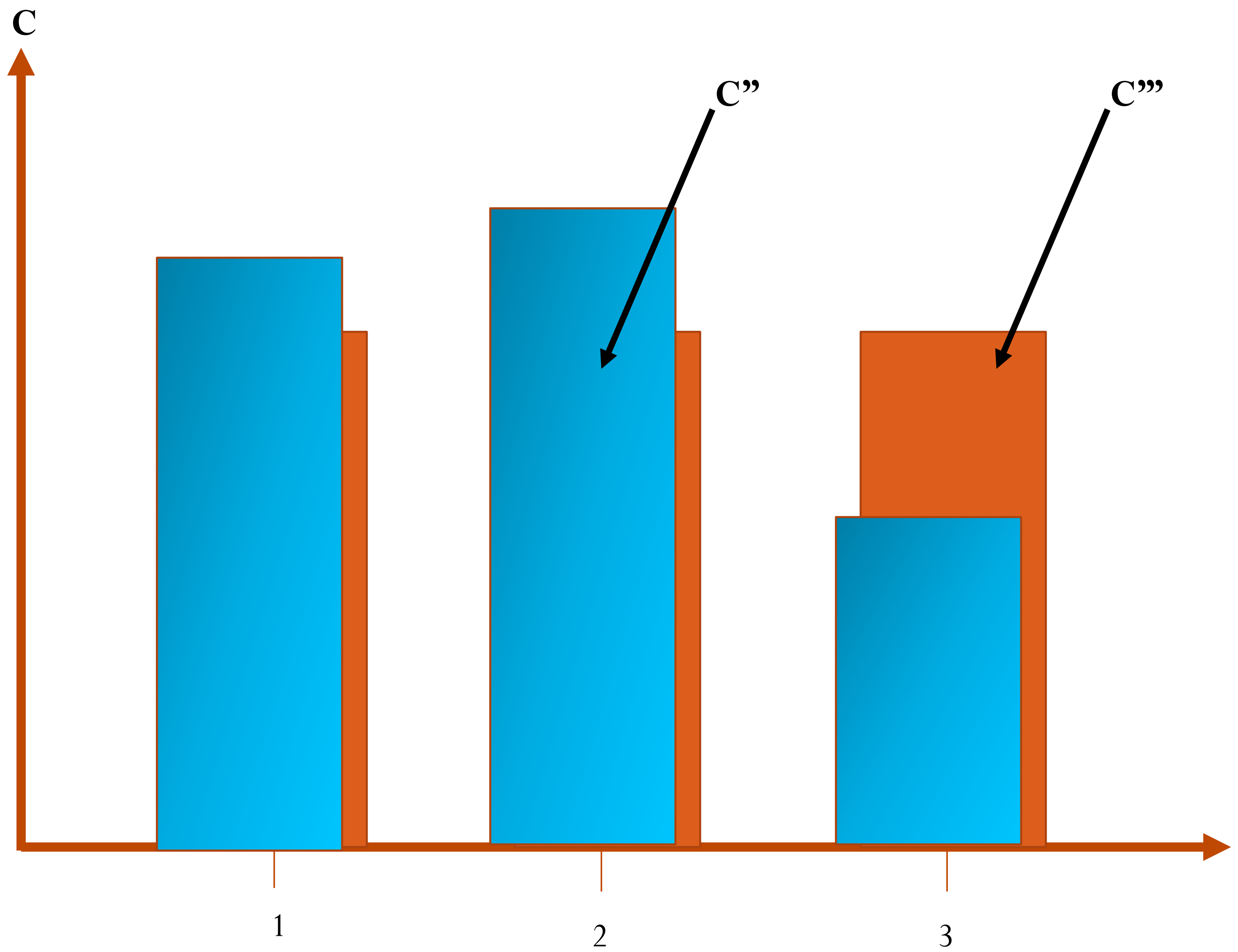
Indifference Curves: $u(c) = \ln(c)$, $g=1$



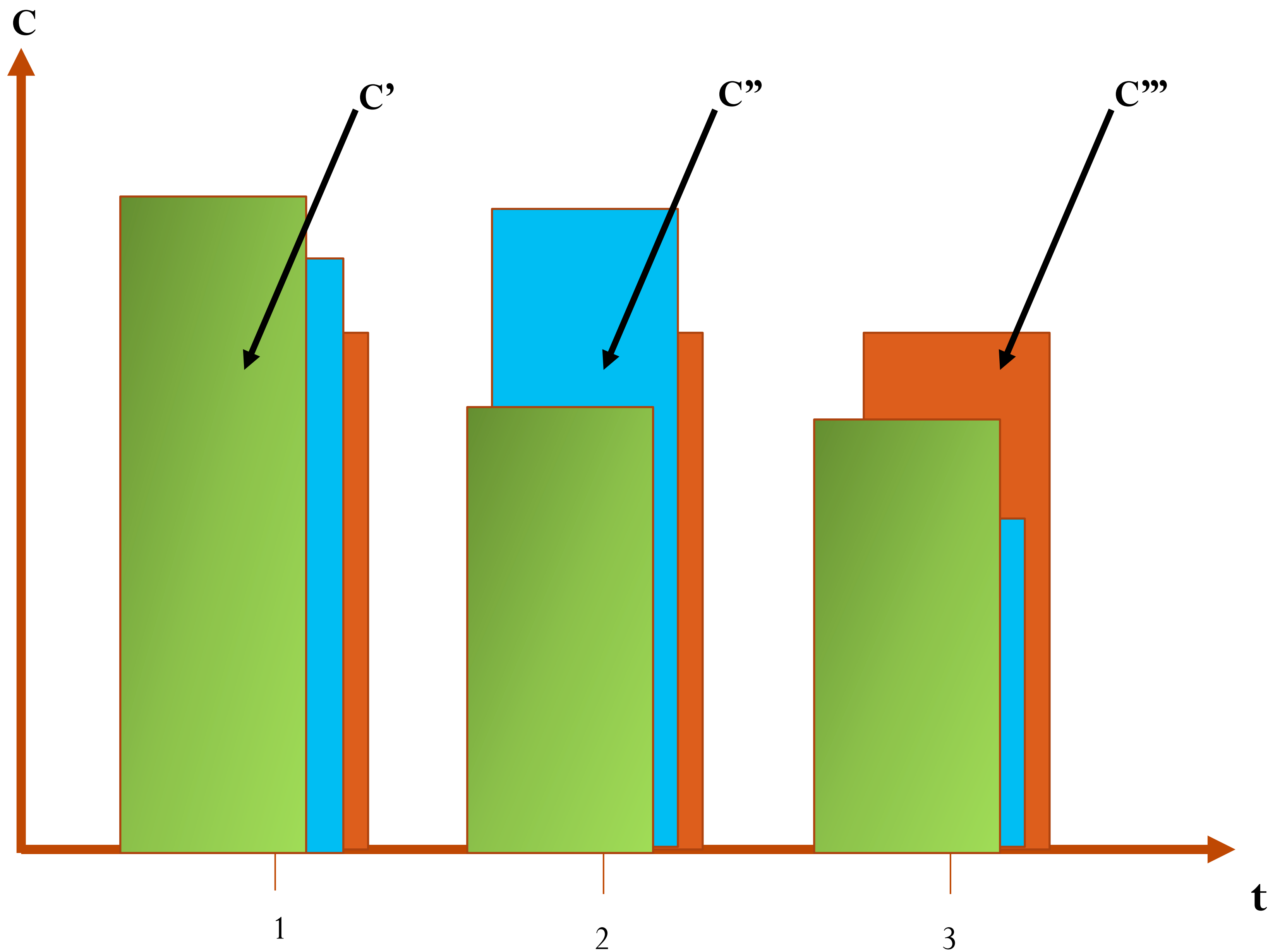
Intransitivities: $\delta_1 < \delta_2 < \delta_3$

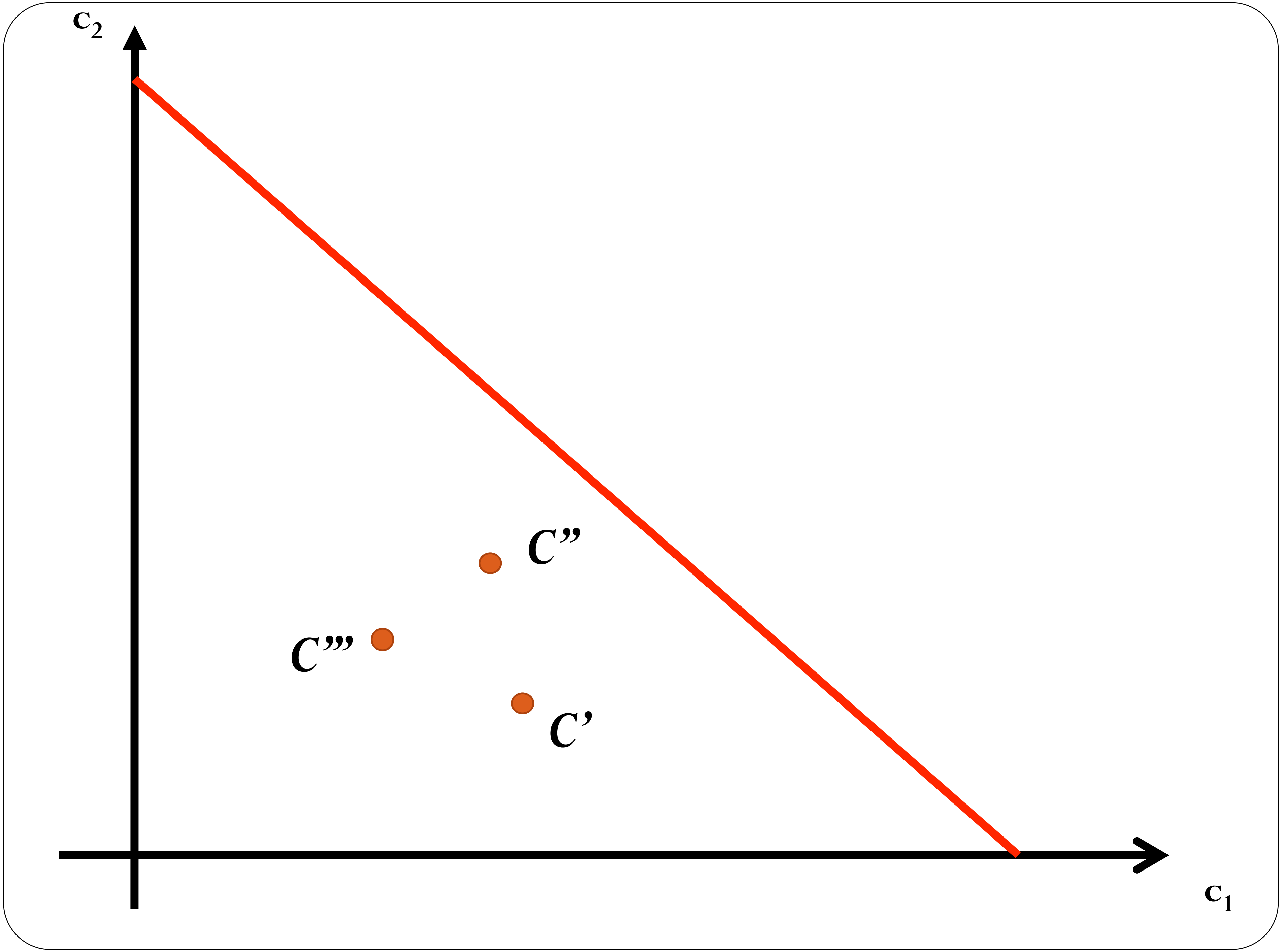
- Consider three consumption streams (can be done locally and constructively):
  - ***C'''*** relatively balanced consumption across periods
  - ***C''*** moves more of the consumption forward to periods 1 and 2 relative to ***C'''***.
  - ***C'*** moves consumption towards periods 1 and 3 relative to ***C''***


c
C'''
1
2
3
t



C
C’’
C’’’
1
2
3
t


c
C’
C’’
C’’’
1
2
3
t



c2
C''
C'''
C'
c1

Intransitivities: $\delta_1 < \delta_2 < \delta_3$

- Patient agent likes balance - prefers ***C'''*** to ***C'***, likes ***C''*** least as it has too little consumption in the third period:

$$C''' \succ_3 C' \succ_3 C''$$

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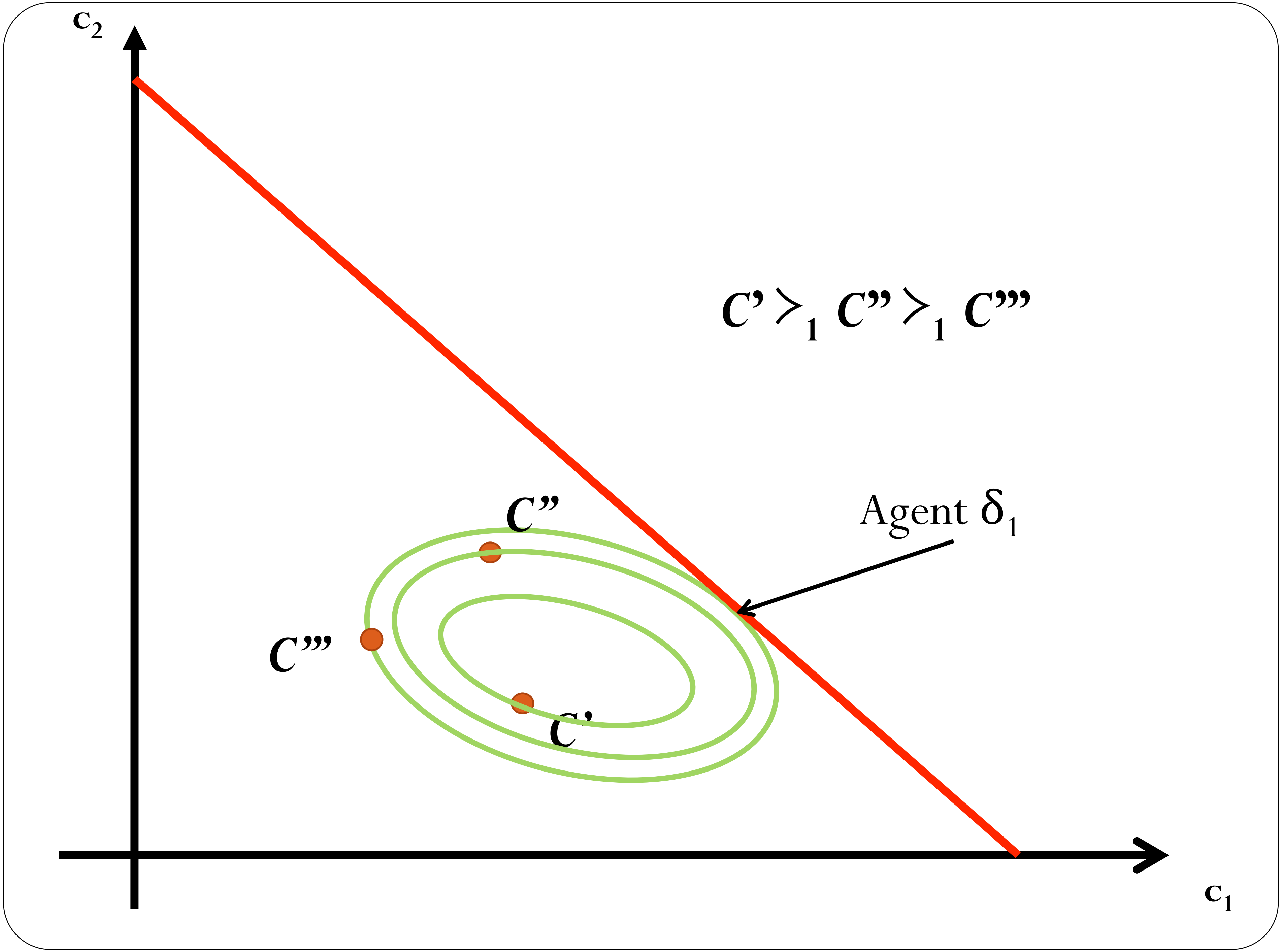
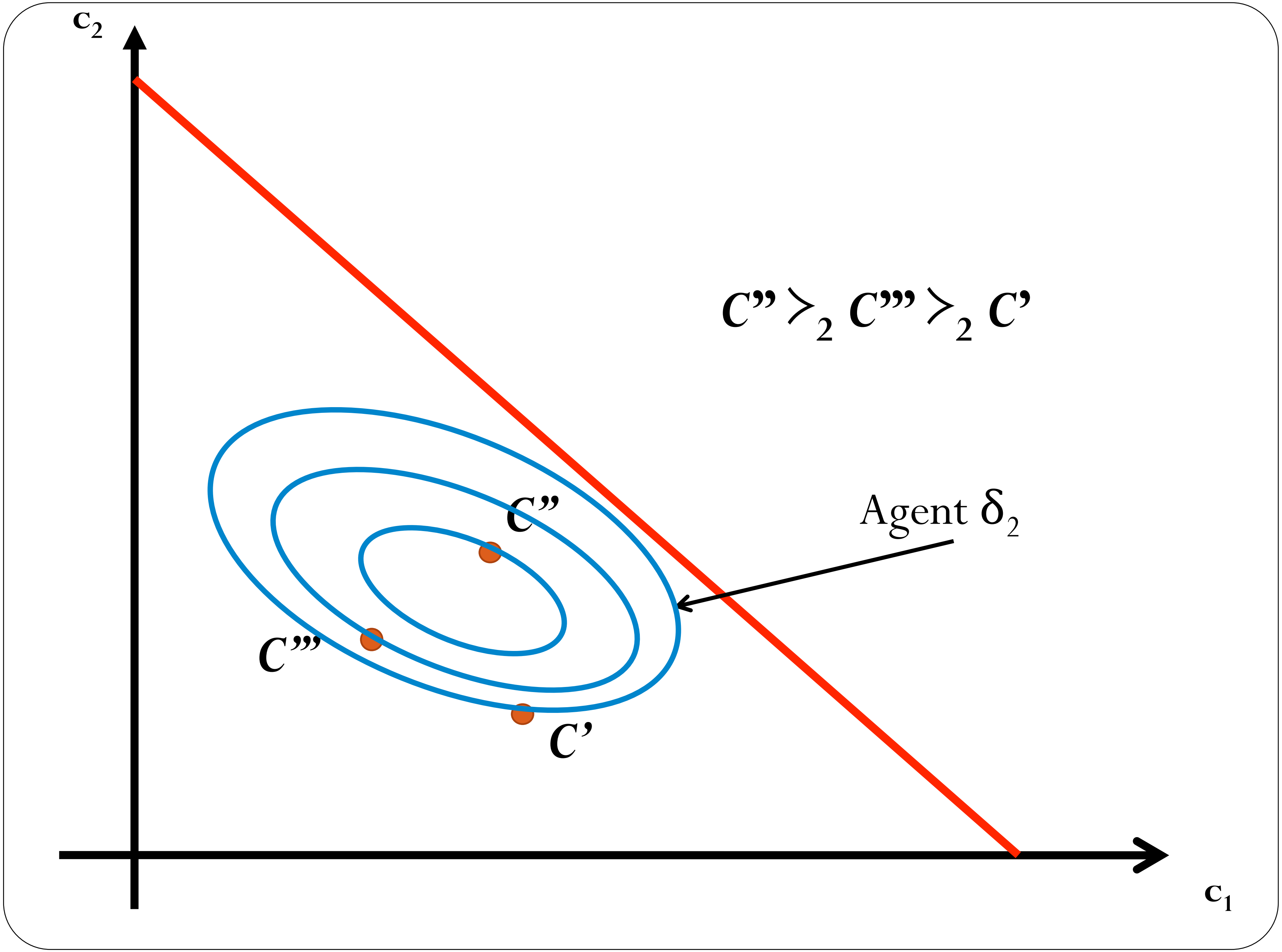
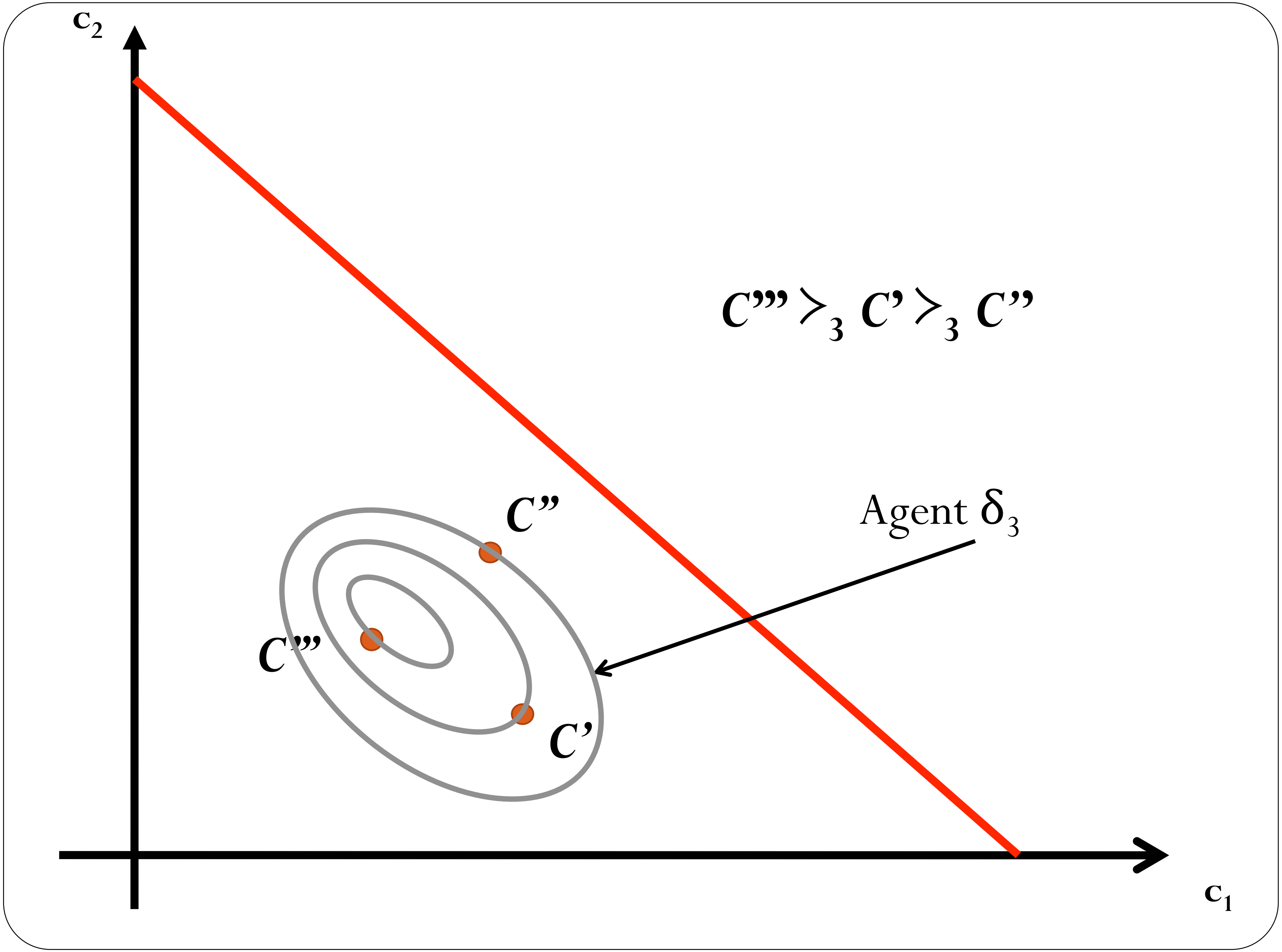
- Impatient agent considers mostly first period consumption:

$$C' \succ_1 C'' \succ_1 C'''$$


c2
C' ≻1 C'' ≻1 C'''
Agent δ1
C''
C'''
C'
c1



c2
C'' ≻2 C''' ≻2 C'
C''
Agent δ2
C'''
C'
c1


c_2
$C''' \succ_3 C' \succ_3 C''$
C''
Agent δ_3
C'''
C'
c_1

Aggregation by Majority Vote

- Rank *C* and *C'* by a vote
  (*for example:* simple majority)

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- Is the resulting ranking standard? (at a minimum transitive?)

Condorcet Cycles

- Alternatives: {a,b,c}
- Agent 1: $U_1(a) > U_1(b) > U_1(c)$
- Agent 2: $U_2(b) > U_2(c) > U_2(a)$
- Agent 3: $U_3(c) > U_3(a) > U_3(b)$
- Majority prefers a to b to c to a

Condorcet Cycles

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- Agent 2: $U_2(b) > U_2(c) > U_2(a)$
- Agent 3: $U_3(c) > U_3(a) > U_3(b)$
- Relies on richness: "unrestricted domain"
- Which underlies Arrow's Theorem, generalizing Condorcet's paradox

Majority Voting: Example

- Three individuals with discounts:

  $$\delta_1 = 0 < \delta_2 = .5 < \delta_3 = 1$$

- Linear utility: $u(c)=c$

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$$\delta_1 = 0 < \delta_2 = .5 < \delta_3 = 1$$

- Linear utility: $u(c)=c$
- Society chooses by voting over alternatives
- Can we order the voters to find a well-defined median, and hence a representative voter?
- If 1 and 3 prefer C to C', does 2 have the same preference?

Majority Voting: Example

$$\delta_1 = 0 < \delta_2 = .5 < \delta_3 = 1$$

$C = (1, 1, 1, 0, 0, \ldots)$ or

$C' = (1+\varepsilon, 1-6\varepsilon, 1+6\varepsilon, 0, 0, \ldots)$

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$C = (1, 1, 1, 0, 0, \ldots)$ or

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$$U_1(C) = 1 < U_1(C') = 1 + \varepsilon$$

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$C = (1, 1, 1, 0, 0, \ldots)$ or

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$U_1(C) = 1 < U_1(C') = 1 + \varepsilon$

$U_3(C) = 3 < U_3(C') = 3 + \varepsilon$

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$C = (1, 1, 1, 0, 0, \ldots)$ or

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$U_1(C) = 1 < U_1(C') = 1 + \varepsilon$

$U_3(C) = 3 < U_3(C') = 3 + \varepsilon$

$U_2(C) = 1.75 > U_2(C') = 1.75 + \varepsilon - 3\varepsilon + 1.5\varepsilon$

Majority Voting: Example (Cycle)

$$\delta_1 = 0 < \delta_2 = .5 < \delta_3 = 1$$

$$C = (1, 1, 1, 0, 0, \ldots)$$
$$C' = (1+\varepsilon, 1-6\varepsilon, 1+6\varepsilon, 0, 0, \ldots)$$
$$C'' = (1+2\varepsilon, 1-6\varepsilon, 1+3\varepsilon, 0, 0, \ldots)$$

$$U_1(C) = 1 < U_1(C') = 1 + \varepsilon < U_1(C'') = 1 + 2\varepsilon$$

$$U_2(C') = 1.75 - .5\varepsilon < U_2(C'') = 1.75 - .25\varepsilon < U_2(C) = 1.75$$

$$U_3(C'') = 3 - \varepsilon < U_3(C) = 3 < U_3(C') = 3 + \varepsilon$$

Majority Voting: Example (Cycle)

$$\delta_1 = 0 < \delta_2 = .5 < \delta_3 = 1$$

2, 3 prefer C to C”

$C = (1, 1, 1, 0, 0, \ldots)$

$C' = (1+\varepsilon, 1\text{-}6\varepsilon, 1+6\varepsilon, 0, 0, \ldots)$

$C'' = (1+2\varepsilon, 1\text{-}6\varepsilon, 1+3\varepsilon, 0, 0, \ldots)$

1,3 prefer C’ to C

1, 2 prefer C” to C’

$U_1(C) = 1 < U_1(C') = 1 + \varepsilon < U_1(C'') = 1 + 2\varepsilon$

$U_2(C') = 1.75 - .5\,\varepsilon < U_2(C'') = 1.75 - .25\varepsilon < U_2(C) = 1.75$

$U_3(C'') = 3 - \varepsilon < U_3(C) = 3 < U_3(C') = 3 + \varepsilon$

Voting?

- Offer various pairs of streams to groups of three for voting
- Get cycles between 80 to 100 percent of the time predicted by selfish voting

How Altrusitic are subjects?

C = NPVs (170, 210, 90) vs C'=NPVs (120, 230, 90)

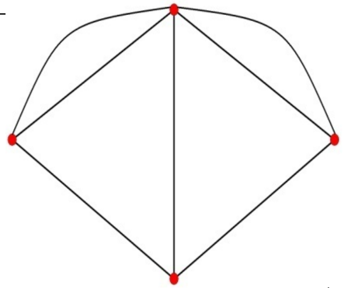
Total =470 (Util., Maxmin, Ineq) Total = 440

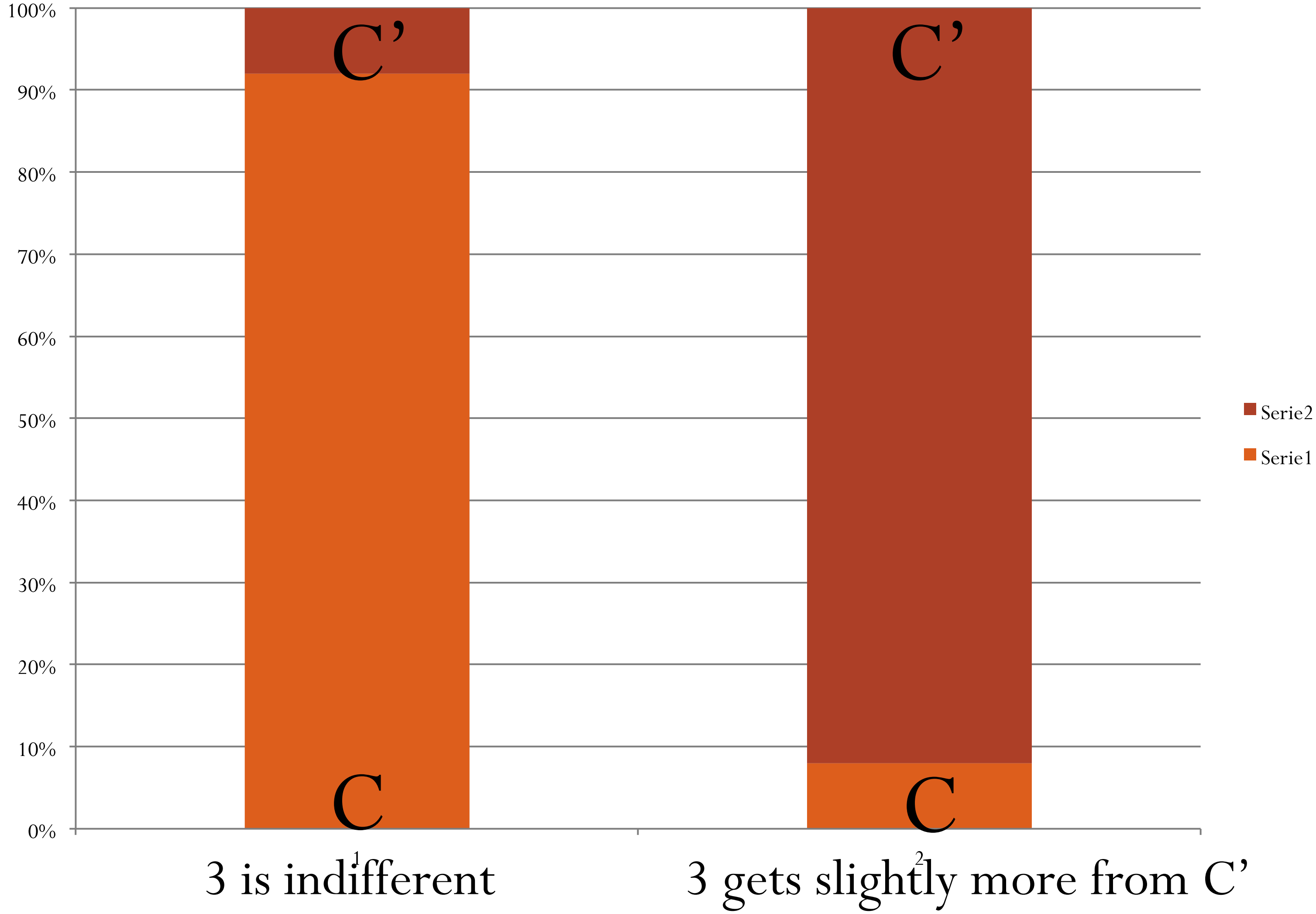
C = NPVs (170, 210, 90) vs C'=NPVs (120, 230, 95)

Total =470 (Util., Maxmin, Ineq) Total = 445

Vote of subject 3:

How Altrusitic are subjects?





Experiments

- Different subjects exhibit different patterns of time consistency

- Some `appear' to pay more attention to inequality than others (in a revealed preference sense)

Individual Scores

Overall Score with individually optimal parameters: 0.83