Fundación BBVA

Documentos de Trabajo

2

2007

Manel Baucells Alibés
Franz H. Heukamp

Stochastic Dominance and Cumulative Prospect Theory

Fundación BBVA

Stochastic Dominance and Cumulative Prospect Theory

Manel Baucells Alibés
Franz H. Heukamp

UNIVERSITY OF NAVARRA

Abstract

We generalize and extend the second order stochastic dominance condition available for Expected Utility to Cumulative Prospect Theory. The new definitions include, among others, preferences represented by S-shaped value and inverse S-shaped probability weighting functions. The stochastic dominance conditions supply a framework to test different features of Cumulative Prospect Theory. In the experimental part of the working paper we offer a test of several joint hypotheses on the value function and the probability weighting function. Assuming empirically relevant weighting functions, we can reject the inverse S-shaped value function recently advocated by Levy and Levy (2002a), in favor of the S-shaped form. In addition, we find generally supporting evidence for loss aversion. Violations of loss aversion can be linked to subjects using the overall probability of winning as heuristic.

Key words

Second order stochastic dominance, cumulative prospect theory, value function, probability weighting function.

Resumen

Este documento de trabajo generaliza y extiende las condiciones de dominancia estocástica de segundo orden disponibles para utilidad esperada a la teoría de prospectiva cumulativa. La nuevas definiciones incluyen, entre otros, las preferencias representadas por una función de valor en forma de S y una función de probabilidad ponderada en forma de S invertida. Las condiciones de dominancia estocástica permiten diseñar test para diferenciar las distintas partes de la teoría de prospectiva cumulativa. En la parte experimental del documento de trabajo se examinan varias hipótesis conjuntas sobre al función de valor y la función de probabilidad ponderada. Asumiendo una forma empíricamente relevante de la función de probabilidad ponderada, podemos rechazar la hipótesis de una función de valor en forma de S invertida, defendida recientemente por Levy y Levy (2002a), a favor de la hipótesis de una función de valor en forma de S. Además, encontramos evidencia a favor de aversión a la pérdida. Violaciones de aversión a la pérdida se pueden atribuir a que algunos sujetos usan como regla heurística la probabilidad de obtener ganancias.

Palabras clave

Dominancia estocástica de segundo orden, teoría de prospectiva cumulativa, función de valor, función de probabilidad ponderada.

La serie Documentos de Trabajo tiene como objetivo la rápida difusión de los resultados del trabajo de investigación entre los especialistas de esa área, para promover así el intercambio de ideas y el debate académico. Cualquier comentario sobre sus contenidos será bien recibido y debe hacerse llegar directamente a los autores, cuyos datos de contacto aparecen en la *Nota sobre los autores*.

The Working Papers series is intended to disseminate research findings rapidly among specialists in the field concerned, in order to encourage the exchange of ideas and academic debate. Comments on this paper would be welcome and should be sent direct to the authors at the addresses provided in the About the authors *section.*

Todos los documentos de trabajo están disponibles, de forma gratuita y en formato PDF, en la web de la Fundación BBVA. Si desea una copia impresa, puede solicitarla a través de publicaciones@fbbva.es.

All working papers can be downloaded free of charge in pdf format from the BBVA Foundation website. Print copies can be ordered from publicaciones@fbbva.es.

La serie Documentos de Trabajo, así como información sobre otras publicaciones de la Fundación BBVA, pueden consultarse en **http://www.fbbva.es**

The Working Papers series, as well as information on other BBVA Foundation publications, can be found at **http://www.fbbva.es**

Stochastic dominance and cumulative prospect theory

EDITA / PUBLISHED BY
Fundación BBVA, 2007
Plaza de San Nicolás, 4. 48005 Bilbao

DEPÓSITO LEGAL / LEGAL DEPOSIT NO.: M-3.428-2007
IMPRIME / PRINTED BY: Rógar, S. A.

Impreso en España – *Printed in Spain*

La serie Documentos de Trabajo de la Fundación BBVA está elaborada con papel 100% reciclado, fabricado a partir de fibras celulósicas recuperadas (papel usado) y no de celulosa virgen, cumpliendo los estándares medioambientales exigidos por la actual legislación.

The Working Papers series of the BBVA Foundation is produced with 100% recycled paper made from recovered cellulose fibre (used paper) rather than virgin cellulose, in conformity with the environmental standards required by current legislation.

El proceso de producción de este papel se ha realizado conforme a las normas y disposiciones medioambientales europeas y ha merecido los distintivos Nordic Swan y Ángel Azul.

The paper production process complies with European environmental laws and regulations, and has both Nordic Swan and Blue Angel accreditation.

C O N T E N T S

1. Introduction

STOCHASTIC Dominance (SD) relations offer an efficient way to compare pairs of prospects. This has been recognized ever since their introduction to economics by Rothschild and Stiglitz (1970). SD conditions are applied in finance, decision analysis, economic modeling, and axiomatic modeling (Levy, 1992). Well known specifications of SD are First-Order SD (FSD) and Second-Order SD (S-SD) which have been presented in the context of Expected Utility (EU). The interpretation of SD conditions is often useful by itself. FSD can be seen as a stochastic version of preferring more money to less, and S-SD as a characterization of riskaversion. Preferences satisfying these SD conditions have a representation that suits this interpretation: FSD is associated with non-decreasing utility, and S-SD with concave utility. SD conditions have other appealing features. Knowing certain qualitative features of the utility function of a decision maker, one can use the corresponding SD condition to eliminate dominated alternatives, which can be useful for management practitioners. More importantly, SD conditions can be used to construct pairs of prospects in which one prospect dominates the other. From the preference of a decision maker between these prospects one can deduce qualitative conditions of her preferences (e.g., risk aversion) and conclude qualitative properties of their representation (e.g., concavity of the value function). SD can thus be a potentially powerful framework for testing qualitative properties of choice models. It is this application that we focus on in this working paper.

EU does not accurately predict empirically observed preferences (Camerer, 1995; Wu, Zhang, and Gonzalez, 2004) while, in contrast, Cumulative Prospect Theory (CPT) (Tversky and Kahneman, 1992) is quite successful at it (see Abdellaoui [2000] and Abdellaoui, Vossmann, and Weber [In Press] for recent tests). Therefore, SD conditions related to CPT are likely to be of practical interest. They can suggest experimental designs that can isolate certain features of CPT without having to estimate all the elements of that theory. CPT proposes that subjects encode outcomes in terms of gains and losses. Furthermore, CPT replaces the traditional utility function by a value function defined over variations of wealth with respect to some reference point, and includes a probability weighting function (pwf) that reflects the subjective probability distortion shown by most individuals.

We begin by offering a general SD result for the EU model, which accommodates several curvature forms for the utility function, namely, S-shaped, inverse S-shaped, concave, and convex. Next, we generalize the previous result to the CPT model by incorporating the pwf. For the value function, this second result encompasses the previous four curvature combinations. For the pwf, we go beyond the case of a constant curvature, investigated in previous generalizations, and consider a broader class that includes the inverse S-shaped form. In addition, we examine SD conditions that capture the remaining important aspect of CPT, namely, loss aversion. Loss aversion plays a central role in behavioral decision research (Benartzi and Thaler, 1995; Langer and Weber, 2001); in CPT it is expressed mathematically as a steeper value function for losses than for gains, capturing the psychological intuition that losses loom larger than gains. We conclude the theoretical part of the working paper by presenting an SD condition that incorporates loss aversion, an S-shaped value function, and an inverse S-shaped pwf, i.e., *Prospect Stochastic Dominance* in the sense of the specification of CPT put forward by Tversky and Kahneman (1992).

Stochastic Dominance conditions are necessary and sufficient to characterize combinations of classes of value functions and pwfs. They can be used to test joint hypotheses on the curvature of the value function and the pwf. This can be done in three ways, the application of which is presented in the experimental part of this working paper. First, if one assumes that the pwf is inverse S-shaped, then the corresponding SD conditions serve the purpose of testing hypotheses about the curvature of the value function and/or about loss aversion. Second, if one assumes that the specifications of CPT for the curvature of the value function hold, then the corresponding SD conditions allow one to test hypotheses on the shape of the pwf. Third, if one assumes that CPT holds under established qualitative specifications for the value function, including loss aversion, and the pwf, then a violation of the corresponding SD condition implies a violation of the CPT model. This suggests ways to compare CPT with alternative heuristics models of decision that have been proposed in the literature. The experiments reported in this working paper involved 277 subjects, consisting of undergraduates, MBA students and executives.

2. CPT Stochastic Dominance Conditions

THROUGHOUT the working paper we will be comparing pairs of prospects F and G, having cumulative distributions F and G. For simplicity, we assume that the outcomes of F and G are contained in an interval $[a, b]$, for some $a < 0$ and $b > 0$, and that F and G are continuous except for finitely many points. Subjects are assumed to abide by the rank- and sign-dependent framework of CPT. This is, their preferences are represented by (1) a non-decreasing value function $v(x)$ defined over a monetary gain or loss (change in wealth with respect to some reference point), with $v(0) = 0$; and (2) a pair of non-decreasing probability weighting functions (pwf) $w^-(p)$ and $w^+(p)$ that transform the objective probabilities into decision-weights, and having $w(0) = 0$ and $w(1) = 1$ (w is short for "both w^+ and w^-"). We also assume that v is continuous and, except for finitely many points, differentiable. w is assumed to be continuous, except for finitely many points.

Cumulative Prospect Theory (CPT) (Tversky and Kahneman, 1992) has become widely accepted as a descriptive theory of choice under risk. Empirical specifications of the model indicate that v is S-shaped. Formally, we define $\mathcal{V}_P$, the family of *Prospect* value functions, as those $v(x)$ that are convex for $x < 0$ and concave for $x \geq 0$. In order to contrast this specification with other alternatives, we define $\mathcal{V}_{P*}$ as the family of *Inverse Prospect* value functions containing those $v(x)$ that are concave for $x < 0$, and convex for $x \geq 0$. Similarly, $\mathcal{V}_{Concave}$ ($\mathcal{V}_{Convex}$) is the family of value functions that are concave (convex) for all x.

The empirically observed probability distortion agrees with a pwf that is "shallow in the open interval and changes abruptly near the end-points where $w(0) = 0$ and $w(1) = 1$" (Tversky and Kahneman, 1992: 282), more specifically, an inverse S-shaped pwf that is concave first, and then convex, has found broad empirical support (see Tversky and Kahneman, 1992; Tversky and Wakker, 1995; Gonzalez and Wu, 1999; Abdellaoui, 2000).

2.1. Accounting for the value function

In this section, we assume the decision maker maximizes the expectation of the value function, and she does not distort probabilities (i.e., $w(p)$ is not part of this model). Depending on the interpretation of x (relative or absolute) we encompass either a reference dependent model or EU. In any such model, the choice between F and G is determined by $V_F - V_G$, the difference between the evaluation of each prospect, where $V_F = \int_a^b v(x)dF(x)$ is the expectation of v under F. If F and G are continuous, then integration by parts produces:

$$\Delta_{[a,b]} \equiv V_F - V_G = \int_a^b [G(x) - F(x)]v'(x)dx \tag{2.1}$$

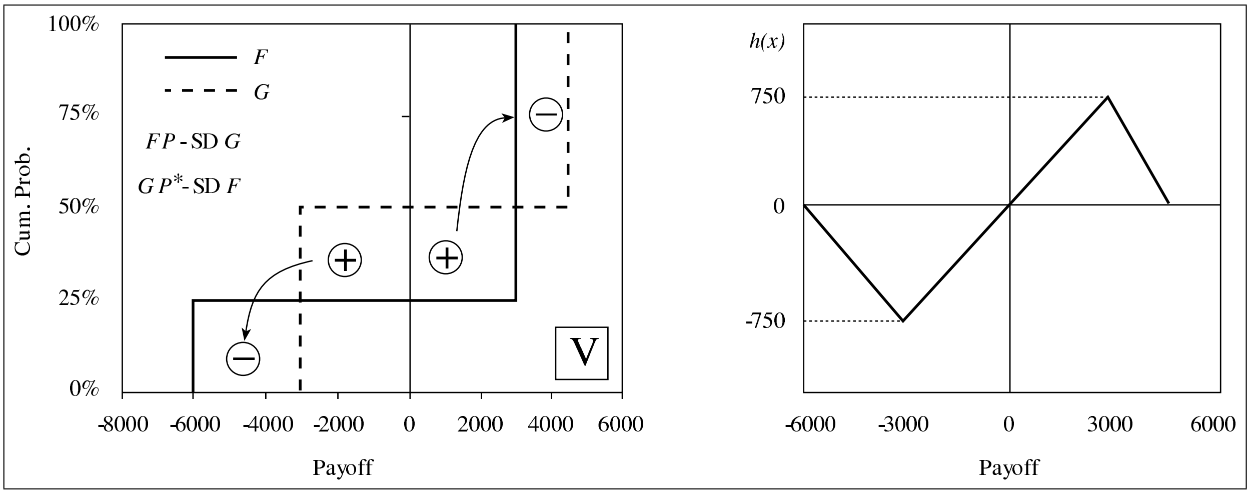
$\Delta_{[a,b]}$ can be seen as the inner product of $G(x) - F(x)$ and $v'(x)$. The relative magnitude of $\Delta_{[a,b]}$ can be seen in the graph of the cumulative distributions of F and G (e.g. graphic 2.1) by first deforming the horizontal payoff axis in proportion to v', and then simply calculating the area between G and F. In this process, segments where the slope of $v(x)$ is high are stretched more relative to segments where it is small. No deformation ($v' = 1$), of course, corresponds to the risk-neutral evaluation $\int_a^b [G(x) - F(x)]dx = E_F - E_G$. In any case, the integration in (2.1) never decreases in intervals where F dominates, $G(x) > F(x)$, and it never increases in intervals where G dominates.

In the left graph of graphic 2.1, the "$+$" and "$-$" signs correspond to the signs of $G(x) - F(x)$, respectively.

Consider some interval $[a_0, a_1]$ in which (i) the value function is convex, and (ii) any "$-$" area is *followed* by some "$+$" area of equal or larger size. By convexity, the horizontal stretching of the "$+$" area will be larger than the horizontal stretching of the "$-$" area, and the evaluation of (2.1) on that interval, called $\Delta_{[a_0,a_1]}$, will be positive. Similarly, consider some interval $[b_0, b_1]$ in which (i) the value function is concave, and (ii) any "$-$" area is *preceded* by some "$+$" area of equal or larger size. By concavity, the horizontal stretching of the "$+$" area will be larger than the stretching of the "$-$" area, and $\Delta_{[b_0,b_1]}$ will be positive.

For a prospect value function, the function is convex in $[a, 0]$ and concave in $[0, b]$. Hence, the required cancelation occurs in graphic 2.1 (left), ensuring that F is preferred over G. Conversely, if v is inverse S-shaped, then the convex and concave intervals are $[0, b]$ and $[a, 0]$, respectively. The area cancelation now finds that G is preferred over F.

GRAPHIC 2.1: **Left: Graphic illustration of stochastic dominance for Task V. Right: *h(x)* for Task V**



For the ensuing definitions, we introduce the function

$$h(x) \equiv \int_a^x [G(y) - F(y)]dy \tag{2.2}$$

Clearly, h is continuous and $h'(x) = G(x) - F(x)$. Moreover, $h(a) = 0$ and $h(b) = E_F - E_G$, so that $h(x)$ can be viewed as the "advantage" in expected value of F over G up to x. The right graph of graphic 2.1 (right) shows an example of this function.

The value of $h(x_1) - h(x_0)$ measures the area contained between G and F in the interval $[x_0, x_1]$. The function h is very convenient to express stochastic dominance conditions. For instance, $h'(x) \geq 0$ for all x is equivalent to $G(x) \geq F(x)$, which is FSD. Similarly, $h(x) \geq 0$ for all x is the second-order stochastic dominance condition. In the context of the previous discussion, the condition that a "$+$" area is canceled by some subsequent "$-$" area on $[a_0, a_1]$ is easily expressed as $h(x) \leq h(a_1)$ for all x, $a_0 \leq x < a_1$. Similarly, the condition that a "$-$" area is canceled by some preceding "$+$" area on $[b_0, b_1]$ is easily expressed as $h(x) \geq h(b_0)$ for all x, $b_0 \leq x < b_1$.

A general SD result can now be presented considering the interplay between the function h and the curvature of v.

Proposition 1. *Let $A = [a_0, a_1]$ and $B = [b_0, b_1]$ be intervals such that*

$A \cup B = [a, b]$. *Given prospects F and G, let h be defined as in (2.2). Then,*

$$h(x) \leq h(a_1) \text{ for } a_0 \leq x < a_1, \text{ and} \quad (2.3)$$
$$h(x) \geq h(b_0) \text{ for } b_0 \leq x < b_1 \quad (2.4)$$

hold if and only if $V_F \geq V_G$ for all v that are convex in A and concave in B.

Notice that the intervals A and B don't have to be disjoint, in which case v is linear in their intersection. We present four specialized versions of proposition 1 tailored to the four relevant curvature combinations of reference-dependent value functions. The corresponding integral conditions will be denoted by P-SD (prospect), P^*-SD (inverse prospect), S-SD (second order), and S^*-SD (inverse second order) stochastic dominance, respectively[1].

Definition 2. *Given prospects F and G, let h be defined as in (2.2). F dominates G according to P-SD, denoted by F P-SD G, if and only if $h(x) \leq h(0)$ for $a \leq x \leq 0$, and $h(x) \geq h(0)$ for $0 \leq x \leq b$. Similarly, we define*

$$\begin{array}{lll} F\ P^*\text{-SD}\ G & \text{iff} & h(x) \geq h(a) \text{ for } a \leq x \leq 0, \text{ and} \\ & & h(x) \leq h(b) \text{ for } 0 \leq x \leq b; \\ F\ S\text{-SD}\ G & \text{iff} & h(x) \geq h(a) \text{ for } a \leq x \leq b; \text{ and} \\ F\ S^*\text{-SD}\ G & \text{iff} & h(x) \leq h(b) \text{ for } a \leq x \leq b. \end{array}$$

Proposition 3. *F P-SD G if and only if $V_F \geq V_G$ for all $v \in \mathcal{V}_P$. Similarly,*

$$\begin{array}{lll} F\ P^*\text{-SD}\ G & \text{iff} & V_F \geq V_G \text{ for all } v \in \mathcal{V}_{P^*}; \\ F\ S\text{-SD}\ G & \text{iff} & V_F \geq V_G \text{ for all } v \in \mathcal{V}_{Concave}; \text{ and} \\ F\ S^*\text{-SD}\ G & \text{iff} & V_F \geq V_G \text{ for all } v \in \mathcal{V}_{Convex}. \end{array}$$

These four particular results are available in Levy and Levy (2002b), albeit not in this unified treatment. To illustrate the proposition, notice that F and G in graphic 2.1 satisfy F P-SD G and G P^*-SD F. We also remark that if any of these conditions apply, then $E_F \geq E_G$. To see this, notice that a is equal to either a_0 or b_0, and b is equal to either a_1 or b_1. If follows from (2.3) and (2.4) that $E_F - E_G = h(b) \geq h(a) = 0$.

1. Combination of letters such as P, P^*, S, S^*, W and L will denote different SD conditions. Although this type of notation might seem cumbersome at the beginning, in the course of the working paper its usefulness will become apparent - please bear with us.

2.2. Accounting for the probability weighting function

In this section (and the rest of the working paper), we assume that the decision maker follows CPT, i.e., she has preferences represented by a value function and a probability weighting function. The introduction of a nonlinear and sign-dependent pwf transforms $V_F - V_G$ in (2.1) into

$$\Delta^w_{[a,b]} = \int_a^0 [w^-(G(x)) - w^-(F(x))]v'(x)dx +$$

$$\int_0^b [w^+(1-F(x)) - w^+(1-G(x))]v'(x)dx \qquad (2.5)$$

In the discrete case, we first label the outcomes as $a < x_1 < ...x_{k-1} < x_k = 0 < x_{k+1} < ... < x_n < b$,[2] and let

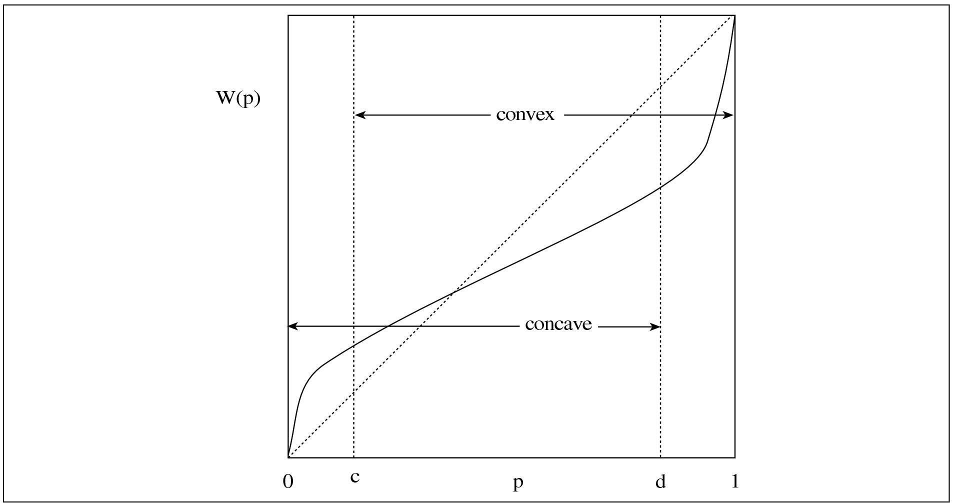
$$\begin{aligned}\Delta^w \;&=\; \sum_{i=1}^{k-1}[w^-(G(x_i)) - w^-(F(x_i))][v(x_{i+1}) - v(x_i)] \\ &+\; \sum_{i=k}^{n-1}[w^+(1-F(x_i)) - w^+(1-G(x_i))][v(x_{i+1}) - v(x_i)]\end{aligned} \qquad (2.6)$$

with the understanding that if $k = 1$ ($k = n$) then the summation over negative (positive) outcomes disappears.

Considering the evaluation of $\Delta^w_{[a,b]}$ according to (2.5), we observe that w adds a non-linear transformation of the vertical axis in proportion to $w'(p)$ (Quiggin, 1993: 150). For concreteness, let's consider an inverse S-shaped pwf. Referring to graphic 2.1 (left) the nonlinearity of the pwf *stretches* the lower and upper ends of the vertical probability axis, relative to the middle range of cumulative probabilities. Accordingly, the gap between $G(x)$ and $F(x)$ in zones near the probability extremes is magnified. As a consequence, CPT remains ambiguous in the case of prospects F and G in graphic 2.1 (left). While F is favored by the stretching of the horizontal axis near the origin enlarging the "+" areas, G is favored by the stretching of the vertical axis for low values of p in the losses domain and for high values of p in the gains domain, enlarging the "−" areas. Thus, according to (2.5), an S-shaped value function is still compatible with $V_G > V_F$. In the following we argue that second order SD conditions require that the value function and the pwf have *conjugate*

2. Notice that to have $x_k = 0$ for some k may require setting $p_k = 0$.

GRAPHIC 2.2: Schematic depiction of the W_c^d class of pwfs. Notice that the W_c^d class grows as c increases and d decreases



curvatures. By *conjugate* we refer to curvature combinations that ensure the monotonicity of $[w^-(G(x)) - w^-(F(x))]v'(x)/[G(x) - F(x)]$ or $[w^+(1 - F(x)) - w^+(1 - G(x))]v'(x)/[G(x) - F(x)]$. In the losses domain, a convex v and a convex w^- are conjugate; or a concave v and concave w^- are also conjugate. In the gains domain, a concave v and a convex w^+ are conjugate; or a convex v and a concave w^+ are conjugate too.

Assuming a constant curvature for the pwf is the simplest possibility, but this would limit the potential for application of our results. With the goal of finding stochastic dominance conditions that can encompass an inverse S-shaped pwf, we consider pwfs that are concave in the range $[0, d)$ and convex in the range $(c, 1]$, for given values of d and c in $[0, 1]$. We denote this class by $\mathcal{W}_c^d$. If $c < d$, then the segment between c and d is necessarily linear; and if $0 < c \leq d < 1$, then the pwf is inverse S-shaped and continuous in $(0, 1)$ (see graphic 2.2). Most of the commonly employed parametric families of pwfs (Prelec, 1998; Tversky and Kahneman, 1992; Lattimore, Baker, and Witte, 1992) fall within the $\mathcal{W}_c^d$ class, with $c = d$ being the inflection point of these inverse S-shaped functions. The family $\mathcal{W}_0^1$, $c = 0$ and $d = 1$, corresponds to pwfs that are linear in $(0, 1)$ and possibly discontinuous at either 0 or 1. Finally, if $c > d$, then w is unrestricted between d and c. Here, we have used c (or d) to denote both c^- and c^+ (or both d^- and d^+), which will apply to w^- and w^+, respectively.

We are interested in the intervals of the payoff line in which the curvatures of v and w are conjugate. To describe their boundaries, we define four characteristic points such as the point x_c^-, which corresponds to the algebraically smallest negative payoff for which both F and G take values greater than c^-. Analogously, x_d^- is the greatest value in the negative domain for which both F and G take smaller values than d^-; x_c^+ the greatest positive payoff for which F and G are smaller than $1 - c^+$; and x_d^+ the smallest positive payoff for which F and G take values greater than $1 - d^+$. Formally,

Definition 4. *If the pwf is continuous at the corresponding points c^-, d^-, d^+, and d^+, then let*

$$\begin{aligned}
x_c^- &= \inf\{x \leq 0 : F(x) \geq c^-,\ G(x) \geq c^-\}, && (2.7)\\
x_d^- &= \sup\{x \leq 0 : F(x) \leq d^-,\ G(x) \leq d^-\}, && (2.8)\\
x_c^+ &= \sup\{x \geq 0 : F(x) \leq 1 - c^+,\ G(x) \leq 1 - c^+\}, \text{ and} && (2.9)\\
x_d^+ &= \inf\{x \geq 0 : F(x) \geq 1 - d^+,\ G(x) \geq 1 - d^+\}, \text{ respectively.} && (2.10)
\end{aligned}$$

If the pwf is not continuous at some of these points, then replace the corresponding weak inequalities (e.g., $F(x), G(x) \geq c^-$) with strict inequalities (e.g., $F(x), G(x) > c^-$). If the sets over which the infimum and the supremum are taken are empty, then set $x_c^- = 0$, $x_d^- = a$, $x_c^+ = 0$, and $x_d^+ = b$, respectively.

The definition ensures that the curvatures of w^-, w^+, and v are conjugate:

Proposition 5. *If $x_c^- < x < 0$ and $x \in A$ ($a \leq x < x_d^-$ and $x \in B$) then $[w^-(G(x)) - w^-(F(x))]v'(x)/[G(x) - F(x)]$ is non-decreasing (non-increasing). If $0 \leq x < x_c^+$ and $x \in B$ ($x_d^+ < x < b$ and $x \in A$), then $[w^+(G(x)) - w^+(F(x))]v'(x)/[G(x) - F(x)]$ is non-increasing (non-decreasing).*

To illustrate the intervals, let $c = 0.1$ and $d = 0.8$, and consider the prospects in graphic 2.1 (left). We have that $x_c^- = -3000$, $x_d^- = 0$, $x_c^+ = 3000$, and $x_d^+ = 0$. In this example, x_d^- and x_d^+ are both zero because F and G never reach a cumulative probability of $d = 0.8$ in the negative domain; and it exceeds $1 - d = 0.2$ in the positive domain.

For the results that follow, we are given prospects F and G, and values of c^-, d^-, c^+, d^+. We then calculate x_c^-, x_d^-, x_c^+, and x_d^+ according to definition 4, and $h(x)$ as in (2.2). We generalize proposition 1 by ensuring first order stochastic dominance in the zones lacking conjugate curvature. The result is rather technical: non-interested readers can skip this statement and move to the particularization that follows.

Proposition 6. *Let* $A = [a_0, a_1]$ *and* $B = [b_0, b_1]$ *be intervals such that* $A \cup B = [a, b]$, *and let* $\hat{a}_0 = \max\{a_0, 0\}$ *and* $\hat{b}_1 = \min\{0, b_1\}$. *Then, conditions (2.3) and (2.4) from proposition 1, together with*

$$\text{if } x \in A, \text{ then } G(x) \geq F(x) \text{ for } a_0 \leq x < x_c^- \text{ and } \hat{a}_0 \leq x < x_d^+, \tag{2.11}$$

$$\text{if } x \in B, \text{ then } G(x) \geq F(x) \text{ for } x_d^- \leq x < \hat{b}_1 \text{ and } x_c^+ \leq x < b_1, \tag{2.12}$$

$$\text{if } a_0 < 0 < a_1, \text{ then } h(x) \leq h(0) \text{ for } a_0 \leq x < 0, \text{ and} \tag{2.13}$$

$$\text{if } b_0 < 0 < b_1, \text{ then } h(x) \geq h(0) \text{ for } 0 \leq x < b_1 \tag{2.14}$$

hold if and only if $V_F \geq V_G$ *for all* v *convex in* A *and concave in* B, $w^- \in \mathcal{W}_{c^-}^{d^-}$, *and* $w^+ \in \mathcal{W}_{c^+}^{d^+}$.

Based on this general result, we present the corresponding extension of proposition 3, now incorporating the non-linear pwf. In terms of notation, we add a "W" to the names of the SD conditions.

Definition 7. F *dominates* G *according to* PW*-SD, denoted by* F PW*-SD* G, *if and only if* F *dominates* G *according to* P*-SD, and* $G(x) \geq F(x)$ *for* $a \leq x < x_c^-$ *and* $x_c^+ \leq x < b$. *Similarly, we define*
F P^*W*-SD* G *iff* F P^**-SD* G, *and* $G(x) \geq F(x)$ *for* $x_d^- \leq x < x_d^+$;
F SW*-SD* G *iff* F S*-SD* G, $G(x) \geq F(x)$ *for* $x_d^- \leq x < 0$ *and* $x_c^+ \leq x < b$, *and* $h(x) \geq h(0)$ *for* $0 \leq x < b$; *and*
F S^*W*-SD* G *iff* F S^**-SD* G, $G(x) \geq F(x)$ *for* $a \leq x < x_c^-$ *and* $0 \leq x < x_d^+$, *and* $h(x) \leq h(0)$ *for* $a \leq x < 0$.

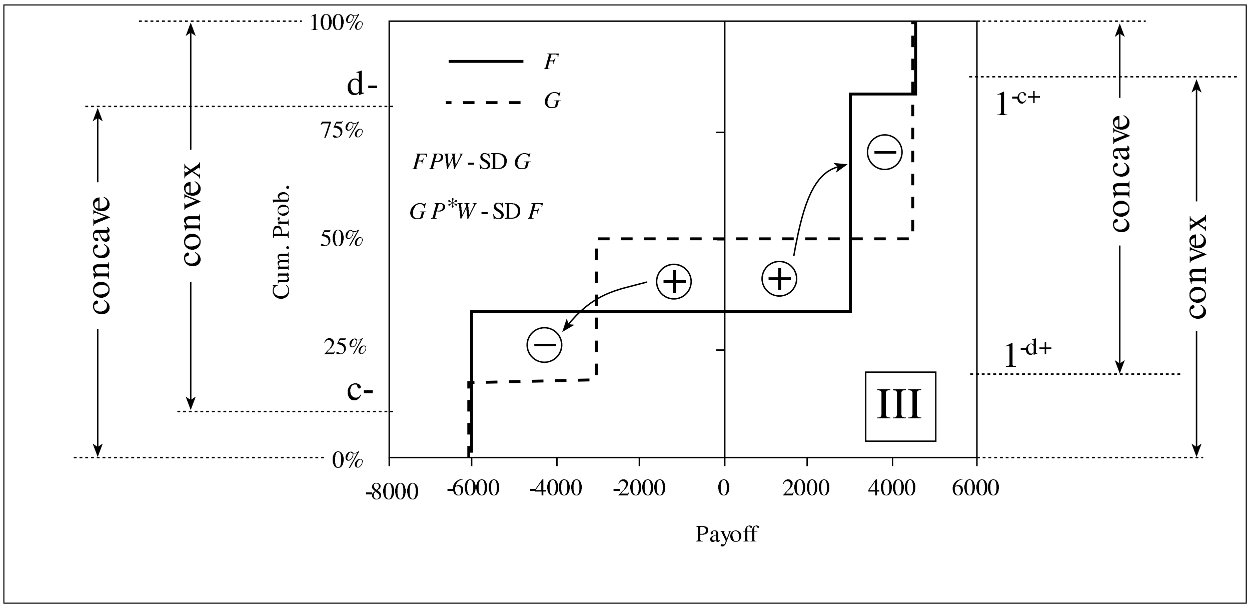
Preferences that agree with these dominance conditions have the following representation.

Proposition 8. F PW*-SD* G *if and only if* $V_F \geq V_G$ *for all* $v \in \mathcal{V}_P$, $w^- \in \mathcal{W}_{c^-}^{d^-}$ *and* $w^+ \in \mathcal{W}_{c^+}^{d^+}$. *Similarly,*

F P^*W*-SD* G	*iff*	$V_F \geq V_G$ *for all* $v \in \mathcal{V}_{P^*}$, $w^- \in \mathcal{W}_{c^-}^{d^-}$ *and* $w^+ \in \mathcal{W}_{c^+}^{d^+}$;
F SW*-SD* G	*iff*	$V_F \geq V_G$ *for all* $v \in \mathcal{V}_{Concave}$, $w^- \in \mathcal{W}_{c^-}^{d^-}$ *and* $w^+ \in \mathcal{W}_{c^+}^{d^+}$; *and*
F S^*W*-SD* G	*iff*	$V_F \geq V_G$ *for all* $v \in \mathcal{V}_{Convex}$, $w^- \in \mathcal{W}_{c^-}^{d^-}$ *and* $w^+ \in \mathcal{W}_{c^+}^{d^+}$.

An example for F PW-SD G, $c = 1/6$, is displayed in graphic 2.3. The fact that FSD has to be imposed in the absence of conjugate

GRAPHIC 2.3: Cumulative distributions for Task III. To the left and the right of the cumulative graph we indicate the curvature of the pwf, according to the $\mathcal{W}_c^d$ class



curvatures is in itself very revealing: If no restriction is placed on the pwf, then all four SD conditions reduce to FSD. In other words, *knowledge of v, together with an unrestricted pwf, is insufficient to predict preferences between stochastically undominated prospects.* Conversely, a pwf with certain constant curvatures allows us to extend with minor modifications the known forms of SD to the CPT framework.

For example, if both w^- and w^+ are convex ($c^- = c^+ = 0$), then P-SD and PW-SD coincide (Levy and Wiener [1998]). To extend S-SD to SW-SD requires w^- to be concave ($d^- = 1$) and w^+ to be convex ($c^+ = 0$), together with $h(0) \leq h(x)$ for all x, $0 \leq x < b$. In general, by decreasing c or increasing d we make the $\mathcal{W}_c^d$ class narrower, but impose FSD on a smaller range, and hence increase the scope of application of the different SD conditions. However, if the class $\mathcal{W}_c^d$ is too narrow, then it might not contain the desired functions. Ultimately, the choice of c and d is the product of resolving this trade-off. From an experimental point of view this trade-off will be addressed in section 3.1.

2.3. Accounting for loss aversion

Loss aversion, an important feature of CPT with strong empirical support (Abdellaoui, Bleichrodt, and Paraschiv, 2004), has not played a role in our definitions so far. To incorporate this third feature of CPT, we define the class of value functions possessing *loss aversion* as follows. The value function v is in $\mathcal{V}_L$ if and only if

$$v'(-x) \geq v'(x) \text{ for all } x > 0 \text{ where } v'(x) \text{ and } v'(-x) \text{ exists.} \qquad \text{(L)}$$

Condition (L) was characterized through preference conditions by Wakker and Tversky (1993) and is essentially equivalent to $v(y) - v(x) \leq v(-x) - v(-y)$ for all $y \leq x \leq 0$ (Bowman, Minehart, and Rabin [1999, Assumption A2]). (L) is more stringent than $-v(-x) \geq v(x)$ for all x, the condition originally put forward in Kahneman and Tversky (1979). Thinking in terms of *deformation* of the cumulative graph, loss aversion guarantees that the stretching of the horizontal axis at $x < 0$ is at least as large as the stretching of the horizontal axis at $|x| > 0$. This allows us to use "+" segments in the losses domain where $G(x) > F(x)$ to counteract "$-$" segments in the gains domain where $G(|x|) < F(|x|)$.

Loss aversion entails comparisons between the negative and the positive domain. Hence, we need to constrain the sign-dependent pwfs. Throughout this section we assume that the given values of c^-, d^-, c^+, and d^+ satisfy $c^+ < d^+$ and $c^- < d^-$. Under this specification, we let s be the slope of the linear segment of w between c and d, respectively. If w is continuous, then $s = (w(d) - w(c))/(d - c)$ [in general $s = \lim_{\varepsilon \to 0}[w(d - \varepsilon) - w(c + \varepsilon)]/(d - c)$]. We will consider pairs of pwfs satisfying $s^- \geq s^+$. That the slope of w^- is larger than the slope of w^+ ensures that zones of positive FSD in the negative domain can counteract zones of negative FSD in the positive domain. The condition $s^- \geq s^+$ is consistent with the empirical finding that w^- exhibits less deformation than w^+ (Tversky and Kahneman [1992] and Abdellaoui [2000]). For the following definition, set $A = B = [0, b]$ and calculate x_c^+ and x_d^+ according to definition 4.

Definition 9. *F dominates G according to PL-SD, denoted by F PL-SD G, if and only if*

$$G(x) \geq F(x) \text{ for } a \leq x < x_d^+ \text{ and } x_c^+ \leq x < b; \text{ and} \qquad (2.15)$$

$$G(-x) - F(-x) \geq F(x) - G(x), \text{ for } x \geq 0 \text{ and } F(x), G(x) \text{ continuous.} \qquad (2.16)$$

Proposition 10. *F PL-SD G if and only if $V_F \geq V_G$ for all $v \in \mathcal{V}_L$, $w^- \in \mathcal{W}_{c^-}^{d^-}$, and $w^+ \in \mathcal{W}_{c^+}^{d^+}$ such that $s^- \geq s^+$.*

Graphic 2.4 (left) shows an example satisfying the conditions of the proposition[3]. Notice that if $d^+ \leq c^+$, then $x_c^+ \leq x_d^+$ and (2.15) would reduce to FSD.

We now take up the more challenging task of defining a condition of stochastic dominance that combines curvature conditions together with loss aversion. For our purposes, it is sufficient to consider the class of prospect value functions which exhibit also loss aversion, or $\mathcal{V}_{PL} = \mathcal{V}_P \cap \mathcal{V}_L$. An S-shaped value function provides the possibility to use "$+$" segments of small gains (small losses) to counteract "$-$" segments of large gains (large losses). We call this gains-gains (losses-losses) cancelation. Loss aversion brings the possibility of losses-gains cancelation, i.e., utilize "$+$" segments of losses to cancel "$-$" segments of appropriately sized gains. To know which parts of positive FSD segments in the losses domain are available to counteract negative FSD parts in the positive domain, we rely again on the function $h(x)$. Recall that the segments where $h(x)$ is increasing are precisely those where F FSD G, so that they are available to counteract segments in which h is decreasing. Decreasing values of $h(x)$ in the negative domain can only be made up for with increasing values of $h(\hat{x})$ for an $\hat{x}$ between x and zero; and this is most efficiently done for an $\hat{x}$ as close to x as possible. All those pairs $(x, \hat{x})$ that can perform the losses-losses cancelation in this way are given by the following set:

$$E^- = \{x < 0 : h(x) \leq h(\hat{x}), \text{for some } \hat{x},\, a \leq \hat{x} < x\}$$

Going from negative to positive, points in E^- are those for which the function h falls below a level that was reached before. Now consider a point $x < 0$ not in E^-. Then, $h(x)$ increases beyond any past value of x, implying that $h'(x) \geq 0$. Hence, points such as x are available for losses-gains cancelation. However, according to (L), for this cancelation to be successful it needs to be applied to a point in the positive domain with value $|x| = -x$ or larger. This will be the case if $h(-x) \geq h(x)$. We are now ready for the full CPT SD definition, which incorporates a slightly modified version of PW-SD, together with this last condition.

Definition 11. *F dominates G according to PWL-SD, denoted by F PWL-SD G,*

3. This example also illustrates that in discontinuity points, such as $x = 1000$, the inequality in (2.16) need not be satisfied.

if and only if

$$h(x) \leq h(0), \text{ for } a \leq x < 0, \tag{2.17}$$

$$h(x) \geq h(0) \text{ for } 0 \leq x < x_d^+, \tag{2.18}$$

$$G(x) \geq F(x) \text{ for } a \leq x < x_c^- \text{ and } x_c^+ \leq x < b, \text{ and} \tag{2.19}$$

$$h(-x) \geq h(x) \text{ for } x < 0 \text{ and } x \notin E^-. \tag{2.20}$$

Proposition 12. *F PWL-SD G if and only if $V_F \geq V_G$ for all $v \in \mathcal{V}_{PL}$, $w^- \in \mathcal{W}_{c^-}^{d^-}$, and $w^+ \in \mathcal{W}_{c^+}^{d^+}$ such that $s^- \geq s^+$.*

Graphic 2.4 (right) shows an example of this kind of stochastic dominance. We invite the reader to follow the arrows that identify the different types of cancelation (losses-losses and losses-gains). In the losses-losses cancelation, arrows have to be oriented right to left. In the losses-gains cancelation, an arrow starting at $x < 0$ can be applied to $|x|$ or beyond, but never to values lower than $|x|$. In the gains-gains cancelation (present in Task III [graphic 2.3] but not in Task XI [graphic 2.4]), arrows have to be oriented left to right.

If F and G are all-gains prospects, then PWL-SD, PW-SD, and SW-SD coincide; and if F and G are all-losses prospects, then PWL-SD, PW-SD, and S^*W-SD coincide. Clearly, PW-SD implies PWL-SD, but the converse is not true unless E^- is the entire negative domain.

We finish this section with table 2.1, which summarizes properties of the value function and the pwf that correspond to the six different forms of stochastic dominance introduced in the working paper for the CPT model.

GRAPHIC 2.4: Tasks VIII and XI. Task VIII involves cancelations based solely on loss aversion (*WL*). Task XI requires in addition a convex value function for losses (*PWL*)

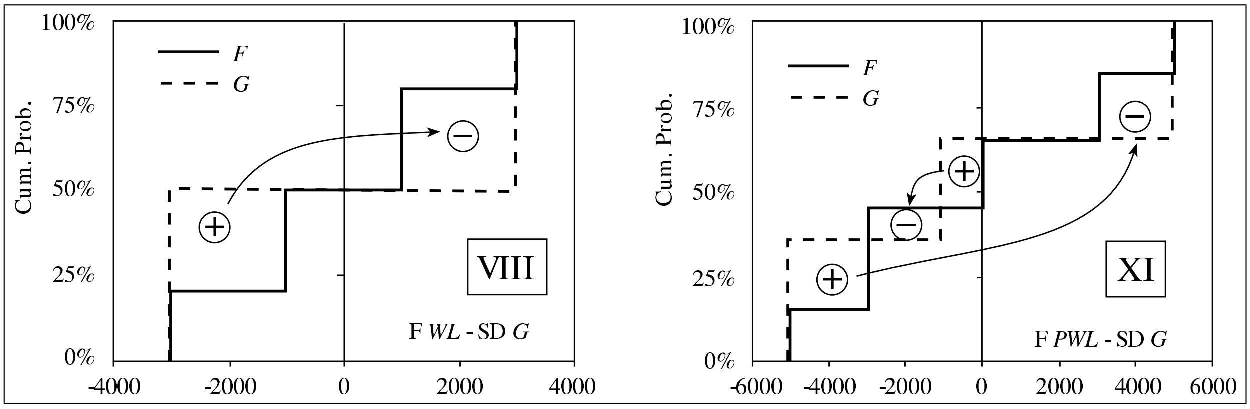


TABLE 2.1: Summary of the different stochastic dominance conditions and their requirements on the value function v and the pwf w

SD Condition	Value Function v	pwf w^-	pwf w^+
PW-SD	S-shaped	Convex on $(c^-, 1]$	Convex on $(c^+, 1]$
P^*W-SD	Inverse S-shaped	Concave on $[0, d^-)$	Concave on $[0, d^+)$
SW-SD	Concave	Concave on $[0, d^-)$	Convex on $(c^+, 1]$
S^*W-SD	Convex	Convex on $(c^-, 1]$	Concave on $[0, d^+)$
WL-SD	Loss Aversion (L)	Inverse S-shaped	Inverse S-shaped
PWL-SD	S-shaped and (L)	Linear on (c^-, d^-)	Linear on (c^+, d^+)

3. Experimental Applications

THE analytical SD conditions introduced above suggest experimental designs of fine-tuned prospects through which different elements of CPT can be tested. This section presents the experimental results of twelve prospect comparisons or *tasks* that were presented to 277 individuals. The subjects were students (undergraduates and MBA) and professionals (business executives and lawyers). The 84 college students are from the University of Navarra (Pamplona, Spain), majoring in economics and most of them from Spain. The 78 MBA students are from IESE Business School (Barcelona, Spain) representing a broad range of countries of origin. The 99 executives were participants at executive education programs at IESE. The professional group also contains a sample of 15 lawyers who were approached through personal contacts. The group of professionals consists almost exclusively of Spaniards. The answers from both types of students and the professionals are very similar so that their results are reported together. The different tasks were divided between different sessions so that the number of valid answers for a given task varies between 177 and 277. Subjects were not paid. According to Camerer and Hogarth (1999), this should not significantly influence the average results for gains in choices among risky gambles. For losses, data for a similar conclusion are not available.

The tasks (and results) are presented in tables 3.3 through 3.6. Prospect F always corresponds to the prospect that is preferred according to the SD condition being tested. In the actual experiments, both the assignment of the preferred prospect to F or G and the order of the tasks were randomized. All tasks were printed on a questionnaire that was handed out to the subjects. The tasks were introduced with the written question: “Suppose that you decide to invest $10,000 either in stock F or in stock G. Which stock would you choose, F, or G, when it is given that the dollar gain or loss one month from now will be as follows:”. Typically, subjects needed 20 minutes to complete the questionnaire.

The experimental part is divided into four sections, depending on which SD conditions are applied and, most importantly, on four possible ways in which SD tests can be interpreted. In the first section we assume that the pwf is inverse S-shaped and the goal is to investigate the curvature of the value function. In contrast, the second section assumes that the value function is S-shaped and tests hypotheses on the pwf. The third section is similar to the first, i.e., we assume that the pwf is inverse S-shaped, but we test whether the value function exhibits loss aversion. In the final section, we assume that all the empirical specifications of CPT for v and w hold, including loss aversion, and interpret the results as a global test of CPT. The hypotheses and findings of the four sections are summarized in table 3.1.

TABLE 3.1: Summary for each task of the joint hypotheses tested on the value function v and the pwf w

		Joint Hypothesis			Not	
Task	**SD Condition**	**Value Fctn**	**pwf**	**Result**	**questioned**	**Conclusion**
I	$F\ SW$-SD G	$v \in V_P$	$w^+ \in W_{0.1}$	Not Rejected	w	Consistent with v S-shaped
	$G\ S^*W$-SD F	$v \in V_{P^*}$	$w^+ \in W^{0.9}$	Rejected	w	$v \notin V_{P^*}, v \notin V_{Convex}$
II	$F\ S^*W$-SD G	$v \in V_P$	$w^- \in W_{0.1}$	Not Rejected	w	Consistent with v S-shaped
	$G\ S^*W$-SD F	$v \in V_{P^*}$	$w^- \in W^{0.9}$	Rejected	w	$v \notin V_{P^*}, v \notin V_{Concave}$
III	$F\ PW$-SD G	$v \in V_P$	$w \in W_{1/6}$	Not Rejected	w	Consistent with v S-shaped
	$G\ P^*W$-SD F	$v \in V_{P^*}$	$w \in W^{2/3}$	Rejected	w	$v \notin V_{P^*}$
IV	$F\ PW$-SD G	$v \in V_P$	$w \in W_{0.02}$	Not Rejected	v	Cannot reject $w \in W_{0.02}$
V	$F\ PW$-SD G	$v \in V_P$	$w \in W_0$	Rejected	v	$w \notin W_0$
VI	$F\ WL$-SD G	$v \in V_L$	$w \in W_{0.1}^{0.5}$	Rejected	w	Loss Aversion fails
VII	$F\ WL$-SD G	$v \in V_L$	$w \in W_{0.2}^{0.5}$	Not Rejected	w	Consistent with Loss Aversion
VIII	$F\ WL$-SD G	$v \in V_L$	$w \in W_{0.2}^{0.5}$	Not Rejected	w	Consistent with Loss Aversion
IX	$F\ PWL$-SD G	$v \in V_{PL}$	$w \in W_{0.1}^{0.5}$	Rejected/Mixed	v, w	CPT fails
X	$F\ PWL$-SD G	$v \in V_{PL}$	$w \in W_{0.1}^{0.7}$	Not Rejected	v, w	Consistent with CPT
XI	$F\ PWL$-SD G	$v \in V_{PL}$	$w \in W_{0.15}^{0.65}$	Not Rejected	v, w	Consistent with CPT
XII	$F\ PWL$-SD G	$v \in V_{PL}$	$w \in W_{0.2}^{0.5}$	Not Rejected	v, w	Consistent with CPT

3.1. Choice of *c* and *d*

It may appear as if the exact knowledge of the parameters c and d were critical for a sound application of the SD relations. However, it is only necessary in the design of the tasks to choose c and d such that they are respectively big (c) or small (d) enough such that the pwf can be safely assumed to be concave up to d and convex from c on. Between c and d, the pwf has to be approximately linear. The higher the c and the lower the d one chooses, the less one assumes about the pwf, but the more restrictive is the set of lotteries one can design. Hence, in order to increase the freedom in choosing the prospects, we want to know how low c and how high d can be. While the parametric forms of the pwf proposed in the literature (Tversky and Kahneman, 1992; Prelec, 1998) have no linear segments, we can consider some range as being approximately linear if the differences in slope between any two points are not too large. Taking the inflection point which has the minimum slope as the reference, we calculate the range, delimited by c and d, for which $w'(p)$ is at most 50% higher. This is a conservative estimate, if we keep in mind that in commonly employed parametric specifications the slope of w (and v) at different points can differ by several orders of magnitude. For the formulation for the pwf proposed by Prelec (1998), the inflection point is at $1/e$, and $w'(1/e) = \gamma$. Table 3.2 shows an overview of the corresponding values for c and d. Values of c greater than 0.10 and d smaller than 0.76 should be sufficient for the empirically plausible range of γ, i.e. 0.5 to 0.7.

TABLE 3.2: Based on the pwf, $w(p)=exp[-(-ln\ p)^{\gamma}]$, the values of c and d are the end points of the interval where $w'(p)$ is less than or equal to 1.5 $w'(1/e)$

γ	c	d
1	0.00	1.00
0.9	0.02	0.98
0.8	0.05	0.90
0.7	0.07	0.84
0.6	0.09	0.79
0.5	0.10	0.76

3.2. Shape of the value function

Design

The first experimental part (Tasks I through III) aims at finding the most representative value function, assuming that the pwf is inverse S-shaped.

Task I presents an all-gains choice, Task II an all-losses choice, and Task III uses a mixed choice (see table 3.3 and, for Task III, see also graphic 2.3). They are designed so that F PW-SD G. In the gains domain, PW-SD becomes the generalized second order stochastic dominance SW-SD. In the losses domain, PW-SD agrees with the reverse condition S^*W-SD. Because $E_F = E_G$, the comparison between G and F in all three tasks exhibits G P^*W-SD F . Hence, we perform two tests of the curvature

TABLE 3.3: Design and Results of Tasks I, II and III

	F		G		Choice		
TASK I	**Gain/Loss**	**Prob.**	**Gain/Loss**	**Prob.**	**N**	**F[%]**	**G[%]**
F SW -SD G, $c = 0.1$	0	10%	0	50%	277	**74**	26
G S^*W -SD F, $d = 0.9$	1000	40%					
	2000	40%					
	3000	10%	3000	50%			
TASK II	**Gain/Loss**	**Prob.**	**Gain/Loss**	**Prob.**	**N**	**F[%]**	**G[%]**
F S^*W -SD G, $c = 0.1$	-3000	50%	-3000	10%	273	**65**	35
G SW -SD F, $d = 0.9$			-2000	40%			
			-1000	40%			
	0	50%	0	10%			
TASK III	**Gain/Loss**	**Prob.**	**Gain/Loss**	**Prob.**	**N**	**F[%]**	**G[%]**
F PW -SD G, $c = 1/6$	-6000	1/3	-6000	1/6	276	**76**	24
G P^*W -SD F, $d = 2/3$	3000	1/2	-3000	1/3			
	4500	1/6	4500	1/2			

of v in each task. Here, we don't question the shape of the pwf, which is assumed to be in $\mathcal{W}_c^d$, for values of c equal to 0.1 (Task I and II) and $1/6$ (Task III); and of d equal to 0.9 (Tasks I and II) and $2/3$ (Task III). Hence, we test whether v is in $\mathcal{V}_P$ for F PW-SD G and whether v is in $\mathcal{V}_{P^*}$ for G P^*W-SD F.

Results of Tasks I - III

The results of Tasks I-III are presented in the rightmost columns of table 3.3. For each task, the number of answers is provided as well as the percentage of individuals that chose the respective prospect. In all three tasks, prospect F is preferred over prospect G. For the joint hypotheses on the value function and the pwf this means the following: In Task I, G is not preferred and, as we assume that the pwf is concave in $[0, 0.9)$, it follows that the most representative value function is not convex for gains. Hence, v cannot be in $\mathcal{V}_{P^*}$ or in $\mathcal{V}_{Convex}$. Similarly, in Task II, the hypothesis that the value function is concave for losses is rejected, concluding that v is neither in $\mathcal{V}_{P^*}$ nor in $\mathcal{V}_{Concave}$. The result of Task III completes the picture for a mixed lottery. Under the assumption that the pwf is concave in $[0, 2/3)$, and given that G is not preferred, the value function cannot be in $\mathcal{V}_{P^*}$.

In all the three tasks, the majority of individuals prefer the PW-SD dominating prospect F. Given the rejected shapes of the value function, the S-shaped value function remains as a specification that is consistent with the results. It also establishes risk aversion for gains and risk seeking for losses as a phenomenon *independent* of the certainty effect.

Certainly, $d = 0.9$ in Tasks I and II is higher than the proposed value of 0.76 in subsection 3.1.. However, in Tasks I and II, the nonlinear pwf has a minor effect because the vertical stretching due to the pwf close to $p = 0$ magnifies a zone of negative FSD, whereas the vertical stretching close to $p = 1$ acts over a zone of positive FSD. In any case, the results of Task III, done under $d = 2/3$, are sufficient to reject that the most representative v is in $\mathcal{V}_{P^*}$, $\mathcal{V}_{Concave}$, or $\mathcal{V}_{Convex}$.

3.3. Shape of the pwf

Design

The second experimental part focusses on the pwf. In the joint hypotheses on the value function and the pwf that our SD relations test, we assume that the value function follows the specifications of CPT, i.e. $v \in \mathcal{V}_P$, and use PW-SD to test the convexity of the pwf. Table 3.4 shows the design of the two corresponding tasks. Again, in both tasks we have F PW-SD G. Task IV tests the convexity of w^- on $[0.02, 0.5)$ and of w^+ on $[0.02, 0.74)$; and Task V tests the convexity of w^- on $[0, 0.5)$ and of w^+ on $[0, 0.75)$.

TABLE 3.4: Design and Results of Tasks IV and V

	F		G		Choice		
TASK IV	**Gain/Loss**	**Prob.**	**Gain/Loss**	**Prob.**	**N**	**F[%]**	**G[%]**
F PW -SD G, $c = 0.02$	-6000	26%	-6000	2%	273	**81**	19
G P^*W -SD F, $d = 0.74$	3000	72%	-3000	48%			
	4500	2%	4500	50%			
TASK V	**Gain/Loss**	**Prob.**	**Gain/Loss**	**Prob.**	**N**	**F[%]**	**G[%]**
F PW-SD G, $c = 0$	-6000	1/4	-3000	1/2	271	37	**63**
G P^*W-SD F, $d = 1$	3000	3/4	4500	1/2			

Results of Tasks IV and V

The results of Tasks IV and V are presented in the rightmost columns of table 3.4 (for Task V, see also graphic 2.1). In Task IV, F is preferred by most subjects. Hence, the hypothesis that the pwf is convex form 0.02 on cannot be rejected. This is remarkable for the low value of c and underscores the earlier considerations of the right choice of c in the experimental design. In Task V, however, the majority of subjects prefers prospect G. Thus rejecting that both w^- and w^+ are convex in the corresponding intervals. Together with the results of Task IV, the reasonable conclusion is that the pwf is not convex near the origin. Notice that Task IV is a slight modification of Task V. In Task IV the extreme outcomes are set equal; in F the maximum outcome has been added and in G the minimum outcome has been added, both with a probability of 2%. This change is sufficient to reverse the preference of the majority of subjects. That a common range is sufficient to account for most of the effect of the pwf suggests that decision makers use the *range of outcomes* as a decision criterion.

Task V is a replication of Experiment 1.III from Levy and Levy (2002a). They use this task to perform a test on the shape of the value function, assuming that the pwf does not produce any significant distortion (i.e., without questioning the shape of the pwf, as we do in Tasks I-III). Hence, they conclude that v is not in $\mathcal{V}_P$ and, instead, endorse the $\mathcal{V}_{P^*}$ class of value functions. The mistake, of course, is that they implicitly assume that the pwf is convex (or linear) around zero. Wakker (2003) and Baucells and Heukamp (2004) show that the experimental results presented by Levy and Levy (2002a) are *compatible* with CPT. But a stronger case for the $\mathcal{V}_P$ class, and a rejection of the $\mathcal{V}_{P^*}$ class, is made in Tasks III and IV, which use the SD conditions suited to test the shape of the value function under the assumption that the pwf is inverse S-shaped.

Notice that in Tasks IV and V, prospect G dominates F according to P^*W-SD. This allow us to perform the following test: assume that v is in $\mathcal{V}_{P^*}$, and test the concavity of the pwf. Thus, the preference for F in Task IV could be interpreted as rejecting the hypothesis that w^- is concave on $[0.02, 0.5)$ and w^+ concave on $[0.02, 0.74)$. However, having just rejected the hypothesis that v is in $\mathcal{V}_{P^*}$, we don't embrace this interpretation.

3.4. Loss aversion

Design

The third experimental part consists of three tasks (VI to VIII) that focus on loss aversion. We assume that the pwf is inverse S-shaped and test whether the value function exhibits loss aversion. The experimental assessment of loss aversion is a complex issue that has not received a lot of attention in the past (see for example Schmidt and Traub, 2002) Here, we test loss aversion employing WL-SD, introduced in proposition 10.

In Tasks VI, VII and VIII prospect F dominates G according to WL-SD (see table 3.5 and, for Task VIII, see also graphic 2.4). The

TABLE 3.5: Design and Results of Tasks VI through VIII

	F		G		Choice		
TASK VI	**Gain/Loss**	**Prob.**	**Gain/Loss**	**Prob.**	**N**	**F[%]**	**G[%]**
$F\ WL$-SD G	-1000	10%	-1000	50%	216	43	**57**
$c = 0.1, d = 0.5$	0	80%					
	1000	10%	1000	50%			
TASK VII	**Gain/Loss**	**Prob.**	**Gain/Loss**	**Prob.**	**N**	**F[%]**	**G[%]**
$F\ WL$-SD G	-3000	20%	-3000	50%	208	**61**	39
$c = 0.2, d = 0.5$	0	60%					
	3000	20%	3000	50%			
TASK VIII	**Gain/Loss**	**Prob.**	**Gain/Loss**	**Prob.**	**N**	**F[%]**	**G[%]**
$F\ WL$-SD G	-3000	20%	-3000	50%	208	**64**	36
$c = 0.2, d = 0.5$	-1000	30%					
	1000	30%					
	3000	20%	3000	50%			

preference for prospect F is based therefore solely on loss aversion if one assumes that the pwf is inverse S-shaped. While Tasks VI and VII include a zero payoff and test loss aversion close to the origin of the value function, Task VIII tests loss aversion beyond $\pm\$1000$. We invite the reader to draw the cumulative distributions of F and G and check the cancelation of the "$-$" areas b "$+$" areas.

Results of Tasks VI to VIII

The results of the tasks are presented in the rightmost columns of table 3.5. In Task VI, the majority of subjects prefers G and the hypothesis of a value function that incorporates loss aversion is rejected. In Tasks VII and VIII the results are consistent with loss aversion because the majority of subjects prefers F. One possible explanation for these results is that the assumption about the pwfs ($c = 0.1$ and $d = 0.5$) in Tasks VI is stronger than in Tasks VII and VIII ($c = 0.2$ and $d = 0.5$). However, $c = 0.1$ is empirically plausible. A more plausible explanation is that subjects might be following the heuristic of choosing the prospect with the highest probability of strictly positive gains, according to which G is more appealing in Task VI. Such an explanation would be in line with work by Payne (2005), who finds data that contradicts a parametric specification of CPT, but is strongly consistent with this heuristic. Still, the results of Task VII are at odds with this heuristic. The stakes in Task VII are three times larger than the stakes in Task VI. Thus, we can hypothesize that the degree of loss aversion increases with stakes. There is some additional evidence supporting this hypothesis. We can safely assume that students and professionals differ in their assessment of the stakes so that the same dollar amount is subjectively perceived as larger by MBAs and undergrads than by professionals. In Task VI, MBAs and undergrads prefer G by a slight margin, 52%, whereas the proportion of professionals violating loss aversion is 65%, which is statistically different from 50%. A similar pattern is observed in Task VII: MBAs and undergrads show strong loss aversion and professionals prefer F by only a slight margin. Hence, the heuristic of considering the probability of positive gains may apply only/specially in the presence of small stakes (Payne, 2005, uses hypothetical choices among graduate and undergraduate students, with outcomes varying between -$90 and +$138).

3.5. CPT

Design

In the fourth experimental part we assume that all the empirical specifications of CPT for the value function and the pwf, including loss aversion hold. We design Tasks IX to XII (see table 3.6) with prospect F dominating G according to PWL-SD. Thus we interpret the tasks as a global test of CPT. Specifically, in Task IX Prospect F is preferred if besides exhibiting loss aversion, a decision maker also has a value function that is concave for gains. Only in that case does the positive FSD zone in the losses range cancel the negative zone in the gains range associated with the highest outcome *and* do the remaining zone in the gains range cancel each other (again, we encourage the reader to draw the cumulative distributions of F and G and check the cancelation of the "$-$" areas by "$+$" areas). Tasks X and XI necessitate a value function which is convex for losses to make prospect F be preferred over prospect G. Tasks X and XI require a sophisticated accounting between the extreme positive and negative zones and the remaining negative zones (for Task XI, see graphic 2.4). Finally, Task XII shows an example where the losses-gains cancelation could be done in several ways.

Results of Tasks IX to XII

The results of the tasks are presented in the rightmost columns of table 3.6. In Task IX, the majority of subjects prefers G although not at a statistically significant level (The overall 46-54 response is not statistically different from 50 percent by a one-tailed binomial test at $\alpha = 0.05$). That

TABLE 3.6: Design and Results of Tasks IX through XII

	F		G		Choice		
TASK IX	**Gain/Loss**	**Prob.**	**Gain/Loss**	**Prob.**	**N**	**F[%]**	**G[%]**
F PWL-SD G	-500	10%	-500	30%	209	46	**54**
$c = 0.1$, $d = 0.5$	0	40%	500	20%			
	1500	40%	1000	20%			
	2000	10%	2000	30%			
TASK X	**Gain/Loss**	**Prob.**	**Gain/Loss**	**Prob.**	**N**	**F[%]**	**G[%]**
F PWL-SD G	-2000	30%	-2000	10%	216	**70**	30
$c = 0.1$, $d = 0.7$			-1000	60%			
	0	60%					
	1000	10%	1000	30%			
TASK XI	**Gain/Loss**	**Prob.**	**Gain/Loss**	**Prob.**	**N**	**F[%]**	**G[%]**
F PWL-SD G	-5000	15%	-5000	35%	209	**74**	26
$c = 0.15$, $d = 0.65$	-3000	30%	-1000	30%			
	0	20%					
	3000	20%					
	5000	15%	5000	35%			
TASK XII	**Gain/Loss**	**Prob.**	**Gain/Loss**	**Prob.**	**N**	**F[%]**	**G[%]**
F PWL-SD G	-1500	20%	-1500	50%	177	**77**	23
$c = 0.2$, $d = 0.5$	1500	60%					
	4500	20%	4500	50%			

subjects are not following CPT in this task might be explained by G having a higher probability of strictly positive gains (Payne, 2005). In the other three tasks, X to XII, F is clearly preferred by most of the subjects, in line with the predictions of CPT.

For the seven tasks that show WL-SD or PWL-SD (Tasks VI through XII) we see strong support for CPT on an individual basis: 24% of the individuals follow CPT in all tasks and 76% follow CPT in half or more of their choices.

4. Conclusions and Extensions

THE SD conditions presented in this working paper can be used to test joint hypotheses on the shape of the value function and the pwf (see table 2.1). Making some assumptions on the shape of the pwf one can easily find a simple test for the shape of the value function and viceversa. Thus we supply a potentially powerful framework to suggest experimental designs that isolate certain qualitative features of either the value function or the pwf without having to estimate these functions. This adds a method to the toolbox of an experimentalist who many times does not need to estimate the entire value function or pwf; rather she is interested only in falsifying a specific hypothesis about the shape of either value function or pwf. Clearly, a different method is needed if the entire value function and pwf needs to be explored (Abdellaoui, Bleichrodt and Paraschiv, 2004). Three ways of testing joint hypotheses based on the SD conditions have been presented in the second part of the working paper (see table 3.1).

a) Assuming that the empirical specifications of CPT for the pwf hold, one can test the curvature of the value function and the presence of loss aversion. Examples for the test of curvature are Tasks I through III and for loss aversion Tasks VI through VIII.

b) Assuming that the value function follows the empirical specifications of CPT, one can test the curvature of the pwf. This is what we have done in Tasks IV and V.

c) Assuming that all the empirical specifications of CPT hold, violations of SD actually imply violations of CPT. This way of applying the SD conditions can be very useful to pit CPT against heuristics. This can be the interpretation of Task VI and IX, in which the violation of SD actually suggests that, in those tasks, individuals resort to ad–hoc heuristics.

A restrictive feature of our method is that the designs of the tasks which use the SD notions always imply adding common extreme outcomes. This requires prospects with at least three outcomes, which precludes the use of our SD conditions to test CPT in simpler prospects. In addition, working with the parameters c and d is in itself a drawback compared to other methods of measuring weighting functions (Abdellaoui, 2000). This is rooted in the nature of the extension of the SD-techniques to CPT which require joint curvature conditions that are at odds with the double curvature observed for the pwf. Even so, the parameters c and d do not have to be known exactly but can be chosen from a quite large interval. Furthermore, in our presentation, we focus on a family of pwfs that includes the inverse S-shaped pwf, but that also contains empirically implausible forms. The pwf class could be tightened, e.g., by restricting $\mathcal{W}$ to inverse S-shaped pwfs that cross the 45-degree line from above. We leave for future exploration the SD conditions that characterize such preferences.

Extending our results to the rank-dependent framework is straightforward: Chew, Karni, and Safra (1987) have shown that S-SD applies to rank-dependent models for concave $w = w^-$.

The experimental results in this paper confirm that it is important to account for the pwf, especially in choices where the lower and upper distortions of the pwf tend to reinforce the same prospect (see for example Task IV). In those cases, very small probability changes in the extreme outcomes (here 2%) suffice to reverse preferences of individuals. SD conditions are very adequate to uncover violations of a choice theory, which is a very active area of research (see Wu and Markle, 2004; Birnbaum, In press; Payne, 2005). For example, while we confirm the existence of loss aversion (which clearly shows if the probability of having positive outcomes is similar or the same in both choices), we document violations of loss aversion attributable to subjects using the probability of strictly positive gains as a criteria. This last effect shows specially in tasks with relatively small stakes.

Appendix: Proofs

Proof of proposition 1. $(\Rightarrow)$ Recall that $h'(x) = G(x) - F(x)$ and $v'(x) \geq 0$, so that, in view of (2.1), the goal is to use (2.3) and (2.4) to show that the integration in the range where $h'(x) < 0$ is compensated by the integration in the range where $h'(x) \geq 0$, i.e., $\Delta_{\{x:h'(x)\geq 0\}} + \Delta_{\{x:h'(x)<0\}} \geq 0$. We now define a cancelation function H, that to each point in $\{x : h'(x) < 0\}$ will associate a point $H(x)$ having $h'(H(x)) \geq 0$. If $x \in A$, then let $H(x) = \inf\{x' > x : h(x') = h(x)\}$; and if $x \in B$, then let $H(x) = \sup\{x' < x : h(x') = h(x)\}$. By continuity of h, right-continuity of h', (2.3), and (2.4), H is a well defined, one-to-one function with points in A (resp. B) having their image in A (resp. B). We now claim that in the domain of H, $v'(H(x)) \geq v'(x)$. If $x \in A$, then $H(x) > x$, which together with the convexity of v in A, proves the claim; and if $x \in B$, then $H(x) < x$, which together with the concavity of v in B, proves the claim.
Let D° be the domain of H after having excluded the following exceptional points: a, 0, b, discontinuity points of F or G, and points x where $h'(H(x)) = 0$. A moment's reflection reveals that if a point is not exceptional, then it is in the interior of D°, i.e., D° is open, and that the integration of $\Delta_{[a,b]}$ in exceptional points is zero (they are isolated point of measure zero). Moreover, if $x \in D^\circ$, then $H(x)$, $h'(x)$, and $h'(H(x))$ are continuous, and $h'(H(x)) > 0$. Using $h(x) = h(H(x))$, we have that $h'(x) = h'(H(x))H'(x)$. Hence $H'(x) = h'(x)/h'(H(x))$ is well defined in D°. We now write

$$\int_{H(D^\circ)} h'(x)dx = \int_{D^\circ} h'(H(x))|H'(x)|dx = -\int_{D^\circ} h'(x)dx \tag{4.1}$$

where the first equality follows from the change of variable formula for Lebesgue integrals [Strichartz (1995, p.719)], and the second from $h'(x) = -h'(H(x))\,|H'(x)|$. Using (4.1) and the claim, we conclude that $\Delta_{H(D^\circ)} + \Delta_{D^\circ} \geq 0$.

($\Leftarrow$) Assume (2.3) fails. Hence, $h(x_1) > h(a_1)$ for some $x_1 \in A$. Let

$$v(x) = \min\{0, \max\{x - a_1, x_1 - a_1\}\} \tag{4.2}$$

which is convex in A. Applying (2.1) we find $\Delta_{[a,b]} = h(a_1) - h(x_1) < 0$, a contradiction.
Finally, assume (2.4) fails. Hence, $h(x_0) < h(b_0)$ for some $x_0 \in B$. Let

$$v(x) = \max\{0, \min\{x - b_0, x_0 - b_0\}\} \tag{4.3}$$

which is concave in B. Thus, $\Delta_{[a,b]} = h(x_0) - h(b_0) < 0$.

Proof of proposition 3. Each of the four results is a particular case of proposition 1, after defining the intervals A and B. For example, conditions (2.3) and (2.4), as applied to $A = [a, 0]$ and $B = [0, b]$, agree with the definition of P-SD. Hence $V_F \geq V_G$ for all v convex in $[a, 0]$ and concave in $[0, b]$, which is precisely the $\mathcal{V}_P$ class. To obtain P^*-SD, S-SD, and S^*-SD, use $A = [0, b]$ and $B = [a, 0]$; $A = \varnothing$ and $B = [a, b]$; and $A = [a, b]$ and $B = \varnothing$, respectively.

Proof of proposition 5. The definition of x_c^- ensures that $w^-(p)$ is convex (concave) in points where p is equal to either $F(x)$ or $G(x)$, $x_c^- < x < 0$ ($a \leq x < x_d^-$). Hence, $[w^-(G(x)) - w^-(F(x))]/[G(x) - F(x)]$ is non-decreasing (non-increasing). Of course, $v'(x) \geq 0$ is non-decreasing (non-increasing) in A (B), and the first part of the result follows.
Similarly, the definition of x_c^+ ensures that $w^-(p)$ is concave (convex) in points where p is equal to either $F(x)$ or $G(x)$, $0 \leq x < x_c^+$ ($x_d^+ < x < b$). Hence, $[w^-(G(x)) - w^-(F(x))]/[G(x) - F(x)]$ is non-increasing (non-decreasing). Of course, $v'(x) \geq 0$ is non-increasing (non-decreasing) in B (A), and the second part of the result follows.

Proof of proposition 6. ($\Rightarrow$) We take the function h and repeat the construction of H as in the proof of proposition 1. By (2.13) and (2.14), points in $(a, 0)$ and $(0, b)$ have their image in $[a, 0]$ and $[0, b]$, respectively. Hence, we can let $D^{\circ-} = D^\circ \cap (a, 0)$ and $D^{\circ+} = D^\circ \cap (0, b)$ and, using a change of variable, write

$$\int_{H(D^{\circ-})} h'(x)dx = -\int_{D^{\circ-}} h'(x)dx, \text{ and} \tag{4.4}$$

$$\int_{H(D^{\circ+})} h'(x)dx = -\int_{D^{\circ+}} h'(x)dx. \tag{4.5}$$

We now wish to combine proposition 5 with (4.4) and (4.5) to produce $\Delta^w_{H(D^{\circ-})} + \Delta^w_{D^{\circ-}} \geq 0$ and $\Delta^w_{H(D^{\circ+})} + \Delta^w_{D^{\circ+}} \geq 0$. To do so, we let $\hat{x} = H(x)$ and check the following. If $x \in A \cap D^{\circ-}$, then $\hat{x} > x$ and, from (2.11), both $x, \hat{x} \in A \cap [x_c^-, 0]$. If $x \in B \cap D^{\circ-}$, then $\hat{x} < x$ and both

$x, \hat{x} \in B \cap [a, x_d^-]$. If $x \in A \cap D^{\circ +}$, then $\hat{x} > x$ and, from (2.11), both $x, \hat{x} \in A \cap [x_d^+, b]$. Finally, if $x \in B \cap D^{\circ +}$, then $\hat{x} < x$ and both $x, \hat{x} \in B \cap [0, x_c^+]$. Hence, the desired monotonicity property holds.

($\Leftarrow$) The proof of proposition 1 shows that both (2.3) and (2.4) are necessary. Assume (2.11) fails. Hence, $h'(x_1) < 0$ for $a_0 \leq x_1 < x_c^-$ or $\hat{a}_0 \leq x_1 < x_d^+$. By right-continuity of $h'(x)$, there is an $\varepsilon > 0$ such that $x_1 + \varepsilon < a_1$ and $h'(x) < 0$ for $x_1 \leq x < x_1 + \varepsilon$. Because $\hat{a}_0 \geq 0$, both x_1 and $x_1 + \varepsilon$ have the same sign. If $x_1 < 0$, then let $p_c = \min\{F(x_1 + \varepsilon), c^-\}$ and $p_d = 1$; and if $x_1 \geq 0$, then let $p_c = 0$ and $1 - p_d = \min\{F(x_1 + \varepsilon), 1 - d^+\}$. Let v be as in (4.2), w^- as

$$w(p) = \min\{1, p/p_c\} \tag{4.6}$$

with the understanding that if $p_c = 0$, then $w(p) = 1$, $0 < p \leq 1$; and w^+ as

$$w(p) = \max\{0, (p - p_d)/(1 - p_d)\} \tag{4.7}$$

with the understanding that if $p_d = 1$, then $w(p) = 0$, $0 \leq p < 1$. We check that v is convex in A, w^- convex in $(c^-, 1]$ and concave throughout, and w^+ concave in $[0, d^+)$ and convex throughout. v ensures that only integration in $[x_1, a_1]$ matters. w ensures that points $x > x_1 + \varepsilon$ having $h'(x) > 0$ do not yield a positive contribution because if $x < 0$, then w^- is constant whenever $G(x) > F(x_1 + \varepsilon) \geq p_c$; and if $x \geq 0$, then w^+ is constant whenever $G(x) > F(x_1 + \varepsilon) \geq 1 - p_d$. That $h'(x) < 0$ in $[x_1, x_1 + \varepsilon]$ implies $\Delta^w_{[a,b]} \leq \Delta^w_{[x_1, x_1+\varepsilon]} < 0$, a contradiction.
If (2.12) fails, then $h'(x_1) < 0$ for $x_d^- \leq x_1 < \hat{b}_1$ or $x_c^+ \leq x_1 < b_1$. As before, there is an $\varepsilon > 0$ such that $x_1 + \varepsilon < b_1$ and $h'(x) < 0$ for $x_1 \leq x < x_1 + \varepsilon$. Let $x_0 = x_1 + \varepsilon$. Because $\hat{b}_1 \leq 0$, both x_1 and x_0 have the same sign. If $x_1 < 0$, then let $p_d = \max\{G(x_1), d^-\}$ and $p_c = 0$; and if $x_1 \geq 0$, then let $p_d = 1$ and $1 - p_c = \max\{G(x_1), 1 - c^+\}$. Let v, w^-, and w^+ be as in (4.3), (4.7), and (4.6), respectively. Notice that v is concave in B, w^- concave in $[0, d^-)$ and convex throughout, and w^+ convex in $(c^+, 1]$ and concave throughout. v ensures that only integration in $[b_0, x_0]$ matters. w ensures that points $x < x_1$ having $h'(x) > 0$ do not yield a positive contribution because if $x < 0$, then w^- is constant whenever $p_d \geq G(x_1) > F(x)$; and if $x \geq 0$, then w^+ is constant whenever $1 - p_c \geq G(x_1) > F(x)$. That $h'(x) < 0$ in $[x_0 - \varepsilon, x_0]$ implies $\Delta^w_{[a,b]} \leq \Delta^w_{[x_0-\varepsilon, x_0]} < 0$, a contradiction.
If (2.13) fails, then $h(x_1) > h(0)$ for $a_0 \leq x_1 < 0$. Let v be as in (4.2), which is convex in A, $w^-(p) = p$, and $w^+(p) = 0$, $0 \leq p < 1$, which are linear throughout. (2.5) produces $\Delta^w_{[a,b]} = h(0) - h(x_1) < 0$, a contradiction. Finally, if (2.14) fails, then $h(x_0) < h(0)$ for some x_0,

$0 \leq x_0 < b_1$. Let v be as in (4.3), which is concave in B, $w^-(p) = 0$, $0 \leq p < 1$, and $w^+(p) = p$, which are linear throughout. (2.5) yields $\Delta^w_{[a,b]} = h(x_0) - h(0) < 0$, a contradiction.

Proof of proposition 8. Each of the four results is a particular case of proposition 6, after defining the intervals A and B. For example, letting $A = [a, 0]$ and $B = [0, b]$, we obtain the conditions (2.3), (2.4) of P-SD, and (2.11) implies that $h'(x) \leq 0$ for $a_0 \leq x < x_c^-$ and $x_c^+ \leq x < b_1$, which yields PW-SD. Notice that conditions (2.13) and (2.14) do not apply to P-SD and P^*-SD. To obtain P^*-SD, S-SD, and S^*-SD, use $A = [0, b]$ and $B = [a, 0]$; $A = \varnothing$ and $B = [a, b]$; and $A = [a, b]$ and $B = \varnothing$, respectively.

Proof of proposition 10. $(\Rightarrow)$ If $w \in \mathcal{W}_c^d$, $c < d$, and $1 > G(x) \geq F(x) > 0$, then

$$\frac{w^-(G(x)) - w^-(F(x))}{G(x) - F(x)} \geq s^-, \; a \leq x < 0; \text{ and} \tag{4.8}$$

$$\frac{w^+(1 - F(x)) - w^+(1 - G(x))}{G(x) - F(x)} \geq s^+, \; 0 \leq x < b, \tag{4.9}$$

with equality if $x_d^+ \leq x < x_c^+$. This property of w, together with $s^- \geq s^+$, $v'(x) \geq v'(-x)$ for $x < 0$, and (2.16), produces

$$\begin{aligned}
\Delta^w_{[a,b]} &\geq \int_a^0 s^-[G(x) - F(x)]v'(x)dx + \int_0^b s^+[G(x) - F(x)]v'(x)dx \\
&\geq \int_a^0 s^-[G(x) - F(x)]v'(-x)dy + \int_0^b s^+[G(x) - F(x)]v'(x)dx \\
&\geq \int_0^b s^+[G(-x) - F(-x) + G(x) - F(x)]v'(x)dx \geq 0.
\end{aligned}$$

$(\Leftarrow)$ Condition (2.15) can fail in the following three independent ways. First, assume $G(x_0) < F(x_0)$ for some x_0, $a \leq x_0 < 0$. By right-continuity of $h'(x)$, there is an $\varepsilon > 0$ such that $x_0 + \varepsilon < 0$ and $h'(x) < 0$ for $x_0 \leq x < x_0 + \varepsilon$. Let $x_1 = x_0 + \varepsilon$ and consider $v(x) = \min\{0, \max\{x - x_1, x_0 - x_1\}\}$ which is in $\mathcal{V}_L$; and $w(p) = p$ which are linear throughout. This produces $\Delta^w_{[a,b]} = \Delta^w_{[x_0,x_1]} < 0$.
Second, assume $F(x_0) > G(x_0)$ for some x_0, $0 \leq x < x_d^+$. There exists ε such that $h'(x) < 0$ for $x_0 \leq x < x_0 + \varepsilon$. Let $1 - p_d = \min\{F(x_0 + \varepsilon), 1 - d^+\}$ and consider $v(x) = x - x_0$ if $x \geq x_0$; $v(x) = 0$ if $-x_0 \leq x < x_0$; and $v(x) = x + x_0$ if $x < -x_0$; $w^-(p) = 0$, $0 \leq p < 1$; and w^+ as in (4.7). v is in $\mathcal{V}_L$, w^- is concave in $[0, 1)$ and convex throughout, and w^+ is concave in $[0, d^+)$ and convex throughout.

Moreover, $s^- = s^+ = 0$. By construction, $\Delta^w_{[a,b]} \leq \Delta^w_{[x_0,x_0+\varepsilon]} < 0$, which yields the desired contradiction.
Third, assume $F(x_0) > G(x_0)$, $x_c^+ \leq x_0 < b$. There exists ε such that $h'(x) < 0$ for $x_0 \leq x < x_0 + \varepsilon$. Let $x_1 = x_0 + \varepsilon$ and consider $v(x) = \max\{-x_1, \min\{x, x_1\}\}$. Let , $w^-(p) = 0$, $0 \leq p < 1$. Finally, let $1 - p_c = \max\{G(x_0), 1 - c^+\}$ and w^+ as in (4.6). v is in $\mathcal{V}_L$, w^- is concave in $[0, 1)$ and convex throughout, and w^+ convex in $(c^+, 1]$ and concave throughout. Moreover, $s^- = s^+ = 0$. By construction, $\Delta^w_{[a,b]} \leq \Delta^w_{[x_0,x_1]} < 0$, a contradiction.

Finally, assume (2.15) holds but (2.16) fails. Then, $0 \leq G(-x_0) - F(-x_0) < F(x_0) - G(x_0)$ for some x_0, $x_d^+ \leq x_0 < x_c^+$. Notice that $x_0 > 0$ and that F and G are continuous at x_0. Hence, there is an $\varepsilon > 0$ such that $G(-x) - F(-x) < F(x) - G(x)$ for $0 < x_0 - \varepsilon \leq x_0 \leq x_0 + \varepsilon]$. Define $v(x) \in \mathcal{V}_L$ using continuity, $v(0) = 0$, $v'(x) = 1$ for $x \in [x_0, x_0 + \varepsilon] \cup [-x_0 - \varepsilon, -x_0]$, and 0 otherwise; and $w(p) = p$. Then, $\Delta^w_{[a,b]} = \int_{-x_0-\varepsilon}^{-x_0} [G(t) - F(t)]dt + \int_{x_0}^{x_0+\varepsilon} [G(t) - F(t)]dt < 0$.

Proof of proposition 12. ($\Rightarrow$) First let

$$E^+ = \{x \geq 0 : h(x) \geq h(\hat{x}) \text{ for some } \hat{x},\ 0 \leq \hat{x} < x\},$$

which will be the set where the gains-gains cancelation takes place (the losses-losses cancelation takes place in E^-). We define the cancelation function $H_L(x)$, whose domain are the points having $h'(x) < 0$, as follows. If $x \in [a, 0]$, then let $H_L(x) = \inf\{x' > x : h(x') = h(x)\}$; if $x \in E^+$, let $H_L(x) = \sup\{x' < x : h(x') = h(x)\}$; and if $x \in [0, b]\backslash E^+$, then let $H_L(x) = \sup\{x' \in [a, 0]\backslash E^- : h(x') = h(x)\}$. As argued in the proof of proposition 1, $H_L'(x)$ is well defined on D°, an open dense set of $\{x : h'(x) < 0\}$. Let $\hat{x} = H_L(x)$. We now claim that $v'(\hat{x}) \geq v'(x)$, $x \in D^\circ$. By (2.17), if $x \in [a, 0]$, then $x < \hat{x} \in E^-$; and if $x \in E^+$, then $x > \hat{x} \in E^+$. Because $v \in \mathcal{V}_P$, the claim holds in $[a, 0] \cup E^+$. Finally, if $x \in [0, b]\backslash E^+$, then $\hat{x} \in [a, 0]\backslash E^-$ and, by (2.20), $-x \leq \hat{x} \leq 0$. Because $v \in \mathcal{V}_L$, the claim holds in $[0, b]\backslash E^+$. If $\hat{x} < 0 \leq x$, then notice that, by (2.18) and (2.19), $x_d^+ \leq x < x_c^+$. Hence, from (4.8) and (4.9),

$$\frac{w^-(G(\hat{x})) - w^-(F(\hat{x}))}{G(\hat{x}) - F(\hat{x})} \geq s^- \geq s^+ = \frac{w^+(1 - F(x)) - w^+(1 - G(x))}{G(x) - F(x)}. \tag{4.10}$$

Using the change of variable formula, $v'(\hat{x}) \geq v'(x)$, and (4.10) produces $\Delta^w_{H_L(D^\circ)} \geq \Delta^w_{D^\circ}$.

($\Leftarrow$) The proof of proposition 1 shows that (2.17) is necessary; and the proofs of propositions 6 and 10 show that (2.19) is necessary for

$a \leq x < x_c^-$ and $x_c^+ \leq x < b$, respectively. If condition (2.18) fails, then $h(x_0) < h(0)$ for some x_0, $0 \leq x_0 < x_d^+$. By continuity, $x_0 > 0$, and by differentiability, one can assume that $h'(x_0) < 0$, so that $F(x_0) > 0$. Let $v(x) = \max\{-x_0, \min\{x, x_0\}\}$, $w^-(p) = 0$, $0 \leq p < 1$, and w^+ as in (4.7) using $1 - p_d = \min\{F(x_0), 1 - d^+\}$. We check that $v \in \mathcal{V}_{PL}$, w^- is linear, and w^+ is concave in $[0, d^-)$ and convex throughout, and that $s^- = s^+ = 0$. If $d^+ < 1$, then $p_d < 1$ and, by construction, $\Delta^w_{[0,x_0]} = [h(x_0) - h(0)]/(1 - p_d) < 0$; and if $d^+ = 1$, then $\Delta^w_{[0,x_0]} = \int_0^{x_0} -F(x)dx < 0$, a contradiction. Finally, assume (2.20) fails. Then, $h(-x_1) < h(x_1)$ for some $x_1 < 0$ and $x_1 \notin E^-$. Consider $v(x) = \max\{x_1, \min\{x, -x_1\}\} \in \mathcal{V}_{PL}$ and $w(p) = p$. By construction, $\Delta^w_{[a,b]} = h(-x_1) - h(x_1) < 0$, a contradiction.

References

ABDELLAOUI, M. (2000): "Parameter-free Elicitation of Utility and Probability Weighting Functions", *Management Science*, 46(11), 1497–1512.

— H. BLEICHRODT and C. PARASCHIV (2004): "Measuring Loss Aversion: A Parameter-free Approach", Working Paper.

— F. VOSSMANN and M. WEBER (In Press): "Choice-Based Elicitation and Decomposition of Decision Weights for Gains and Losses under Uncertainty", *Management Science*.

BAUCELLS, M. and F. HEUKAMP (2004): "Reevaluation of the Results of Levy and Levy (2002a)", *Organizational Behavior and Human Decision Processes*, 94(1), 15–21.

BENARTZI, S. and R. THALER (1995): "Myopic Loss Aversion and the Equity Premium Puzzle", *Quarterly Journal of Economics*, 110(1), 75–92.

BIRNBAUM, M. (In press): "Three New Tests of Independence that Differentiate Models of Risky Decision Making", *Management Science*.

BOWMAN, D., D. MINEHART and M. RABIN (1999): "Loss Aversion in a Consumption-Savings Model", *Journal Economic Behavior and Organization*, 38(2), 155–178.

CAMERER, C. (1995): "Individual Decision Making", in *Handbook of Experimental Economics*, ed. by J. Kagel, and A. Roth, 617–703, Princeton University Press.

— and R. HOGARTH (1999): "The Effects of Financial Incentives in Experiments: A Review and Capital-Labor-Production Framework", *Journal of Risk and Uncertainty*, 19(1-3), 7–42.

CHEW, S., E. KARNI and Z. SAFRA (1987): "Risk Aversion in the Theory of Expected Utility with Rank Dependent Probabilities", *Journal of Economic Theory*, 42(2), 370–381.

GONZALEZ, R. and G. WU (1999): "On the Shape of the Probability Weighting Function", *Cognitive Psychology*, 38(1), 129–166.

KAHNEMAN, D. and A. TVERSKY (1979): "Prospect Theory: An Analysis of Decision under Risk", *Econometrica*, 47(2), 263–291.

LANGER, T. and M. WEBER (2001): "Prospect Theory, Mental Accounting and Differences in Aggregated and Segregated Evaluation of Lottery Portfolios", *Management Science*, 47(5), 716–733.

LATTIMORE, P., J. BAKER and A. WITTE (1992): "The Influence of Probability on Risky Choice - A Parametric Examination", *Journal of Economic Behavior and Organization*, 17, 377–400.

LEVY, H. (1992): "Stochastic Dominance and Expected Utility: Survey and Analysis", *Management Science*, 38(4), 555–593.

— and M. LEVY (2002a): "Experimental Test of the Prospect Theory Value Function: A Stochastic Dominance Approach", *Organizational Behavior and Human Decision Processes*, 89(2), 1058–1081.

— and M. LEVY (2002b): "Prospect Theory: Much ado about nothing?", *Management Science*, 48(10), 1334–1349.

— and Z. WIENER (1998): "Stochastic Dominance and Prospect Dominance with Subjective Weighting Functions", *Journal of Risk and Uncertainty*, 16(2), 147–163.

PAYNE, J. (2005): "It is whether you win or lose: The importance of the overall probabilities of winning or losing in risky choice", *Journal of Risk and Uncertainty*, 30, 5–19.

PRELEC, D. (1998): “The Probability Weighting Function”, *Econometrica*, 66(3), 497–528.
QUIGGIN, J. (1993): *Generalized Expected Utility Theory/The Rank-Dependent Expected Utility model*, Kluwer-Nijhoff, Amsterdam.
ROTHSCHILD, M. and J. STIGLITZ (1970): “Increasing Risk. I. A Definition”, *Journal of Economic Theory*, 2, 225–243.
SCHMIDT, U. and S. TRAUB (2002): “An Experimental Test of Loss Aversion”, *Journal of Risk Uncertainty*, 25(3), 233–249.
STRICHARTZ, R. (1995): *The Way of Analysis*, Jones and Bartlett Publishers.
TVERSKY, A. and D. KAHNEMAN (1992): “Advances in Prospect Theory - Cumulative Representation of Uncertainty”, *Journal of Risk and Uncertainty*, 5, 297–323.
— and P. WAKKER (1995): “Risk Attitudes and Decision Weights”, *Econometrica*, 63(6), 1255–1280.
WAKKER, P. (2003): “The Data of Levy and Levy (2002), "Prospect Theory: Much Ado about Nothing?" Actually Support Prospect Theory”, *Management Science*, 49(7), 979–981.
WAKKER, P. and A. TVERSKY (1993): “An Axiomization of Cumulative Prospect Theory”, *Journal of Risk and Uncertainty*, 49(7), 147–176.
WU, G. and A. MARKLE (2004): “An Empirical Test of Gain-Loss Separability in Prospect Theory”, Working Paper.
— J. ZHANG and R. GONZÁLEZ (2004): “Decision under Risk”, in *Blackwell Handbook of Judgment and Decision Making*, ed. by D. Koehler, and N. Harvey, 399–423. Blackwell, Oxford.

ABOUT THE AUTHORS*

MANEL BAUCELLS ALIBÉS is associate professor at IESE Business School (Barcelona). His education includes a degree in mechanical engineering (Polytechnic University of Catalonia, Barcelona), an MBA (IESE, Barcelona), and a PhD in management (University of California at Los Angeles, 1999). For two academic years (2001-03), he held a double appointment as assistant professor at IESE and adjunct professor at the Fuqua School of Business, Duke University. He won the student paper competition of the Decision Analysis Society in 2001, and mentored the winner of the competition in 2003. He is a council member of the Decision Analysis Society of INFORMS. He has taught decision analysis in MBA and executive education programs in Spain, the United States, Germany and Nigeria. His research interests cover both normative and descriptive aspects of decision making. He is associate editor for the journals *Management Science* and *Operations Research*.
E-mail: mbaucells@iese.edu

FRANZ H. HEUKAMP received his PhD from the Massachusetts Institute of Technology (MIT) and has been assistant professor of management at IESE Business School. His research has focused on engineering mechanics and behavioral decision making under uncertainty. He has published articles in leading professional journals, such as *Organizational Behavior and Human Decision Processes* and *International Journal of Solids and Structures*. He received the 2003 student paper award of the Decision Analysis Society and was a Hugh Hampton Scholar during his PhD at MIT.
E-mail: fheukamp@iese.edu

Any comments on the contents of this paper can be addressed to Manel Baucells at mbaucells@iese.edu.

* The authors wish to thank Robin Hogarth (UPF), the Decision Sciences group at Duke, the EURO/INFORMS Istanbul conference, the INFORMS meeting Atlanta and the BDRM 2004 conference participants for helpful discussions; and Javier Gómez-Biscarri, Jordi Utgés, César Beltrán de Miguel, Tony Salvia (all IESE Business School) and Javier Vidal-Quadras for their support in conducting the experiments. Manel Baucells is also grateful to the BBVA Foundation for financial support.

Fundación BBVA

D O C U M E N T O S   D E   T R A B A J O

NÚMEROS PUBLICADOS

DT 01/04 *Métodos de solución de problemas de asignación de recursos sanitarios*
Helena Ramalhinho Dias Lourenço y Daniel Sierra de la Figuera

DT 01/05 *Licensing of University Inventions: The Role of a Technology Transfer Office*
Inés Macho-Stadler, David Pérez-Castrillo, Reinhilde Veugelers

DT 02/05 *Estimating the Intensity of Price and Non-Price Competition in Banking: An Application to the Spanish Case*
Santiago Carbó Valverde, Juan Fernández de Guevara Radoselovics, David Humphrey, Joaquín Maudos Villarroya

DT 03/05 *Sistemas de pensiones y fecundidad: Un enfoque de generaciones solapadas*
Gemma Abío Roig y Concepció Patxox Cardoner

DT 04/05 *Análisis de los factores de exclusión social*
Joan Subirats i Humet (Dir.), Ricard Gomà Carmona y Joaquim Brugué Torruella (Coords.)

DT 05/05 *Riesgos de exclusión social en las Comunidades Autónomas*
Joan Subirats i Humet (Dir.), Ricard Gomà Carmona y Joaquim Brugué Torruella (Coords.)

DT 06/05 *A Dynamic Stochastic Approach to Fisheries Management Assessment: An Application to some European Fisheries*
José M. Da-Rocha Álvarez y María-José Gutiérrez Huerta

DT 07/05 *The New Keynesian Monetary Model: Does it Show the Comovement between Output and Inflation in the U.S. and the Euro Area?*
Ramón María-Dolores Pedrero y Jesús Vázquez Pérez

DT 08/05 *The Relationship between Risk and Expected Return in Europe*
Ángel León Valle, Juan Nave Pineda y Gonzalo Rubio Irigoyen

DT 09/05 *License Allocation and Performance in Telecommunications Markets*
Roberto Burguet Verde

DT 10/05 *Procurement with Downward Sloping Demand: More Simple Economics*
Roberto Burguet Verde

DT 11/05 *Technological and Physical Obsolescence and the Timing of Adoption*
Ramón Caminal Echevarría

DT 01/06 *El efecto de la inmigración en las oportunidades de empleo de los trabajadores nacionales: Evidencia para España*
Raquel Carrasco Perea, Juan Francisco Jimeno Serrano y Ana Carolina Ortega Masagué

DT 02/06 *Inmigración y pensiones: ¿Qué sabemos?*
José Ignacio Conde-Ruiz, Juan Francisco Jimeno Serrano y Guadalupe Valera Blanes

DT 03/06 *A Survey Study of Factors Influencing Risk Taking Behavior in Real World Decisions under Uncertainty*
Manel Baucells Alibés y Cristina Rata

DT 04/06 *Measurement of Social Capital and Growth: An Economic Methodology*
Francisco Pérez García, Lorenzo Serrano Martínez, Vicente Montesinos Santalucía y Juan Fernández de Guevara Radoselovics

Documentos de Trabajo

2

Fundación BBVA

Gran Vía, 12
48001 Bilbao
Tel.: 94 487 52 52
Fax: 94 424 46 21

Paseo de Recoletos, 10
28001 Madrid
Tel.: 91 374 54 00
Fax: 91 374 85 22

publicaciones@fbbva.es
www.fbbva.es