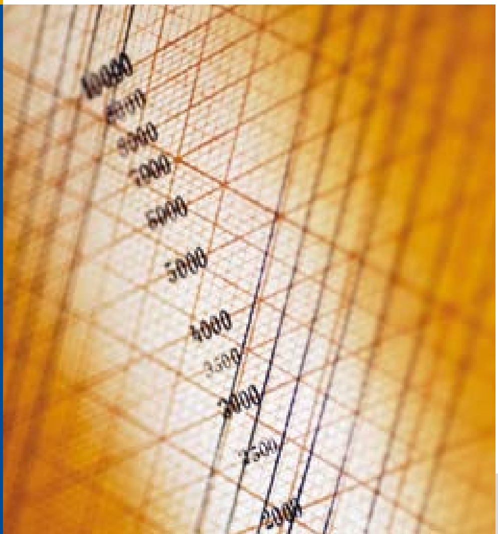
MATERIALES DE REFLEXIÓN



The Political Economy of Democracy

Enriqueta Aragonès
Carmen Beviá
Humberto Llavador
Norman Schofield (Eds.)

Fundación BBVA

THE POLITICAL ECONOMY OF DEMOCRACY

THE POLITICAL ECONOMY OF DEMOCRACY

Edited by
Enriqueta Aragonès
Carmen Beviá
Humberto Llavador
Norman Schofield

Fundación BBVA

First edition, 2009

PUBLISHED BY:
Fundación BBVA
Plaza de San Nicolás, 4. 48005 Bilbao

ISBN: 978-84-96515-91-8
LEGAL DEPOSIT NO.: M-27485-2009

TYPESETTING AND LAYOUT: Ibersaf Editores
PRINTED BY: Ibersaf Industrial, S. L.

Printed in Spain

The books published by the BBVA Foundation are produced with 100% recycled paper made from recovered cellulose fibre (used paper) rather than virgin cellulose, in conformity with the envorinmental standards required by current legislation.

The paper production process complies with European envorinmental laws and regulations and has both Nordic Swan and Blue Angel accreditation.

Contents

Introduction

There are reasons to think that a fourth wave of democratization is coming. There are now more democracies on earth than ever before. Since 1991, no fewer than 40 governments have undertaken the transition to democracy. All these newly democratizing nations and redemocratizing nations, as well as the efforts to create suprastate constitutions, especially that of the European Union, have made more relevant and necessary than ever to understand legislative procedures and alternative political constitutions. The historical formal split into the distinct studies of political sciences biased the way economists and political scientists approached many questions and placed artificial constraints on the study of many important social issues. Thus, the importance of a unified study of political economy that explores the frontiers of the interaction between politics and economics has become nowadays an unavoidable necessity. The characterization of political economy as a synthesis of fields will provide sparks and an exciting research agenda for enlightening our understanding of democracies.

The workshop on "The Political Economy of Democracy", held in Barcelona on 5-7 June 2008 under the sponsorship of the BBVA Foundation, brought together intellectual leaders from economics and political science to obtain a balanced understanding of common topics of analysis, such as pre-electoral maneuvering, elections, coalition building and governance, within a single comprehensive framework. Particular attention was devoted to fields of active development such as endogenous entry of candidates, politicians' and voters' behavior, negotiations and agreements, and political regimes.

In the current political economy literature, citizen-candidate models provide a framework to address the issue of endogenous entry of candidates. Since the seminal papers of Osborne and Slivinski (1996) and Besley and Coate (1997), citizen-candidate models have typically suffered from multiple equilibria and thus, they lack clean empirical predictions. Grosser and Palfrey study a citizen-candidate model with *private information* about the candidates' preferred policies. Introducing private information has the advantage of providing sharp predictions and allows the making of comparative statics. These show that as the entry costs increase or the benefits from holding office decrease, there are fewer entrants and the candidates are more extreme on average.

With a different approach, Elbittar and Gomberg argue that many of the problems with testing the citizen-candidate model in the field can be easily overcome in an experimental lab. They report some preliminary results of a laboratory study of the citizen-candidate model. In particular, they observe a certain degree of excess entry even from the hopeless positions.

Also in a model of endogenous candidates, Levy and Razin consider the role that money plays in elections. In their case candidates must compete and win an all pay auction in order to have the possibility of advertising their position to voters. Only advertised policies are relevant for voters, determining the agenda of the electoral debates. Extremist politicians will face a fierce competition once they advertise their position. This deters them from investing too much and allows moderates to gain an advantage.

Iaryczower and Mattozzi develop a simple theoretical framework in which not only the number of candidates running for office but also the candidates' quality are endogenous. Analyzing proportional representation electoral systems, they provide conditions under which elections would result in a positive association between the quality and the number of candidates running for office.

The idea that political contenders are characterized by the "valence" or electorally perceived quality is key to Schofield's stochastic model of elections. In his model, resources provided by activist groups may be used to enhance the contenders' valence. This model is used to explain party positions in the Netherlands, Britain and the United States. It is suggested that the plethora of parties in polities based on proportional representation is a consequence of the low returns to activist group coalescence in such systems. The model can also be used to examine the possibility of transition from an autocratic system to democracy.

Understanding autocratic regimes and how they may be transformed into democratic ones is the topic of Boix and Svolik. They explore how power is structured and exercised in different authoritarian arrangements, going beyond tyrannical rule to describe a broad range of outcomes in the universe of authoritarian regimes. They argue that the autocrat's use of elections to construct a parliamentary support coalition can make the autocracy more durable and less susceptible to economic downturn.

Madison, in *The Federalist No. 10* hoped that the concerns of the people would engender a probability of a fit choice, when transforming votes to seats, rather than being perverted by selfish interests. Bouton and Castanheira construct a model where there are swing (or independent) voters whose preferences depend on the state of the world. The other voters are stalwarts (or partisans) who prefer one or other of the parties independently of the state of the world. They show that, as Duverger (1954) postulated, under plurality rule there is an incentive for voters to coordinate on a strong candidate.

On the other hand, Colomer and Llavador argue that politicians may disregard important issues for the electorate. They present a model in which two parties compete by choosing the issues that will key out their campaigns. Parties trade off the issues with high salience in voters' concerns and those with broad consensus on some policy proposal. They show that the most important issue for a majority of the electorate may not be given political salience if there is not sufficient consensus on the alternative to the status quo.

Both electoral candidates and office holders devote substantial resources to gathering information about voters through private polling. Bernhardt, Duggan, and Squintani review some of the recent growing literature on rational choice theory exploring the implications of polling in elections and policy choice of office holders. They address the strategic incentives of polled citizens to report honestly, and the possibly adverse welfare effects of public polling.

In the 2008 Democratic Party primary race in the United States, Barack Obama obtained relatively more delegates through caucuses than his opponent, Hillary Clinton. A caucus is a more time consuming process than a usual election, and it can be expected that voters with a higher intensity of support will be willing to accept the higher caucus costs. Epstein, Morelli and O'Halloran examine the impact of higher variance of intensity of support to a candidate's spending patterns and to his or her electoral success in such a system and show that the theoretical predictions capture the differentials in the Democratic party primary race.

The essence of democracy is that the opinions or beliefs of the electorate are turned into votes. Electoral systems transform these votes into a set of representatives of the people. The comparison of the outcomes obtained with different electoral systems is analyzed in the next two contributions.

Lizzeri and Persico compare proportional and winner-take-all systems by the degree to which candidates are willing to offer risky proposals to the electorate. They show that in general candidates under plurality rule are more likely to offer risky and non-centrist proposals. Because the proportional system gives voters less variety in policy platforms, it is more likely to give rise to the "excessive sameness" hypothesized by Hotelling (1929).

Colomer returns to the classical Montesquieu-Madison arguments on the "probability of a fit choice" in the small as against the large Republic. He argues that globalization has meant that relatively small states can be economically viable. Observing that half of the democratic regimes are either presidential or semi-presidential and a third have proportional electoral systems, he infers that proportional representation leads to a stable, risk-avoiding political strategy for both leaders and voters, particularly in small polities.

An important aspect of democratic theory is the attempt to understand political actors' conduct and its impact on outcomes. The analysis of the politicians' behavior is the focus of the contributions that follow.

Querubin and Snyder present evidence on the actual magnitudes of endogenous "rents" from office. Using data on wealth of individuals who served in the U.S. House of Representatives during the period 1848-1875, they find no evidence of large returns for the 1850s or late 1860s. However, they do find evidence of significant returns for the early 1860s, suggesting that although returns to a seat in the House were low during "normal" times, they increased during the Civil War years—perhaps coincidentally, a period of dramatically higher federal spending.

Analysis of roll call data is one of the standard techniques used to infer legislators' preferred policy positions. It is usually the case that missing votes are treated as being identical to abstentions. Rosas and Shomer warn that such an assumption can lead to bias in interpretation, and thus misleading inferences about true preferences.

Roemer considers competition between two parties where the policy space can be interpreted in terms of taxation. Working on an infinite-dimensional space of possible policies and ruling out simplifying mathematical assumptions, such as linearity, he justifies the use of certain rules of thumb by politicians in the tax policy game as a way of dealing with complicated policy choices.

The assumption that the winner chooses the policy is the baseline case of most political models. However, in legislative bodies, legislators typically negotiate over coalition formation and policymaking. The next set of authors contributes to this literature.

In particular, Eguia studies the incentives of an assembly to coalesce, forming voting blocs to vote together under a common ideology to affect the policy outcome. Working on a two dimensional policy space, he finds that if one agent holds the monopoly of forming blocs, a single voting bloc forms and the policy outcome moves away from the Condorcet winner and away from the status quo policy. On the other hand, if any subset of agents can form a voting bloc at least two voting blocs form in equilibrium.

Aragonès and Dellunde combine bargaining models with the theory of automata in an attempt to understand the complex process of negotiation over coalition bargaining in a legislature. As is usual in such bargaining models, a *formateur* starts the process by offering particular combinations of policies and cabinet positions to a proto-coalition. The recipients of the proposal may either accept it, in which case the government forms, or reject it, in which case there is another round of negotiation. The advantage of using automata theory to simulate this process is that it allows to understand the effect of different negotiation rules and behaviors on the final outcome.

Within a classical one dimensional bargaining game with a randomly chosen proposer or *formateur*, Cardona and Ponsati show existence of an equilibrium set that depends on the weighted rule chosen. They also examine the stability of super majority rules finding that, in symmetric environments, the unanimity rule is uniquely stable. More generally they provide support to the pervasive prevalence of strict super majority requirements.

Negotiation and agreement analysis have also proved useful in the analysis of international politics. Beviá and Corchón apply it to a peace-war model to answer the question: how is it that countries come up to a peaceful coexistence when war is an option? Peace treaties might be signed, but in general, it is not clear why players have incentives to stick to the agreement. They study the validity of transfers to stop war when agents cannot commit to any action post-transfer, finding that the transfer mechanism embeds incentives to fulfill a peace treatment without any need for external enforcement.

Finally, Gailmard, Patty and Penn show that, restricted to the domain of single-peaked preferences, satisfaction of independence of irrelevant alternatives implies that the preference aggregation rule is neutral. Neutrality means that the rule must treat all alternatives equally. In particular, if there is a privileged outcome, such as a status quo, then the rule cannot be neutral. They conclude by pointing out that several institutional features violate neutrality, including bicameralism, gatekeeping powers, supermajority requirements, and veto power.

1

A Citizen-Candidate Model with Private Information

Jens Großer
Departments of Political Science and Economics
Florida State University

Thomas R. Palfrey
Division of the Humanities and Social Sciences
California Institute of Technology

1.1. Introduction

We study a citizen-candidate model with *private information* about the candidates' preferred policies (or, ideal points). By contrast, in the seminal models of Osborne and Slivinski (1996) and Besley and Coate (1997), and most citizen-candidate models that have followed, the candidates' and all citizens' ideal points are assumed to be common knowledge. In the baseline model, a community is about to elect a new leader to implement a policy decision. Each citizen may enter the electoral competition as a candidate at some commonly known cost. Because each candidate's preferred policy is public information, she cannot credibly promise any other than this policy in case of being elected. Anticipating this, citizens prefer the candidate whose ideal point is closest to their own ideal point, possibly themselves. Osborne and Slivinski assume a continuum of citizens (i.e., potential candidates) and sincere voting. That is, citizens vote for the most preferred candidate. Besley and Coate assume a finite number of citizens and strategic voting (i.e., a Nash equilibrium in undominated strategies for the voting subgame). They identify a variety of different kinds of equilibria supporting different numbers of entrants, and show how the set of equilibria depends on the distribution of ideal points as well as the entry costs and benefits from holding office. For most environments, there are multiple equilibria. Both median and non-median policy outcomes can be supported in equilibrium.

The citizen-candidate model makes an important departure from the Hotelling-Downs model of spatial competition because it provides a framework to address questions of endogenous entry of candidates (or parties) when these candidates have preferences over policy outcomes.[1] Importantly, in this model the configuration of equilibrium candidate policies must resist the potential entry of any citizen as a candidate, given the restrictions on the entry costs and spoils of office. Moreover, standard spatial competition models assume candidates without own ideal points and let them float in the policy space in order to maximize their chances of being elected. By contrast, in the baseline model, citizen-candidates have their own ideal points and these coincide with their policy promises.

However, the assumption of common knowledge about citizen-candidates' ideal points is restrictive. For example, it seems to be common that candidates' stands on issues that are not yet foreseen (e.g., unexpected outbreaks of conflicts) are uncertain, and the community may observe unexpected policy decisions when these issues come up. Moreover, extremist candidates may have strong incentives to disguise their actual preferences until they are in power. For example, after becoming the leader of the Communist Party of the Soviet Union, Mikhail Gorbachev surprised many of his comrades with a policy that opened up for the West. In this paper we study such uncertainties by introducing private information about citizen-candidates' ideal points. This approach has another advantage in that it has sharp predictions: citizen-candidate models typically suffer from multiple equilibria and do not have clean empirical predictions.[2]

In this paper we develop a citizen-candidate model with a finite (possibly small) number of citizens whose ideal points are iid draws from a continuous uniform distribution on the policy space and private information. We look at symmetric equilibria in the entry stage of the model, and prove that they always exist and are always unique. There is never a symmetric equilibrium with only "moderate" types entering (moderate in the sense of smaller distances between their ideal points and the median ideal point, as compared to "extreme" types). The equilibrium has the property that if a citizen enters if her ideal point is x, then she also enters when her ideal point is more extreme than x. This unique equilibrium implies a unique probability distribution of the number of entrants, and we are able to obtain comparative statics about how this distribution changes with the underlying parameters of the model: community size, entry costs,

[1] The citizen-candidate models have their roots in the earlier work on strategic entry, models related to Duvergers' law, and models with policy motivated candidates. See for example Palfrey (1984), Wittman (1983), Palfrey (1989), Feddersen et al. (1990), Feddersen (1992), and Osborne (1993).

[2] See for example Roemer (2003), Dhillon and Lockwood (2002), and the references they cite.

and benefits from holding office. As the entry costs increase or the benefits from holding office decrease, there are fewer entrants in the sense of first order stochastic dominance, and the candidates are more extreme on average. As the number of citizens increases, candidates are again more extreme on average but the effect on the number or entrants is ambiguous. A more general account of our citizen-candidate model with private information, including complete proofs is given in Großer and Palfrey (2009), where various symmetric and asymmetric distributions of ideal points and other extensions are analyzed.

Several papers have begun to explore the effects of uncertainty on citizen-candidate equilibria, in several different ways. Due to space constraints in this volume, we can only mention some relevant literature. Eguia (2007) allows for uncertain turnout and shows how this can reduce somewhat the set of equilibria in the Besley and Coate model. Fey (2007) uses the Poisson game approach to study entry where there is an uncertain number of citizens. Brusco and Roy (2007) add aggregate uncertainty, allowing for shifts in the distribution of ideal points. Casamatta and Sand-Zantman (2005) study a model with private information and three types of citizens, and analyze the asymmetric equilibria of the resulting coordination game. Osborne et al. (2000) present a model, though not a citizen-candidate model, where extreme types participate in costly meetings.

Section 1.2 describes our citizen-candidate model with private information and a uniform distribution of ideal points. Section 1.3 characterizes the equilibrium of the model. Section 1.4 derives comparative statics, and illustrates these using some examples with specific parameter values. Section 1.5 briefly discusses concave payoff functions. Section 1.6 discusses possible extensions and concludes.

1.2. The model

A community of $n \geq 2$ citizens is electing a leader to implement a policy decision. The policy space is represented by the $[-1,1]$ interval of the real line. Each citizen, $i = 1,..., n$, has preferences over policies, which are represented by a utility function that is linearly decreasing in the Euclidean distance between the policy decision and her ideal point, $x_i \in -[1,1]$. An individual's ideal point is private information, so only citizen i knows x_i. Ideal points are uniformly distributed according to the continuous cumulative distribution function F, with $F(x) = \frac{1+x}{2}$, $x \in [-1,1] \subset R$, and this distribution is common knowledge. The game for implementing a policy decision proceeds in four stages. In the first stage (*Entry*), all citizens decide simultaneously and independently on whether to bear the entry cost $c \geq 0$ and run for office, $e_i = 1$, or not run, $e_i = 0$. The number of citizen-candidates is denoted by $m \equiv \sum_{i=1}^{n} e_i$. In the second stage (*Policy promises*), each candidate publicly announces a non-binding policy promise. If $m = 0$, a default policy, δ, is implemented according to a commonly known (possibly stochastic) procedure, Δ. In the third stage (*Voting*), each citizen i makes a costless decision about whether to abstain from voting or to vote for one of the candidates, possibly for herself. The new leader is determined by simple majority rule (with random tie breaking) and announced publicly. In the final stage (*Policy decision*), the leader implements a policy $\gamma \in -[1,1]$. Then, each citizen 's total payoff in the game is given by

$$\pi_i\,(x_i, \gamma, e_i, w_i) = -\,|\,x_i - \gamma\,| - ce_i + bw_i, \tag{1.1}$$

where $w_i = 1$ if citizen i is elected as the new leader, in which case she receives private benefits from holding office, $b \geq 0$. If citizen i is not the new leader, then $w_i = 0$. We assume that citizens maximize their own expected payoffs and face identical entry costs, c, and leadership benefits, b.

1.3. Political equilibrium

To solve our citizen-candidate model with private information, we use sequential equilibrium (henceforth 'political equilibrium') and consider behavioral strategies (Kuhn 1953) for each information set. Our theoretical analysis starts with policy promises, voting, and policy decisions before we proceed in more detail with the citizens' decisions on whether or not to run for office.

"Throughout our analysis, we assume that when a voter is indifferent between two candidates he or she votes for the candidates with equal probability."

Lemma 1.1 *(Policy promises, voting, and policy decisions). In any political equilibrium, (i) policy promises are 'cheap talk'; (ii) each entering candidate is elected with equal probability of* $\frac{1}{m}$*; and (iii) the new leader implements her ideal point,* $\gamma^* = x_i$.

Proof. □ *(iii)*: The only credible policy choice of a new leader is to implement her ideal point, $\gamma^* = x_i$, yielding her a zero payoff loss, $-\,|x_i - \gamma^*| = 0$. *(i):* If there is only one candidate, her policy promise is irrelevant because she will be elected anyway (at least she will vote for herself; see below). If there are two or more candidates, their policy promises are not credible because each candidate has an incentive to increase her chances of being elected by misrepresenting her ideal point (recall that promises are non-binding and preferences are private information). Thus, policy promises are 'cheap talk'. *(ii):* Then, each non-candidate is indifferent between the candidates because she cannot distinguish among their ideal points. Thus, she either abstains from voting or votes for any of the candidates. Moreover, each candidate prefers herself to any other candidate (whose ideal points she cannot distinguish either). This is because any other's ideal point yields her a strict payoff loss with probability one. Thus, each candidate votes for herself.[3]

Lemma 2.1 greatly simplifies the equilibrium analysis in the *entry* stage, to which we turn next. We focus on equilibria in *symmetric* cutpoint strategies, defined by

$$\breve{e}_i = \begin{cases} 0 & if \;\; |x_i| < \breve{x} \\ 1 & if \;\; |x_i| \geq \breve{x}, \end{cases} \tag{1.2}$$

where the cutpoint $\breve{x}$ represents a pair of cutpoint policies $(\breve{x}_l, \breve{x}_r)$ with $\breve{x}_l \leq 0 \leq \breve{x}_r$ (i.e., the subscripts denote their relative positions *l* eft and *r* ight, respectively) and $\breve{x} = |\breve{x}_l| = \breve{x}_r \in 0,1] \in \mathrm{R}$ (i.e., the cutpoint policies are symmetric around $x = 0$). In words, the symmetric cutpoint strategy, $\breve{e}$, determines that all citizens with ideal points equally and more 'extreme' than $\breve{x}$ run for office, and all citizens with ideal points more 'moderate' than $\breve{x}$ do not run.

To derive the equilibrium cutpoint policy, $\breve{x}^*$, we must compare a citizen *i*'s expected payoffs as both a candidate and a non-candidate, given the equilibrium decisions in subsequent stages. Then, citizen *i*'s expected payoff for *entering,* $\breve{e}_i = 1$, is

$$E[\pi_i \mid \breve{e}_i = 1] = \breve{x}^{n-1}\, b$$
$$+ \sum_{m=2}^{n} \binom{n-1}{m-1} (1-\breve{x})^{m-1}\, \breve{x}^{n-m} \left[\frac{b}{m} + \frac{m-1}{m} E[-|x_i - \gamma| \mid \breve{x}] \right] - c, \tag{1.3}$$

where $\Pr(|x_i| \geq \breve{x}) = 1 - \breve{x}$ and $\Pr(|x_i| < \breve{x} = \breve{x}$, for our $F(x) = \dfrac{1+x}{2}$, $x \in [-1,1]$. Moreover, assuming without loss of generality that $x_i \geq 0$, *i*'s expected payoff loss if not being elected is given by

---

[3] Note that the *policy promises* and *voting* stages do not demand any particular decision structure. Specifically, lemma 1 holds for any sequence of decisions and information about these decisions. Also note that voting equilibria exist in which some candidates have larger probabilities of being elected than others. However, our model rules out any kind of coordination prior to entry decisions and, hence, ex ante each candidate has an equal probability of becoming the new leader.

$$E[|x_i - \gamma| \mid \breve{x}] = \frac{1}{2} \frac{\int_{-1}^{-\breve{x}} \frac{1}{2} |x_i - x| dx}{F(-\breve{x})} + \frac{1}{2} \frac{\int_{-\breve{x}}^{-1} \frac{1}{2} |x_i - x| dx}{1 - F(\breve{x})} \quad (1.4)$$

$$= \frac{1}{4(1-\breve{x})} \Bigg[-|x_i + \breve{x}|^2 + |x_i + 1|^2 + |x_i - 1|^2$$

$$+ |x_i - \breve{x}|^2 \times \begin{cases} -1 & if \;\; 0 \le x_i < \breve{x} \\ 1 & if \;\; 0 \le \breve{x} \le x_i \end{cases} \Bigg]$$

$$\text{for } \breve{x} \in [0,1),$$

which accounts for the possibility that the expected policy decision lies in the left or right direction, γ_l or γ_r respectively, with equal probability of one half for each. Note that the default policy takes effect if $\breve{x} = 1$. The first term in expression (1.3) gives the case where i receives b because she is the only candidate, which occurs with probability $\breve{x}^{n-1}$. The second term gives the cases where $m - 1 \ge 1$ candidates enter in addition to herself, which occurs with probability $\binom{n-1}{m-1}(1-\breve{x})^{m-1}\,\breve{x}^{n-m}$ and yields her expected leadership benefits of $\frac{b}{m}$. The summation accounts for all possible $2,\ldots,n$. Moreover, i will not be elected with probability $\frac{m-1}{m}$ and her expected payoff loss for this event is $E[|x_i - \gamma|| \breve{x}]$, given in expression (1.4). Finally, i bears the entry costs, c, independent of how many other candidates enter, which gives the third term in expression (1.3).

In contrast, citizen i's expected payoff for *not entering*, $\breve{e}_i = 0$, is

$$E[\pi_i \mid \breve{e}_i = 0] = \breve{x}^{n-1} E[-|x_i - \delta| \breve{x}]$$

$$+ \sum_{m=2}^{n} \binom{n-1}{m-1} (1-\breve{x})^{m-1}\, \breve{x}^{n-m} E[-|x_i - \gamma||\breve{x}]. \quad (1.5)$$

The first term corresponds to the event where, as herself, no other citizen runs for office, which occurs with probability $\breve{x}^{n-1}$. In this paper, we assume for simplicity that the default policy $\delta = 0$ takes effect. This leads to a very simple expression for payoff losses in the no-entry event, which is independent of $\breve{x}$:

$$E[|x_i - \delta|| \breve{x}] = |x_i|. \quad (1.6)$$

Note that for $\breve{x} = 0$ the default policy is irrelevant, because all citizens enter. The remaining terms in expression (1.5) correspond to the events where $m - 1 \geq 1$ other citizens choose to enter. In contrast to expression (1.3), b does not appear in these terms because i does not enter and therefore never wins.

Finally, it is readily verified that relating expressions (1.3) and (1.5) and rearranging yields the best response entry strategy for a citizen with ideal point x_i if all other citizens are using cutpoint strategy $\breve{e}$, which is to enter if and only if[4]

$$\breve{x}^{n-1}\,[b + |\,x_i\,|] + \sum_{m=2}^{n} \binom{n-1}{m-1} (1-\breve{x})^{m-1}\,\breve{x}^{n-m}\,\frac{1}{m}\,[b + E[|x_i - \gamma|\,|\,\breve{x}]] \geq c, \quad (1.7)$$

where the left-hand and right-hand sides (henceforth *LHS* and *RHS*, respectively) give citizen i's expected net benefits and costs from running for office, respectively. We can use this condition, and our assumptions stated above, to derive the following proposition:

Proposition 1.1 *(Equilibrium entry) There always exists a political equilibrium with a unique symmetric cutpoint policy, $\breve{x}^*$, where each citizen i with $|x_i| \geq \breve{x}^*$ enters the electoral competition as a candidate, $\breve{e}^*_i$, and each citizen i with $|x_i| < \breve{x}^*$ does not enter, $\breve{e}^*_i = 0$. This cutpoint policy is characterized by the following necessary and sufficient conditions:*

(i) If $c \leq \underline{c} \equiv \frac{1}{n}\left[b + \frac{1}{2}\right]$, then $\breve{x}^* = 0$ and $\breve{e}^*_i = 1$, $\forall i$ ("every citizen enters", or $m = n$);

(ii) If $c \geq \bar{c} \equiv b + 1$, then $\breve{x}^* = 1$ and $\breve{e}^*_i = 0$, $\forall i$ ("no citizen enters", or $m = 0$);

(iii) If $\underline{c} < c < \bar{c}$, then $\breve{x}^* \in (0,1)$ and some citizens are expected to enter (or $m \in [0, n]$), where $\breve{x}^*$ is determined by

$$\left(\breve{x}^*\right)^{n-1}\left[b + \breve{x}^*\right] + \sum_{m=2}^{n} \binom{n-1}{m-1} \left(1 - \breve{x}^*\right)^{m-1}\left(\breve{x}^*\right)^{n-m} \times \frac{1}{m}\left[b + \frac{1+\breve{x}^*}{2}\right] = c. \quad (1.8)$$

Proof. We give a sketch here (details are in Großer and Palfrey 2009). Recall our assumptions $F(x) = \frac{1+x}{2}$, $x \in [-1,1]$, $n \geq 2$, $c \geq 0$, and $b \geq 0$. First, we show

[4] We assume, without loss of generality, that indifferent citizen types choose to enter.

that (*i*) to (*iii*) give sufficient conditions for an equilibrium cutpoint strategy, $\breve{e}^*$, to exist. To do so, consider *LHS*(1.7) and note that a change in x_i may only affect $|x_i|$ and $E[|x_i - \gamma|| \breve{x}]$, but no other term. Observe that *LHS*(1.7) is strictly increasing in $x_i \in [0,1]$ unless $x_i = \breve{x} = 0$, in which case it does not change in x_i (this is a situation where everyone enters anyway). This proves that (*i*) to (*iii*) provide sufficient conditions for an equilibrium cutpoint strategy, $\breve{e}^*$, to exist. In fact, because this holds for *any* $\breve{x}$, this establishes that *any* symmetric equilibrium is in cutpoint strategies. This leads to three possible situations.

(*i*): If *LHS*(1.7) is equal to or greater than c for all values of x_i and $\breve{x}$, then the unique equilibrium is for all n citizens to enter. This corresponds to an equilibrium cutpoint policy $\breve{x}^* = 0$. Thus, for this to hold, we simply set $\breve{x} = 0$ and $x_i = 0$ and have only to consider the term $m = n$ in *LHS*(1.7). The inequality condition (7) reduces to

$$\frac{1}{n}\left(b + E[|\breve{x}^* - \gamma||\breve{x}^* = 0]\right) = \frac{1}{n}\left[b + \frac{1}{2}\right] \equiv \underline{c} \geq c.$$

Thus, there is universal entry if and only if $c \leq \frac{1}{n}\left[b + \frac{1}{2}\right]$.

(*ii*): If *LHS*(1.7) is less than or equal to c for all values of x_i and $\breve{x}$, then the unique equilibrium is for no citizen to enter. This corresponds to an equilibrium cutpoint policy $\breve{x}^* = 1$. Thus, for this to hold, we simply set $\breve{x} = 1$ and $x_i = 1$. The inequality condition (1.7) changes and reduces to

$$b + E[|\breve{x}^* - \delta||\breve{x}^* = 1] = b + 1 \equiv \bar{c} \leq c$$

Thus, there is zero entry if and only if $c \geq 1 + b$ (note that the probability that any citizen has an ideal point $x_i = 1$ is equal to zero).

(*iii*): If neither boundary condition in (*i*) or (*ii*) hold, then we have an equilibrium with an interior cutpoint, $\breve{x}^* \in (0,1)$. Note first that equilibrium condition (1.8) is the same as equation (1.7) except substituting $x_i = \breve{x}$ and noticing that $E[|\breve{x} - \gamma|| \breve{x}] = \frac{1 + \breve{x}}{2}$. Next, observe that *LHS*(1.8) is continuous on $x \in [1,1]$ because ideal points are distributed continuously, and it is strictly increasing in $\breve{x} \in (0,1)$. This proves that $\breve{x}^* \in (0,1)$ is unique, because *LHS*(1.8) and *RHS*(1.8) can intersect at most once. To see that this equilibrium exists, recall that $\breve{x} = 0$ yields $\underline{c}$ and $\breve{x} = 1$ yields $\bar{c}$ and note that $\underline{c} = \frac{1}{n}\left[b + \frac{1}{2}\right] < b + 1 = \bar{c}$ for $n \geq 2$ and $b \geq 0$. Thus, for any $c \in (\underline{c}, \bar{c})$ there exists a unique interior equilibrium, $\breve{x}^* \in (0,1)$, according to condition (1.8).

1.4. Comparative statics

In this section, we turn to the comparative statics regarding the effects of changes in n, c, and b on $\breve{x}^*$ when we are in a region of the parameter space where the solution is interior, i.e., $\breve{x}^* \in (0,1)$.

Proposition 1.2 *(Comparative statics). The interior symmetric equilibrium cutpoint policy,* $\breve{x}^* \in (0,1)$*, is strictly increasing in the number of citizens, n, and the entry costs, c; and it is strictly decreasing in the benefits from holding office, b. An increase in* $\breve{x}^*$ *implies that, on average, candidates and policy outcomes become more extreme. It also implies a decrease in expected entry when caused by changes in c and b.*

Proof. See Großer and Palfrey (2009).

Proposition 1.2 states that candidates and policy outcomes are, on average, more extreme in larger communities. It is also a straightforward exercise to show that $\lim_{n\to\infty} \breve{x}(n) = 1$. That is, in very large electorates, only the most extreme citizens will throw their hat in the ring. Of course, this does not imply there is zero entry! The limiting distribution of the number of entrants is fully characterized in Großer and Palfrey (2009).

Proposition 1.2 does not give a comparative static result about the expected number of entrants as a function of the number of citizens, n. In contrast to the increases in the expected number of entrants when c and b change, an increase in the number of citizens can yield either more entry or less entry, on average. To see this, notice that there are two effects on entry that result from increasing from n to $n + 1$. First, there is the direct effect that the number of *potential* candidates (i.e., citizens) has increased by 1. This effect works to increase entry. The second effect is an indirect equilibrium effect, namely that $\breve{x}(n + 1,c,b) > \breve{x}(n,c,b)$, and this goes in the opposite direction. I.e., there is one more potential entrant, but each citizen now enters with a lower probability. Which term dominates will depend on n, c, and b.

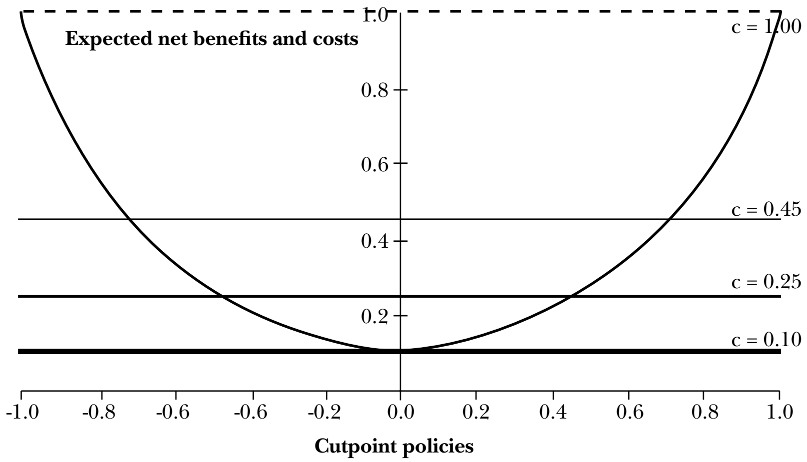
1.5. Examples

We next use specific parametric examples of the uniformly distributed ideal points, x_i, with $F(x) = \frac{1+x}{2}$, $x \in [-1,1]$, to illustrate graphically the key equilibrium properties of our citizen-candidate model with private information.

1.5.1. Variations in the costs of entry, *c*

To illustrate our comparative statics results for changes in the costs of entry, this example uses $n = 5$ and $b = 0$ and varies the costs between $c = 0.10$, 0.25, 0.45, and 1. Figure 1.1 gives the cutpoint policies $\breve{x}_l \in -1,0]$ and $\breve{x}_r \in 0,1$ on the horizontal axis and the expected net benefits and costs from entering as a candidate on the vertical axis (i.e., *LHS*(8) and *RHS*(8), respectively). Expected net benefits are represented by the U-shaped curve and the various costs by horizontal lines.

FIGURE 1.1: **Symmetric cutpoint policy equilibria and variations in entry costs, *c*, for $n = 5$ and $b = 0$**

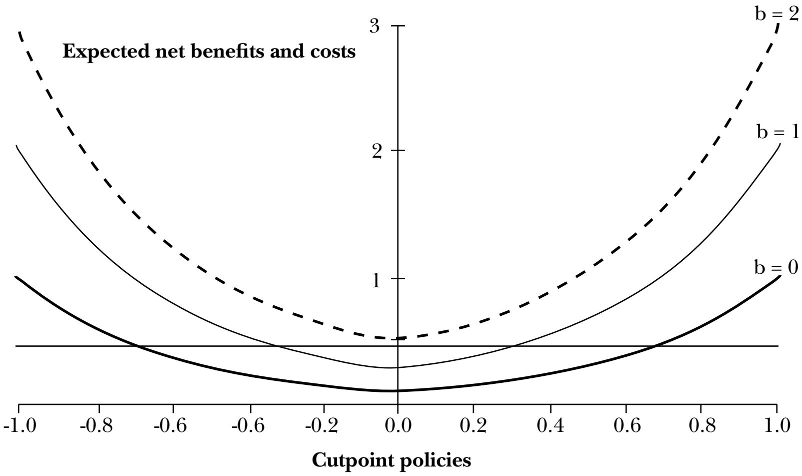


The symmetric cutpoint policy equilibria for the various costs, $\breve{x}^*(c) \in [0,1]$, are determined by the intersections of the expected net benefits curve and the respective cost lines. These equilibria are increasing in *c*, where $\breve{x}^*(c = 0.10) = 0$, $\breve{x}^*(c = 0.25) = 0.455$, $\breve{x}^*(c = 0.45) = 0.713$, and $\breve{x}^*(c = 1) = 1$. Note that $\underline{c} = \frac{1}{5}\left[0 + \frac{1}{2}\right] = 0.1$ and $\bar{c} = 0 + 1 = 1$ for the limit cutpoint policies 0 and 1, respectively (recall from proposition 1 (*i*) and (*ii*) that everyone enters if $c \leq \underline{c}$ and no one enters if $c \geq \bar{c}$). Finally, as a consequence of the increasing $\breve{x}^*$ in *c*, expected policy outcomes in each direction left and right become more extreme (minus and plus 0.5, 0.727, 0.857, and 1 for our ascending *c*, respectively) and expected entry decreases (5, 2.726, 1.434, and 0, respectively).[5]

1.5.2. Variations in the spoils of office, *b*

Here, we demonstrate our comparative statics results for changes in the benefits from holding office. The example uses $n = 5$ and $c = 0.45$ and varies the spoils between $b = 0,1$ and 2. Figure 1.2 shows that $\breve{x}^*$ decreases in b, where $\breve{x}^*(b = 0) = 0.713$ and $\breve{x}^*(b = 1) = 0.276$ (cf. the intersections of the respective net benefits curves and the cost line) and $\breve{x}^*(b = 2) = 0$ (because the respective net benefits curve lies above the cost line). Finally, this decrease yields more moderate expected policy outcomes in each direction left and right (minus and plus 0.857, 0.638, and 0.5 for our ascending *b*, respectively) and raises expected entry (1.434, 3.618, and 5, respectively).

FIGURE 1.2: Symmetric cutpoint policy equilibria and variations in the benefits from holding office, *b*, for n = 5 and c = 0.45

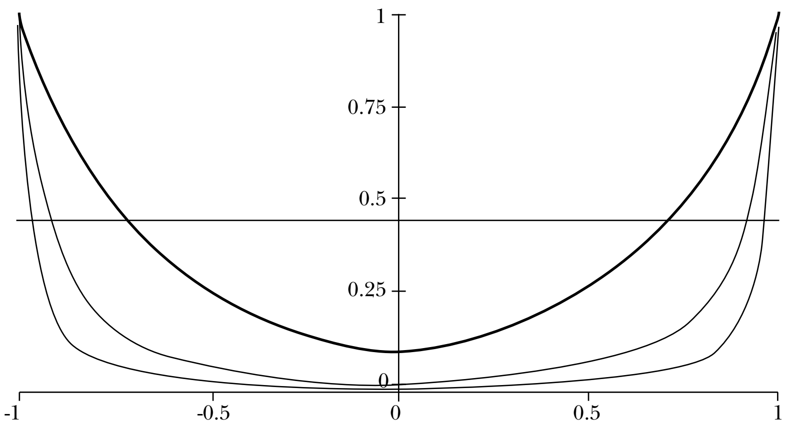


[5] Expected policy outcomes in each direction left and right are derived as $E[\gamma_l \mid \breve{x}_l] = \frac{\breve{x}_l - 1}{2}$ and $E[\gamma_r \mid \breve{x}_r] = \frac{\breve{x}_r + 1}{2}$, respectively, and expected entry is derived as $E[m \mid \breve{x}] = n(1 - \breve{x})$

1.5.3. Variations in the number of citizens, n

The final example illustrates our comparative statics results for changes in the size of the community. It uses $b = 0$ and $c = 0.45$ and varies the number of citizens between $n = 5,20$, and 50. Figure 1.3 shows that $\breve{x}^*$ increases in n, where $\breve{x}^*(n = 5) = 0.713$, $\breve{x}^*$ $(n = 20) = 0.912$, and $\breve{x}^*(n = 50) = 0.963$, respectively (once again, cf. the intersections of the respective net benefits curves and the cost line). As a consequence, this increase yields more extreme expected policy outcomes in each direction left and right (minus and plus 0.857, 0.956, and 0.982 for our ascending n, respectively) and raises expected entry (1.434, 1.757, and 1.832, respectively).[6]

FIGURE 1.3: Symmetric cutpoint policy equilibria and variations in the number of citizens, n, for $c = 0.45$ and $b = 0$



1.6. Concave payoff functions

The model can also be extended in a straightforward way to allow for a class of utility functions that are a concave function of the Euclidean distance between

---

[6] Recall that expected entry does not necessarily increase in n, but depends on the specific parameters in this example.

a citizen's ideal point and the policy outcome. Particularly simple is the special case of power utility functions (which includes the commonly-used specification of quadratic payoffs), and one can think of these utility functions as measuring the risk aversion of the players. The payoff function is

$$\pi_i\,(x_i, \gamma, e_i, w_i, \alpha) = -\,|x_i - \gamma|^{\alpha} - ce_i + bw_i, \tag{1.9}$$

where $\alpha \geq 1$

Formally, the only difference from the piecewise linear utility specification is that the condition for the best response strategy for a voter with ideal point x_i if all other citizens are using cutpoint strategy $\breve{e}$ is now to enter if and only if

$$\breve{x}^{n-1}\Big[b + E[|x_i - \delta|^{\alpha} \mid \breve{x}]\Big]$$

$$+\sum_{m=2}^{n} \binom{n-1}{m-1} (1-\breve{x})^{m-1}\, \breve{x}^{n-m}\, \frac{1}{m}\Big[b + E\big[|x_i - \gamma|^{\alpha} \mid \breve{x}\big]\Big] \geq c \tag{1.10}$$

See Großer and Palfrey (2009) for the equilibrium characterization for strictly concave utility functions.

1.7. Conclusion

We presented our basic citizen-candidate model with private information. The paper specializes the results of Großer and Palfrey (2009) to the case of uniformly distributed ideal points and the simple default policy, $\delta = 0$. We showed that equilibria with symmetric cutpoint policies always exist *and are always unique.* In these equilibria, all citizens with ideal points equally or more extreme than the cutpoint enter the electoral competition as candidates and all citizens with more moderate ideal points do not enter. And, we showed that the equilibrium cutpoint policy is increasing in the entry costs and the number of citizens, and it is decreasing in the leader's benefits from holding office. Moreover, an increase in the equilibrium cutpoint policy through changes in the entry costs and benefits from holding office decreases the expected number of citizen-candidates and the expected policy outcome becomes more extreme. An increase in the equilibrium cutpoint policy through an increase in the number of citizens also results in a more extreme expected policy outcome, however, the effect on the number of expected entrants can be either positive or negative.

The results can be extended and generalized in several directions. First, one can relax the assumption of uniformly distributed ideal points. This assumption made the computations quite easy and allowed us to illustrate the results graphically. In Großer and Palfrey (2009), we obtain similar results for arbitrary symmetric distributions. This allows us to address questions about the effect of polarization of the electorate's preferences on candidate entry. There, we also investigate the effects of distributional asymmetries of ideal points on the characterization of equilibrium entry strategies and more general specifications of the default policy. The model can also accommodate a stochastic default policy, given by a distribution G.

Ideally one would like to endogenize the default policy as part of the equilibrium of the model. Here we used a simple exogenously specified constant default policy. In general, one would expect the default outcome to depend in some way of the electoral the process. One way to endogenize this is to have one of the citizens randomly appointed the new leader, in case no one runs as a candidate. This is considered in Großer and Palfrey (2009).

Another interesting possibility for endogenizing the default policy is to allow multiple rounds in the entry stage: if no citizen chooses to run as a candidate in the first entry round, another round starts and this continues until finally there is at least one citizen-candidate. Such a model of "default" policy has the virtue of guaranteeing endogenous entry of at least one candidate, provided entry costs are not prohibitively large. The effect of this is that after the first entry round the community can update that there are no ideal points that are equally or more extreme than the equilibrium cutpoint policy in the first round. In the second entry round, the game will be solved as the original, only that the truncated probability distribution is used, and so forth. See Großer and Palfrey (2009) for further elaboration and a specific model.

Several other directions extending the model could add some additional insights. For example, in the present formulation of the model citizens do not learn anything useful about a candidate's ideal point. As a first step it would be interesting to look at a model where citizens can learn whether this ideal point is to the left or the right of the median ideal point, as might happen for example if there are interest group endorsements or party labels. Along a similar vein, one could introduce nominating procedures or party formation of left and right candidates, with each side nominating one as their running candidate. One could add partial credibility to the policy promises stage, as in Banks (1990). And finally, it might be possible to extend the model to multiple dimensions, for example where the policy space is the closed unit ball. A natural conjecture for well-behaved symmetric distributions is that there will exist a unique equilibrium with similar features to the one-dimensional model: citizens enter if and only if their ideal point is sufficiently far from the origin.

References

BANKS, J. "A Model of Electoral Competition with Incomplete Information." *Journal of Economic Theory* 50 (1990): 309-325.

BESLEY, T., and S. COATE. "An Economic Model of Representative Democracy." *Quarterly Journal of Economics* 112 (1997): 85-114.

BRUSCO, S. and J. ROY. *Aggregate Uncertainty in the Citizen Candidate Model Yields Extremist Parties.* Working paper, SUNY Stony Brook, 2007.

CASAMATTA, G. and W. SAND-ZANTMAN. "Citizen Candidacy with Asymmetric Information." Topics in Theoretical Economics 5 (2005), Berkeley Electronic Press, Article 3.

DHILLON, A. and B. LOCKWOOD. "Multiple Equilibria in the Citizen-Candidate Model of Representative Democracy." *Journal of Public Economic Theory* 4, no. 2 (2002): 171-184.

EGUIA, J. X. "Citizen Candidates Under Uncertainty." *Social Choice and Welfare* 29, no. 2 (2007): 317-331.

FEDDERSEN, T. "A Voting Model Implying Duverger's Law and Positive Turnout." *American Journal of Political Science* 36 (1992): 938-962.

FEDDERSEN, T., I. SENED and S. WRIGHT. "Rational Voting and Candidate Entry under Plurality Rule." *American Journal of Political Science* 34 (1990): 1005-1016.

FEY, M. "Stability and Coordination in Duverger's Law: A Formal Model of Pre-election Polls and Strategic Voting." *American Political Science Review* 91, no. 1 (1997): 135-147.

—. *Duverger's Law Without Strategic Voting.* Working paper, University of Rochester, 2007.

GROSSER, J. and T. PALFREY. *Strategic Entry in Elections with Private Information.* Working paper in preparation, California Institute of Technology, 2009.

KUHN, H. W. "Extensive Games and the Problem of Information." In Contributions to the *Theory of Games I: 193-216*, eds. Harold W.Kuhn and Albert W. Tucker. Princeton, New Jersey: Princeton University Press, 1953.

OSBORNE, M. "Candidate Positioning and Entry in a Political Competition." Games and Economic Behavior 5 (1993): 133-151.

OSBORNE, M. and A. SLIVINSKI. "A Model of Political Competition with Citizen Candidates." *Quarterly Journal of Economics* 111 (1996): 65-96.

OSBORNE, M., J. ROSENTHAL and M. TURNER. "Meetings with Costly Participation." *American Economic Review* 90, no. 4 (2000): 927-943.

PALFREY, T. "Spatial Equilibrium with Entry." *Review of Economic Studies* 51 (January 1984): 139-156.

—. "A Mathematical Proof of Duverger's Law." In *Models of Strategic Choice in Politics*, ed. Peter C. Ordeshook. Ann Arbor: University of Michigan Press, 1984.

ROEMER, J. *Indeterminacy of Citizen-Candidate Equilibrium.* Cowles Foundation Discussion Paper 1410, 2003.

WITTMAN, D. "Candidate Motivation: A Synthesis of Alternative Theories." *American Political Science Review* 77 (1983): 142-157.

2

An Experimental Study of the Citizen-Candidate Model

Alexander Elbittar
Economics Division
Center for Economic Research and Study (CIDE)

Andrei Gomberg
Center for Economic Research
Autonomous Technological Institute of Mexico (ITAM)

2.1. Introduction

The citizen-candidate model of elections (Osborne and Slivinski 1996; Besley and Coate 1997) is commonly used to endogenize the number and the identity of political candidates and proposals. The model considers a society of agents with publicly known preferences in some policy space who vote to decide on a common policy. Crucially, only alternatives explicitly proposed (nominated) by somebody can be considered for voting and the nomination decision is strategic: citizens choose to nominate themselves, based on their predicted impact on the policy outcome, the cost of running for office and benefits accruing to office-holders. Once the set of candidates is fixed, the entire society votes and the elected candidate implements his/her favorite policy (as the individual preferences are public, candidates cannot commit to implementing any policy at variance with their ideal).

Unfortunately, the citizen-candidate model is not easy to test, as it heavily relies on exact public knowledge of the distribution of policy preferences in the society, including the policy preferences of potential candidates, even before the nomination and the campaign. Thus, in order to test it fully, we need to know not only preferences of actual candidates in the election, but also of those who chose not to get nominated. The predictions of the model are, furthermore, dependent on parameters (such as the cost of running for office and the benefits of holding it) that might be difficult to measure empirically and even harder to vary in real political systems. A direct test of the model's prediction for the differential impact of different electoral systems is complicated by the relative rarity of electoral system changes. The substantial multiplicity of equilibria for many parameter values in the model makes designing a satisfactory empirical test even harder.

Many of the problems with testing the citizen-candidate model in the field can be easily overcome in an experimental lab. Thus, an experimentalist would have no difficulty varying office-holder benefits or nomination costs, changing the distribution of citizens in the policy space or even the electoral system. In the lab it is also possible to design environments that minimize the problems with equilibrium multiplicity, allowing explicit tests of the model predictions.

Surprisingly, in the dozen years since the publication of the original theoretical papers there has been little work on trying to test the model experimentally. The experimental literature on candidate behavior in elections has concentrated on candidate location decisions.[1] However there has been comparatively little research on candidate entry. In fact, Palfrey (2005) in his recent survey of the field, noted that "to date there have been no experiments" on entry by policy-motivated candidates. That same year Cadigan (2005) published the only previous experimental study of the model that we are aware of. Though an important advance, for being the first to attempt a laboratory testing of the model, Cadigan's work is somewhat limited in scope. It reports results of two treatments of an adaptation of the citizen-candidate model that are distinguished by the value of the cost of nomination parameter. In the high-cost treatment the unique predicted equilibrium involves a single candidate entering at the median of the voter distribution, while the low-cost treatment has, in addition to the median-candidate equilibrium, a two-candidate equilibrium with distinct policy proposals.

We propose an experimental design which varies both cost parameters and electoral systems. In particular, in addition to the simple plurality elections, we consider the two-round runoffs.[2] At least in some environments, Osborne and Slivinski (1996) results may be interpreted as implying stronger pull for entry by politicians at the median of the voter ideal point distributions. It is this implication that we would like to test.

---

[1] See, for instance, the early work by McKelvey and Ordeshook (1982) on two-candidate competition in environments with and without Condrocet winners, or the recent study by Aragonès and Palfrey (2004) on policy platform choice by candidates of different quality. For a recent survey of the literature see Palfrey (2005).

[2] In the future we also intend to test the empirical implications of introducing the proportional representation into the citizen-candidate framework, derived by Hamlin and Hjortlund (2000).

Like Cadigan (2005), we impose sincere voting, in order to concentrate on individual entry decisions by potential candidates. At the same time, we want to stay close to the large-electorate spatial model of Osborne and Slivinski (1996). To do this, while keeping the number of participants in an experimental game small, we decouple the potential candidates (whom we shall call "politicians") from the entire society of citizens. Only politicians may choose to run for office, while the set of voters (implemented in our experiments by a computer) is larger. The restriction is not wholly unrealistic, as, in practice, not every voter would have name recognition and/or funding lined up to make him a viable candidate in a given election and only politicians are under a sufficient public scrutiny to make the assumption that their political views are known empirically plausible. In most elections, at least some of the potential "pre-candidates", though credible enough to be considered, choose not to enter the campaign. It is this entry decision that we study. The distribution of politicians' policy preferences is the third variable, crucial to the predictions of the model, that we choose to vary in the lab.

Cadigan's results, which serve as our benchmark, are twofold. Firstly, he observes that the low-cost treatment results in relatively high entry by symmetric off-median subjects, compared with the high-cost treatment. In addition, he claims to confirm observations of over-participation in some of the earlier studies on market entry (such as Camerer and Lovallo (1999) and Fischbacher and Thöni (2008)). It should be noted, though, that since Kahneman (1988), fast convergence to theoretically predicted entry rates of entry has been commonly observed (for a recent survey, see, for instance, Camerer 2003). Given the asymmetry of players in the citizen-candidate model (due to the difference in their ideal points), a more relevant observation in this context may be that of Rapoport *et al.* (2002), who, in a market-entry game with asymmetric entry costs, find that subjects tend to over-enter, when the pure-strategy equilibrium implies they should be staying out, and under-enter, when the equilibrium implies they should enter.[3] Of course, the present model is substantially more complex than the market-entry games, as the candidates' payoffs depend not only on the number of entrants, but also on their location, so a direct analogy may not be appropriate.

Our preliminary conclusions may be summarized as follows. Firstly, we do observe subjects reacting to treatment variable changes. In particular, both the asymmetry of politician ideal point distribution and the run-off electoral system

[3] This does, in fact, seem to hint at a better interpretation of Cadigan's result, since he observes substantial deviation from certain entry predicted by equilibrium for some ideal points.

are conducive to greater entry frequencies at the median of the voter ideal points. Secondly, we seem to confirm Cadigan's observation of comparatively high entry in situations, when equilibrium predicts no entry. In fact, entry rates remain non-negligible even from, essentially, hopeless positions. Other than that, the subjects' entry decisions seem to be reasonably close to best responding to the empirically observed entry frequencies.

The rest of this paper is organized as follows. Section 3.2 develops the benchmark model and the experimental treatments, section 3.3 presents the experimental design, section 3.4 discusses the results, section 3.5 concludes.

2.2. The experimental model

Our model is an adaptation of the one introduced in Osborne and Slivinski (1996). Though Besley and Coate (1997) provide a similar model which allows for a small number of agents, which would seem to be easier to implement in a laboratory experiment, we concentrate on the Osborne and Slivinski approach, as we are interested in large elections, where voting may be assumed to be non-strategic (allowing for strategic voting would introduce additional equilibrium multiplicity which we are trying to avoid). In addition, as in Osborne and Slivinski (1996), an important concern for us is the performance of the model under distinct voting rules.

We consider a society that has to implement a single policy x on a unidimensional [0,100] continuum. Heterogenous voters have single-peaked preferences, with ideal points distributed over the continuum according to some distribution F. As noted in the introduction, our main departure from Osborne and Slivinski is in limiting the set of possible candidates to a small finite subset of citizens (we consider treatments with 3 and 5 potential candidates).

The potential candidates (henceforth, politicians, or agents) may choose to nominate or not to nominate themselves for office. The rest of the voters are assumed to never run for office, but simply to vote for the candidate whose ideal policy is the closest to their own (in experimental treatments the role of these non-politician voters is performed by a computer). Following the classic citizen-candidate models, it is assumed that agent preferences are known by everyone and that there is no commitment, so that the politicians can only promise that if elected they would implement their ideal policies.

Thus, in the experimental game there are $N = \{1,2,...n\}$ agents. Each agent i has a 2-point strategy space $S_i = \{0,1\}$, where $s_i = 1$ means the agent nominates him/herself, and $s_i = 0$ means the agent stays out of the election. Each agent has single-peaked preferences over the policy space, with an integer ideal point $0 \leq q_i \leq 100$ (here and elsewhere we choose to consider only integer points for the purposes of experimental implementation).

Unlike the potential candidates, the voters in our experimental are computerized robots, who always vote sincerely. We assume there are 101 such voters, with a single voter having an ideal point at every integer between 0 and 100 (the discretization of the voter space is done here in order to avoid explaining the notion of a continuous distribution to largely pre-calculus experimental subjects). The robot voters always vote for a nominated candidate whose ideal point is closest to their own (in case $m > 1$ candidates are at the same distance from a given voter, s/he shall randomly select a candidate, with every one of the closest candidates having a probability $\frac{1}{m}$ of being chosen).

The winner of the election is determined by the voting of a larger society. In this paper we consider two voting rules: simple plurality and the two-round runoff. In the former the candidate with the largest number of votes wins outright (ties are resolved randomly, with every one of the top candidates having equal probability of winning). In the latter system, all candidates but the top two get eliminated in the first round, so that the second round is a simple majority vote among the top two contenders (in both rounds, once again, ties are randomly resolved). In this way for any election we have a (stochastic) winner function w_i, which equals to 1 if the agent wins the election, and equals to 0 if he doesn't (whether he does not win due to loosing the vote or due to not entering). In all cases, there is a unique winner in the election, and the implemented social outcome is his/her favorite policy: $x = q_i$ whenever $w_i = 1$.

The agents have standard Euclidean preferences over a single-dimensional policy space, they have to pay a cost $c > 0$ to nominate themselves as candidates and they receive a benefit $b > 0$ if elected. Formally, an agent i with an ideal point $q_i \in [0,100]$ in a society that implements a policy $x \in [0,100]$ receives the payoff

$$u_i\,(x; q_i) = -\alpha\left\|x - q_i\right\| - cs_i + bw_i$$

(where $\alpha > 0$ is a scale parameter, that in the theoretical model may be normalized to 1).

Finally, if no candidate chooses to enter an election, following Osborne and Slivinski we assume every agent receives a large negative shock (this can be viewed as a major disruption of the political system). Formally, if no candidate is nominated the implemented policy is assumed to be $x = d$, which is equally disliked by all participants:

$$u_i\ (d; q_i) = -D$$

where $D > 0$ is large.

In general, the citizen-candidate model leads to multiple equilibria. The exact structure of the equilibrium set may be rather complicated, with existing characterizations focusing on the number of agents entering in an election. The key determinants in this respect are the parameters of cost of running for an election c and the benefit from holding office b, as well as the specific of the voting rule chosen. In our case, where the set of potential candidates is restricted, the availability of candidates at certain points of the political spectrum is also crucial. In particular, there are important implications of having candidates at the median m of the voter ideal point distribution (which in the case of the uniform voter distribution is 50). Following the bulk of the earlier literature, we shall concentrate on the pure strategy Nash Equilibrium. The following proposition, which follows from the results of Osborne and Slivinski (1996), describes some of the equilibrium possibilities in our setting. It is these implications of the model that we shall try to test in the lab.

Proposition 2.1. *a) If there is a unique politician closest to m, then for both voting rules there exists an equilibrium in which he is the only candidate.*

b) If there exist two or more politicians closest to m, then the single-candidate equilibrium exists under either voting rule if and only if $b \leq 2c$. *In such an equilibrium, one of the politicians closest to the median is the only entrant.*

c) In every two-candidate equilibrium under the plurality rule the candidates are located symmetrically around the median of the voter distribution. Furthermore, such an equilibrium will exist only if there are symmetric politicians located between 100/6 and 500/6, or if the symmetric politicians are the closest ones to the median.

d) If there are exactly two potential candidates closest to the median, then under the run-off system there exists an equilibrium in which they are the only entrants if only if $2c \leq b$.

e) If there is more than one politician at each occupied position to one side of the median and $b > 4c$, *then under the run-off in every two-candidate equilibrium only the politicians closest to the median would be candidates.*

It should be noted that, except at high values of the cost parameter c compared to the benefit parameter b and when there is only one candidate at the median of the voter ideal point distribution, it is hard to achieve equilibrium uniqueness in the citizen-candidate model. In choosing the parameters of the model to be tested on, we attempt to, at least, ensure uniqueness of pure-strategy equilibrium candidate ideal point distributions.

2.3. Experimental design

All experimental sessions were conducted at the Autonomous Technological Institute of Mexico (ITAM) in Mexico City and the subjects were undergraduates recruited in introductory economics courses. The experiments were computer-administered. A total of 10 experimental sessions were conducted and each session had between12 and 30 participants.

In each experimental session we consecutively ran 30 elections in groups with three or five candidates, with both group membership and subject ideal points randomly changed before each election. If the total number of subjects in the room was not divisible by 3 (respectively, 5), in each round some subjects would be randomly selected to sit it out. Therefore, up until the last round the termination time effectively remained random.

The distribution of subjects' ideal points was either constant, or varied only once during a session, but each subject's location was randomly chosen for each period, which corresponded to an election. In each election subjects, having observed their positions, had to decide whether to nominate themselves as possible candidates. All voter decisions were taken by the computer. After each election subjects got the feedback about the ideal points of the entrants and the winner in their election, as well as the vote shares received by every candidate and their own monetary payoff.

All payments were in Mexican Pesos ($MN\$11 = USD\$1$). We started each experimental session by allocating every agent MN140 pesos of initial capital, to which the payments corresponding to the model parameter values were added and subtracted. In all treatments we take $D = -MN\$40, b = MN\$25$ and $\alpha = MN\$0.1$. There are both high-cost (HC) and low-cost (LC) treatments: in the former we set $c = MN\$20$ or $MN\$17$[4] and in the latter $c = MN\$5$. If subjects' balance was reduced to below 0 pesos, they were excluded from further election rounds (in practice this only occurred in high-cost treatments).

[4] We reduced the cost to $17 pesos in a later session in an attempt at reducing subject bankruptcy rates.

We have conducted treatments with three—and five-subject groups. For the three-subject groups, we considered two configurations of the politician ideal points: 30, 50, 70 and 30, 50, 80. For the plurality rule, in the former treatment at low cost there are two pure-strategy Nash Equilibria: in one only agent at 50 enters, whereas in the other s/he stays out and agents at 30 and 70 enter the election. At high cost, in both configurations only the agent at 50 may enter in equilibrium (at this point we have only run the low-cost treatments). In order to analyze the impact of prior learning we have run the two ideal point configurations sequentially with the same agents, varying the order: in one treatment the 30, 50, 70 configuration was run before 30, 50, 80, and in the other treatment the ordering was reversed (two session of each order treatment has been run).

Unfortunately, in the three-person treatments with distinct ideal points of candidates it is impossible to eliminate the equilibrium in which the only entrant is the candidate closest to the median of the voter distribution (indeed, conditional on such candidate being the lone entrant, nobody else would be better off by entering either under the plurality rule, or under the run-off rule). This motivated our decision to consider treatments with five agents, some of which coincide in their ideal positions.

For the five-subject groups, we kept the distribution of candidate ideal points fixed at 25, 25, 50, 50, 75. In this treatment for the plurality rule at low cost there is only one pure-strategy equilibrium candidate configurations possible, with the agent at 75 entering together with one of the two agents at 50. In contrast, under the run-off system there is a unique equilibrium in which both agents at 50 enter (and nobody else does). At high cost in any pure-strategy equilibrium under both voting rules *only one* agent can enter at 50. We have run two sessions of each low-cost treatment and two sessions of the high-cost plurality rule treatment. It should be noted, that even though equilibrium entry predictions are unique, only the low-cost run-off treatment has a unique equilibrium. The other treatments, therefore, may present substantial coordination problems. The summary of the equilibrium predictions for the five-subject treatments is given in table 2.1:

TABLE 2.1: Equilibrium entry positions for five-subject treatments

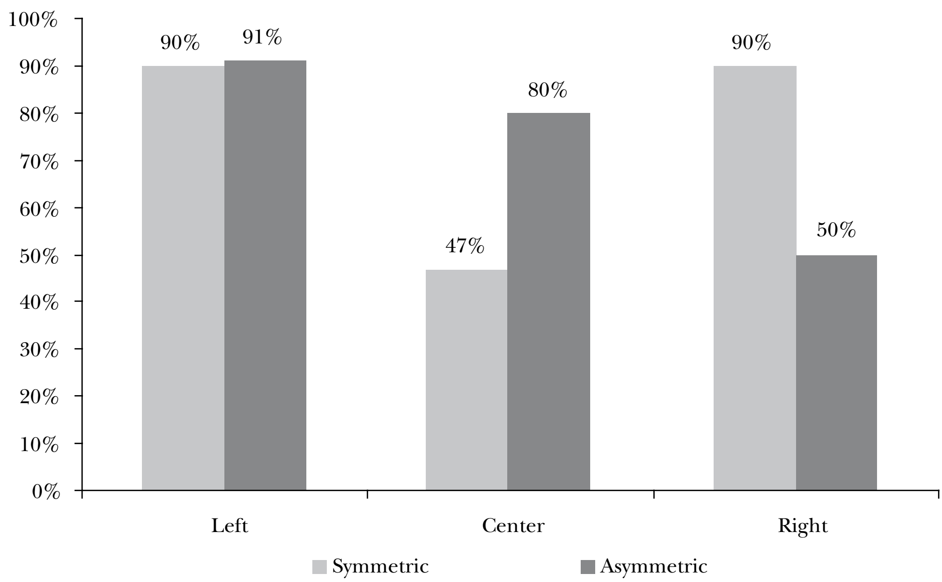
	Plurality rule	**Run-off**
High cost	50	50 (not run)
Low cost	25,75	50,50

2.4. Results

Our results can be summarized as follows. In every treatment there is a substantial "floor" on entry probability from every position. In fact, almost always we observe candidates from each position entering with, at least, a 25% probability. Except for this, agents come close to best-responding to the empirically observed entry distributions. The degree of excess entry in the largely "hopeless" positions was sufficiently high to force us to modify the original experimental design: since the first high-cost session (with entry cost c = 20 pesos) had to be terminated early due to the "bankruptcy" of the majority of subjects, in the subsequent session we lowered the entry cost parameter to c = 17 (lowering it further would have come close to changing the equilibrium predictions). Unfortunately, this session, likewise, ended early due to a mass bankruptcy.

Graph 2.1 presents the entry probabilities by the ideal point in the three-subject treatments. In total, we have an observation of 330 "symmetric" elections (with subjects' ideal points at 30,50 and 70) and an equal number of "asymmetric" elections (with candidates at 30,50, and 80).

GRAPH 2.1: Entry frequencies in three subject treatments



It can be observed that when the distribution of ideal points is symmetric the "extremist" subjects enter with probabilities close to 90%. Since whenever both extreme subjects enter, the centrist subject looses the election, whereas he wins whenever no more than one opponent competes, the empirical distribution implies that such a candidate, if nominated, has just about a 20% probability of winning, with the extremist candidates winning about 40% of the time each. Notably, the empirical entry distribution is not too far from the best response for all candidates: given the value of the parameters, the centrist subjects come close to being indifferent between entering and not entering, while both extremist candidates should enter.

The asymmetric case entry pattern is very distinct. The striking feature is the high entry rate of the nearly hopeless candidates at 80. The "far rightists" have negligible winning probabilities, as they can only win if nobody else enters (in fact, their wins are almost never observed in the data). Nevertheless, they enter half the time. This high entry rate by the ultra-right frequently ensures the victory of the left: whenever the subject at 80 enters, the candidate at 30 is guaranteed a win. This does seem to induce extremely high entry rates on the left. The centrists, however, win whenever the right does not enter, which likewise serves to elevate their entry probabilities.

In order to study the differential effects of treatments on the entry probabilities we ran a random effect logit regressions for entry probability. Table 2.2 presents results of these for each entry position. In each regression the coefficients refer to changes in entry probabilities between the symmetric and the asymmetric treatments. We also introduced two distinct period variables, to account for the possibility of different learning patterns for the symmetric and asymmetric elections.[5]

TABLE 2.2: Random effect logit regressions for the entry probabilities in three-person treatments

	Left	Center	Right
Constant	3.336**	0.083	2.863**
Std. error	–0.625	–0.43	–0.479
Asym	–0.401	0.526	–2.245**
Std. error	–0.63	–0.449	–0.494
Order	–0.721	0.511	–0.394
Std. error	–0.565	–0.459	–0.322

[5] We also considered introducing the separate period variables to account for the order of conducting the symmetric and asymmetric elections, but the corresponding coefficients were never statistically significant.

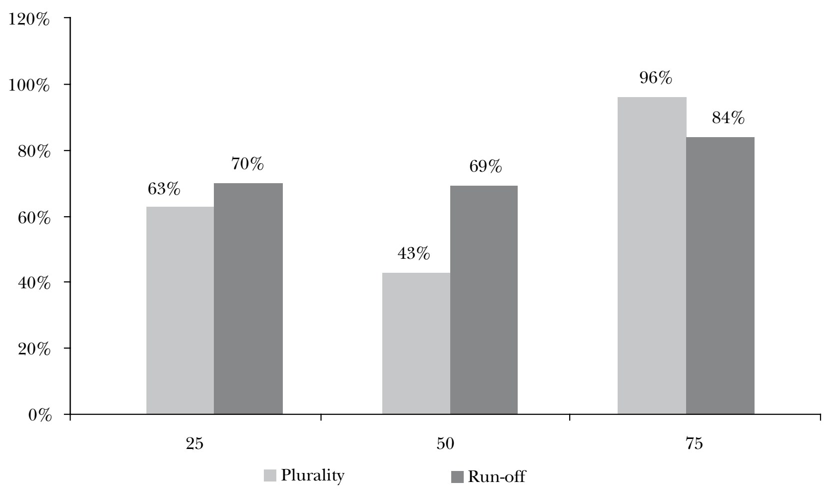
TABLE 2.2 *(cont.)*: **Random effect logit regressions for the entry probabilities in three-person treatments**

	Left	Center	Right
Period	0.009	0.05	–0.017
Std. error	–0.049	–0.033	–0.045
Period-asym	0.064	0.201**	0.025
Std. error	–0.073	–0.052	–0.053
** –p <0.01			

The regressions show that there is little dynamics going on during the sessions, except that in the asymmetric elections the centrist candidates quickly increase their entry probabilities (no observable dynamics occurs in other positions and in symmetric elections). This may suggest a learning argument (as, given the empirical entry frequency by the ultra-rightists at 80, the entry by the leftists is still, in expectation, profitable, no learning occurs in this position). The position asymmetry has a further large effect on the rightist entrants, who sharply decrease their entry probabilities when moved further to the right (this entry rate, however, remains rather high, with no observable dynamics during the session). There are no obvious order effects due to symmetric or asymmetric elections being run first.

Graph 2.2 presents entry probabilities for the low-cost five-person treatments.

GRAPH 2.2: Entry frequencies in low-cost five-subject treatments



In the plurality rule elections subjects nearly always enter at 75 (as is predicted by the equilibrium), and enter with a probability of 63% at 25. Adjusting for the obvious coordination difficulties between the subjects at 25, these results seem to be close to the predicted equilibrium. Indeed, the implied probability that there is nobody entering at either 25 or 75 is under 15%. Thus, the entrance at 50 does not present many chances of winning (to win the centrist candidate has to be alone at his position, with at least one of the remaining positions being unoccupied). Strikingly, 43% of the time the subjects choose to enter here, which implies that an agent considering entry at this position would be expecting to loose with a probability of over 90%, which cannot justify incurring the entry cost. The most notable difference across treatments is a substantial increase in entry at the center in the run-off treatment compared with the simple plurality treatment. A smaller decrease in entry at 75 is likewise notable, as it comes in comparison with nearly certain entry under the plurality rule.

Table 2.3 presents the results of the random-effect logit regressions for entry probabilities for the run-off and plurality elections.

TABLE 2.3: Random effect logit regressions for the entry probabilities in five-person treatments

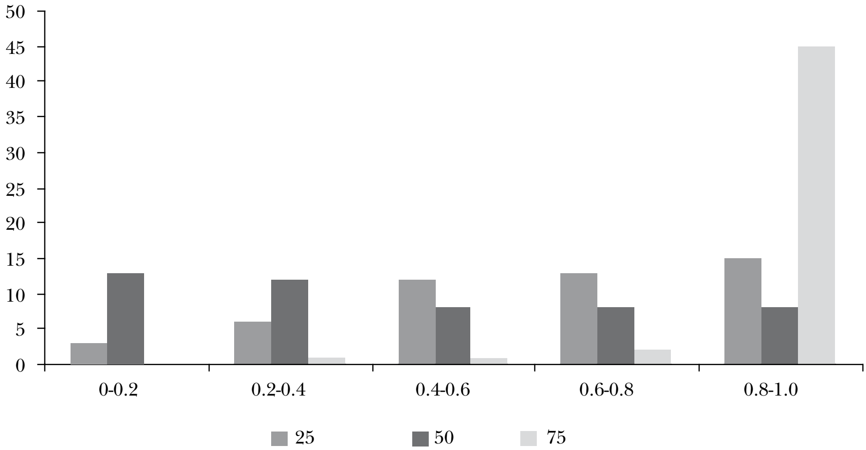
	Left	Center	Right
Constant	1.019**	0.012	3.854**
Std. error	–0.282	–0.334	–0.838
Runoff	–0.444	0.879	–1.389
Std. error	–0.428	–0.513	–0.941
Period	–0.019	–0.026*	–0.036
Std. error	–0.012	–0.013	–0.041
Period/runoff	0.054**	0.043*	–0.028
Std. error	–0.019	–0.02	–0.049
** –p < 0.01	* –p < 0.05		

The results seem to indicate significant increase over time of entry probabilities both at 25 and at 50 during the runoff sessions, and a smaller significant decrease in entry probabilities in the center during the plurality rule sessions. Furthermore, even ignoring the dynamics, the run-off may result in a somewhat higher entry probability in the center (the corresponding coefficient is significant at a 10% level). At this point, the decrease in entry at 75 in the run-off has not been shown to be statistically significant.

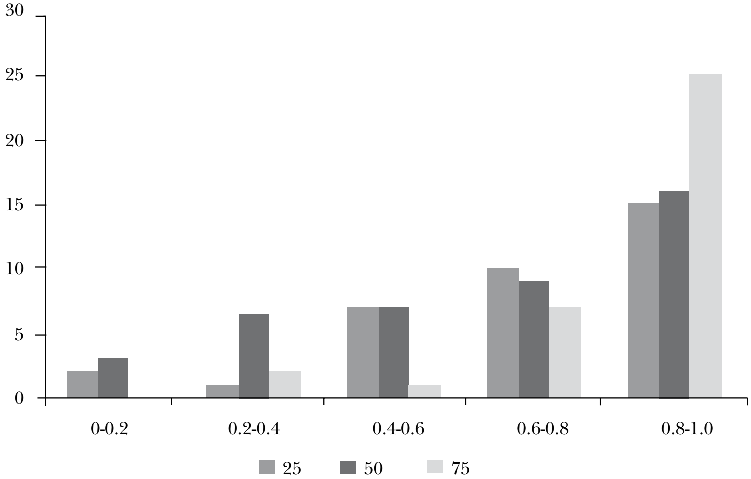
As a total of 49 and 35 distinct subjects participated, respectively, in the plurality and run-off rule treatments we may try to look for individual entry patterns.

Graph 2.3 and Graph 2.4 present the distribution of agents by average entry frequency.

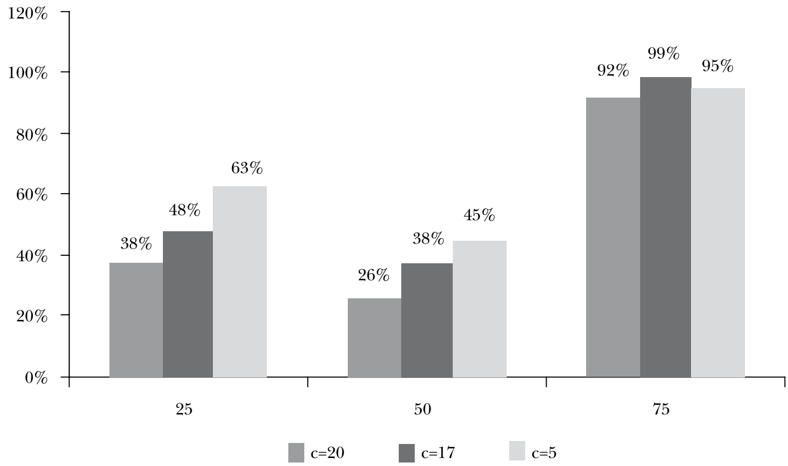
GRAPH 2.3: Individual entry frequencies under the plurality rule



GRAPH 2.4: Entry frequency under the run-off rule



GRAPH 2.5: Entry frequency in plurality elections for various entry costs



It can be immediately seen that in both plurality and run-off elections from every position at least some subjects enter always or nearly always (in fact, we did observe certain subjects entering always, irrespective their position). This accounts for a substantial chunk of overall over-entry in loosing positions. It should be also noted that whereas nearly all subjects always enter at 75 under the plurality rule, the distribution is more diffuse under the run-off rule. Furthermore, whereas many subjects almost never enter at 50 under the plurality rule (or do so only rarely), almost every subject enters with, at least, a 40% probability of under the run-off rule.

The results from the high-cost treatment, at this point, can, at best, be considered tentative. The main reason for this is that neither experimental session for this treatment was properly concluded due to a massive subject bankruptcy (in fact, in an unsuccessful attempt to reduce bankruptcy we reduced in the later session the entry cost from MN$20 to MN$17 pesos).

It is striking, that varying entry cost has no effect on subjects located alone at 75. In contrast, the subjects both 50 and 25 decrease their entry rates when costs are high. Still, these rates remain high, considering that both the leftists and the centrists are loosing with probabilities far above $\frac{1}{2}$, repeatedly incurring the huge entry cost. The equilibrium prediction (entry only at 50) does not come close to being observed. To make any definitive conclusions here, though, we would need to properly implement the high-cost treatment.

2.5. Conclusion

This paper presents the preliminary results of a series of experiments on the candidate entry in the citizen-candidate framework. Overall, we believe the theory does a reasonable job in predicting the consequences in changes of the control variables. The three-person treatments seem to both provide evidence for the theoretically predicted divergent equilibria and suggest a pattern of possible learning behavior. The comparative plurality rule and run-off rule treatments with five subjects provide evidence for the predicted strengthening of the candidates near the median in the latter electoral system. On the other hand, our results provide strong evidence for consistent excess entry by subjects in "hopeless" or "nearly hopeless" positions (an occasional failure to enter by subjects even in the most advantageous positions may be noted as well). Furthermore, the multiplicity of equilibria leads to important coordination problems that require further study. Finally, the definitive conclusions about the high-cost treatments would have to be postponed, until these are run properly, in order to avoid mass subject bankruptcy.

The more detailed analysis of the data remains to be done. In particular, we would need to present a more detailed analysis of individual entry patterns. In addition, the non-negligible entry frequencies by nearly inevitably loosing candidates, combined with what informally appears to be close to the best response behavior by subjects otherwise, leads us to believe that a model of quantal response equilibria of McKelvey and Palfrey (1995) might be useful in explaining our observations. Furthermore, the tentative evidence for the importance of learning that comes out of the three-subject treatments (and recalls some of the observations of Rapoport et al. 2002) suggests another avenue for further research. Finally, as the multiplicity of equilibria is nearly inevitable in the citizen-candidate model (unless the only equilibrium involves entry by only Condorcet winners), the resultant coordination problems would have to be studied.

References

Aragonès, E. and T. Palfrey. "The Effect of Candidate Quality on Electoral Equilibrium: An experimental Study." *American Political Science Review* 98 (2004): 77-90.

Besley, T. and S. Coate. "An Economic Model of Representative Democracy." *Quarterly Journal of Economics* 112 (1997): 85-114.

Cadigan, J. "The Citizen-candidate Model - an Experimental Analysis." *Public Choice* 123 (2005): 197-213.

CAMERER, C. *Behavioral Game Theory.* New York: Russel Sage Foundation, 2003.

CAMERER, C. and D. LOVALLO. "Overconfidence and Excess Entry: An Experimental Approach." *American Economic Review* 89 (1999): 306-318.

FISCHBACHER, U. and C. THÖNI. "Inefficient Excess Entry in an Experimental Winner-Take-All Market." forthcoming in *Journal of Economic Behavior and Organization* (2008).

HAMLIN, A. and M. HJORTLUND. "Proportional Representation with Citizen Candidates." *Public Choice* 103 (2000): 205-230.

KAHNEMAN, D. "Experimental Economics: a psychological perspective." In R. Tietz, *et al.* (editors), *Boundedly Rational Behavior in Experimental Games and Markets.* New York: Springer Verlag (1988): 11-18.

MCKELVEY, R. and P. ORDESHOOK. "Two-candidate Elections Without Majority Rule Equilibria." *Simulation and Games* 13 (1982): 311-335.

MCKELVEY, R. D. and T. R. PALFREY. "Quantal Response Equilibria in Normal Form Games." *Games and Economic Behavior* 10 (1995): 6-38.

OSBORNE, M. and A. SLIVINSKI. "A model of Political Competition with Citizen Candidates." *Quarterly Journal of Economics* 111 (1996): 65-96.

PALFREY, T. *Laboratory Experiments in Political Economy.* CEPS Working Paper No. 111, Princeton University, 2005.

RAPOPORT, A., D. SEALE and E. WINTER. "Coordination and Learning Behavior in Large Groups with Asymmetric Players." *Games and Economic Behavior* 39 (2002): 111-136.

3

Bidding for Attention and Competing for Votes in Political Debates

Gilat Levy
Department of Economics
London School of Economics and Political Science

Ronny Razin
Department of Economics
London School of Economics and Political Science

3.1. Introduction

It has long been established that political markets are not perfect in implementing the preferences of the median voter. The reasons for this may vary from having the need to compromise on multidimensional issues, uncertainty about the median's preferences, or from the ability of some politicians to use campaigns to sway voters away from the median. In this paper we want to further explore the interaction between election, supposedly a mechanism to elect moderates, and money, a tool which may allow extreme politicians (who might care more about winning) to divert outcomes away from the median and outcast moderates. Specifically, we are interested in the ability of money (or more generally, of effort), to *buy the attention of voters* and to *determine on what issues vote*. That is, while the literature has focused on fixed candidates but allowed these candidates to spend resources to sway voters in their favor, we instead explore how money allows politicians to be on the agenda in the first place, and possibly to block other candidates from being on it.[1]

[1] BBC News report on 23/12/2006 on the (then) upcoming Italian election describes exactly such efforts by Berlusconi: "*In the run-up to Italy's 9 April (2006) elections Prime Minister Silvio Berlusconi has been making an unprecedented number of appearances on Italian TV to get his message across and woo voters. It is not hard for Mr. Berlusconi to do so: his company, Mediaset, owns three TV stations - Canale Cinque, Italia Uno and Rette Quattro. Two of the three publicly-run stations: Rai Uno and Rai Due, are now run by his supporters—as they are traditionally controlled by the government of the day. Only one TV station, Rai Tre—the least watched and most*

We analyze the following simple model. There are two politicians whose ideal policy is their private information. The politicians care about the policy that will be chosen by a voter or a decision maker at the end of the campaign. The voter however can only choose among policies which are "on the agenda". Thus, prior to the actual decision, politicians engage in an all-pay auction whose winner can advertise his position to the voter. We allow for two periods of campaigning in order to explore the dynamics of the political debate. We also allow for some uncertainty, so that the politician is not sure whether in the beginning of the campaign, the other candidate has enough resources to bid at all.

The main features of political competition that the model focuses on are therefore as follows. First, what is on the agenda is endogenous, and depends on efforts of politicians and counter-efforts of their rivals. Second, the agenda is dynamic, and once one position is on it, it changes the incentives of other politicians to be on the agenda and hence their efforts. This can allow us to determine whether it is moderate or extremists politicians who take the initiative in such campaigns. Finally, the voter does have his say over the policies or candidates on the agenda.

We first consider a benchmark model in which there is only one round of campaigning, which implies that only one policy can be on the agenda and it therefore wins. We find that in this case there is a unique equilibrium in which the more extreme a politician is (compared with the median of the politicians' distribution of policies), the more he bids. As a result, it is the extreme politician who wins, a result which is detrimental to the voters. Intuitively, extremists are more afraid of losing to a politician whose position they do not know, and hence are willing to pay more.[2]

When we consider the model with two rounds of campaigning, elections become meaningful, as there is a possibility for both politicians' ideal policies to be on the agenda.[3] We find that in the first round, it is moderates who bid more, so that the moderate politician exposes his position first. Moreover, when the more moderate politician is exposed, he is not challenged. Thus, even when campaigns are fairly short, extreme politicians who care more about losing are still not able

---

cerebral of the three state channels—takes a different stance, critical of the Italian leader. Even this limited defiance has angered Mr. Berlusconi. He attacked one of Rai Tre's political discussion programss, Ballaro, as being "scandalous" left-wing propaganda...Mr. Berlusconi has been on TV or the radio every day, even popping up unannounced on chat shows. At one point, he spent half-an-hour discussing football on a talk show on one of his own TV stations." See http://news.bbc.co.uk/1/hi/world/europe/ 4744196.stm.

[2] This result is related to the town meetings model of Osborne *et al* (2000). In that paper agents invest a fixed amount in order to voice their opinions and the outcome is that extremists, who care more about losing, are the ones who invest.

[3] The results can extend to allow for more rounds of campaigning.

to buy the attention of voters outright and at each period in order to crowd out competition.

The key intuition for our result is that once an agent is exposed after the first round, he encounters a fierce competition in which he is disadvantaged as the information about his position is public. The gain of information rent by the rival politician implies that it is very costly to maintain himself solely on the agenda, and it is particularly costly for extremists who by exposing themselves encourage the rival politician to pay even more to avoid losing. Attention is therefore harder to sustain than to achieve in the first place. The more moderate the politician is though, the more he is willing to expose himself. Moreover, moderate politicians are indeed willing to pay in order to be exposed and win outright, rather than to engage in a bidding war in the second round against an exposed extremist.

Our conclusion therefore is that even when the voter's attention is somewhat limited, and is influenced by either money or effort, then the voter still fares relatively well. When he is more likely to make the right choice, as with a higher probability, he has only the moderate option, or both options, on the agenda.

Our work is related to several papers in political economy which consider money and election. Grossman and Helpman (2001) assume that money buys votes, that is, that some voters are impressionable and are affected by advertising. Thus, the more a candidate spends, the more likely he is to gain these voters. A second strand of this literature assumes that voters are rational and that campaign spending provides information to voters, either directly (as in Ashworth (2003), Coate (2004), or Schultz (2003)), or indirectly, as in Prat (2002). In Coate's paper, political advertising is informative as politicians use it to reveal their quality. In Prat (2002), advertising is a way of burning money which allows-politicians to signal their quality.

Our approach falls in between the two approaches discussed above. Voters are not rational in the sense that they do not update their beliefs on the type of politicians given the messages they hear. They are restricted to vote for policies on the agenda. On the other hand, votes are not guaranteed to the one who spends more, as money only buys the attention of voters. Such votes may arise endogenously. Finally, in the papers mentioned above, politicians receive money from interest groups who in turn expect political favors. We abstract from this and do not consider the source of the money that is available to politicians.

Our model is also related to the citizen-candidate literature (Osborne and Slivinski 1996; Besley and Coate 1997). In that literature, politicians are ideological, their positions are known, and they endogenously choose whether to offer them to voters or not. In our model, politicians also choose whether to offer their position to voters, but to do so they must compete and win an all-pay-auction first.

Our analysis, although motivated by political campaigns, should apply more generally to debates in other contexts. In debates in committees or in judicial courts, it is often the case that individuals need to put their positions on the table for the decision maker to consider it. To do so demands investment, such as exerting effort in producing a coherent position or in arguing for it persuasively. Thus, in more general environments, decision making processes may be characterised by a "debate" phase which involves an all-pay-auction in which different agents compete for the decision maker's attention, and an "election" phase, involving the decision maker choosing among the items on the agenda. In these more general contexts, our paper is related to the (small) literature on the dynamics of debates. Glazer and Rubinstein (2001) focus on optimal debate rules when there are time constraints. Ottaviani and Sorensen (2001) ask who should speak first when heterogeneously informed agents have career concerns and may therefore be subject to herd effects.

The remainder of the paper is organized as follows. In the next section we describe the model. In section 3.3 we analyze a benchmark in which the election mechanism plays no role. We analyze the model and compare it to the benchmark in Section 3.4. We conclude in Section 3.5. All proofs are relegated to an Appendix.

3.2. The model

There are two agents (politicians), denoted by $i \in \{1,2\}$, each with an ideal policy x_i in $[-1,1]$. Ideal policies are distributed according to $g(x)$ which is symmetric around 0, and agents' types are drawn independently. We assume that $g(x)$ is continuous, atomless and positive for any x. Ideal policy is private information. There is one voter with ideal policy at 0.

There are three periods to the game. The first two periods are the "campaign" periods, and the third period, is the "election", i.e., the voter casts his vote. The voter can only cast his vote for an ideal policy which is "on the agenda", i.e., that was revealed in the campaigning stage. The chosen policy is denoted by y. It is important to consider at least a two-period campaign, as we will show below. To have longer campaigns will not change the qualitative results reported in this paper.

At each of the first two periods $t \in \{1,2\}$, agents $i \in \{1,2\}$ simultaneously bid $b_i^t \geq 0$ for the voter's attention, in an all-pay auction. Bids are not observed. The agent with the highest bid can reveal his ideal policy x_i (if both agents bid the same amount each is heard with a positive probability).[4] The agent with the lowest bid is not heard. Agents can keep on bidding even though their ideal policy was revealed. By bidding the highest amount at each round, an agent insures that only his ideal policy is on the agenda.[5]

We introduce some uncertainty in the game by assuming that in the first campaigning round, for each politician, there is a positive probability α that he has no resources to bid. This probability is exogenous and represents imperfect campaigning or fund raising. Technically, this uncertainty insures that an equilibrium exists.

The voter chooses the closest policy to his ideal policy 0, given the set of policies that was revealed in the campaign Note that the voter is constrained to choose among policies in the agenda. This implies therefore that politicians are not engaged in signalling their type to voters using their bids. The agents' utility from the outcome and their bids is:[6]

$$-|y - x_i| - \sum_{t=1,2} b_i^t \tag{3.1}$$

We solve the game played by the politicians by backward induction using a (weak) Perfect Bayesian equilibrium. We will focus on continuous bidding functions, and, when reasonable, on symmetric equilibria, i.e., bidding functions which depend only on the distance of the ideal policy from 0, so that $b_i(x) = b_i(-x)$.

3.3. A benchmark: one round of campaign

In this section we consider a benchmark in which the campaigning period consists of only one round. This environment can represent two institutional features. One is a literal interpretation of the model, in which time (or possibly) cognitive constraints play a role, and the voter is therefore restricted to observe one message only. A second interpretation is that in such a society, election or the voter's preferences play no role; as only one policy will be on the agenda, and

[4] A politician may prefer to be silent even if he has the attention of the voter. Our qualitative results should be sustained even if we allow for this possibility.

[5] It is possible to allow politicians to send a message which is different than their ideal policy. We leave this (more complicated) possibility for future research.

[6] For exposition purposes we focus on a simple utility function. The results can be generalized to other concave functions decreasing in the Euclidean distance.

this policy is the ideal policy of the highest bidder, the voter's choice is essentially determined as the ideal policy of the politician who is willing to pay more.

The analysis of this game reveals that in the unique equilibrium, the more extreme the politician is, the higher his bid is:

Proposition 3.1. *There is a unique symmetric equilibrium in the game with only one round of campaigning, in which b(0)=0 and the bidding function strictly increases in the distance of a politician from the median. Thus, the politician with the more extreme ideal policy wins.*

The intuition for this result is straightforward. The closer an agent is to *the median of the distribution of the politicians*, which is also at 0, the less afraid he is of losing to another politician. Thus, moderate politicians are less willing to bid highly. Conversely, the intensity of preferences of extremists is higher, and they are willing to fiercely compete in the bidding wars. Extremists are therefore more likely to win.

With one round of campaigning it is the intensity of preferences that plays a role, and extremists gain the upper hand as they care more about the outcome and are therefore willing to pay more. We now proceed to analyze the model in which there are two campaigning rounds and check whether this intuition carries through, or whether the election phase, which gives voice to the preferences of the voter, can override it.

3.4. Equilibrium analysis

Once we analyze the game in full, we can investigate not only whether moderates or extremists bid highly, but also when they choose to bid, i.e., whether moderates talk first and extremists fight back, or whether it is extremists that open the debate. We first analyze the second round of campaigning.

3.4.1. Bidding for attention: 2nd round

The key feature of the second round of campaigning, is that the ideal policy of one of the agents is *already exposed*. The two politicians are not symmetric any more. On one hand, the exposed politician has the advantage that he might be the only one on the agenda, which is beneficial for extremists and may induce them to expose themselves in the first round. On the other hand, the exposed agent's ideal policy is known to the two players whereas that of the politician who lost is still his private information. He therefore has an information advantage over the exposed agent. Thus, agents may prefer to wait and let the other one reveal himself to gain some information rents.

We proceed with the following Lemma that will help us characterize the 2nd round equilibrium, where we assume that some x was exposed in the first stage.

Lemma 3.1. *The exposed agent x must play a mixed strategy, which includes a bid of zero in its support.*

Note that if the exposed agent uses a pure strategy with some positive bid b, then the other type either bids zero to lose, or bids $b + \varepsilon$ for some small ε. This implies that bidding b cannot be optimal as the exposed agent wins in a wasteful manner and can decrease b to ε.[7]

To see the intuition behind why it includes a bid of zero, note that his lowest bid, if positive, cannot be lower than the bid of the unexposed type (as otherwise he can lower his bid even further), and vice versa. Thus they must have the same lowest bound on their bids. But the lowest bound must be zero as this lowest bid loses to all so any positive amount is wasteful.

Lemma 3.2. *The unexposed agent places a strictly positive bid if his type is in (-x,x) and a zero bid otherwise.*

As there is always a positive probability of winning for all types in $(-x,x)$, the bid of such types is positive, whereas all other types, who are more extreme than x, place a bid of zero.

As all (more moderate) unexposed types bid positive amounts, where the exposed agent x plays a mixed strategy with zero in its support, and as the expected utility for any bid in the support of x has to be equal, we have the following corollary:

Corollary 3.1. *The expected utility of an exposed agent x in the second round is the expected utility from bidding zero and hence losing to a more moderate unexposed agent.*

3.4.2. Bidding for attention: 1st round

We now show that the equilibrium behavior in the first stage of the game is very different compared with the one-round campaign game analyze in the

[7] A similar argument shows why x cannot use a bid of zero in with probability 1 in equilibrium.

benchmark case. The key feature of the analysis is the following. When a player considers lowering his bid, it means that instead of exposing himself in the first round, he lets others be exposed. Whether therefore moderates or extremists wish to bid higher or lower in the first stage, will depend on the relative gain (or loss) of exposing others compared with exposing oneself.

Note however that when an extremist bids low, and lets a more moderate type than him win the first round, he will not bid against him in the second round as he has no chance of winning. On the other hand, if he bids high and exposes himself, then the more moderate type will engage in a bidding war with him in the second stage, and the extremist's utility from that bidding war is the utility from losing.

Thus, for an extremist, bidding high and exposing himself is just a waste of money in the first round, as he will "lose" in any case in the second round.[8] This already implies that an equilibrium as in the benchmark model cannot exist; in such an equilibrium a politician wins in the first round against those who are more moderate than him, only to "lose" to them in the second stage. He will therefore rather reduce his bid (all the way to zero) and let those moderates win outright, to save the cost of his bid. In fact, we show that the unique type of equilibrium is the one in which it is moderates who pay more so they can win outright:

Proposition 3.2. *In any equilibrium, the first round of campaigning is characterised by bidding functions which decrease in the distance of the politician from the median, where the most extreme politician pays zero.*

For a moderate, bidding high can be beneficial; he can win against an extremist straight away without the extremist challenging him. If he on the other hand lets the extremist win, he will have to engage in a bidding war in the second round, in which he wins only with some probability as well as invests resources. His relative gain from winning outright is therefore strictly positive and we can find a small enough bid that each politician pays such that he does not wish to reduce his bid and forgo the opportunity to win outright against agents who are more extreme. The bid must be decreasing in one's type to reflect the relative gain of a more moderate politician from exposing himself. Note that this is the only constraint to satisfy (as politicians never want to increase the price they pay and win against more moderates).

[8] Note that the politician does not actually lose with probability 1, but has the utility of losing, as stated in Corollary 1.

Thus, even when campaigns are relatively short and last for a few periods only, the possibility of an agent responding to another, already exposed, politician implies that moderates have the advantage. Extreme politicians, who care more about losing, are not able in equilibrium to buy attention outright and at each period to crowd out competition. The reason is that attention is much harder to sustain than to achieve in the first place for an extreme politician. Even if attention were cheap to buy in the first round, once the extreme politician is exposed, he will have to pay a lot in order to sustain it in the second round as he is facing fierce competition and losing information rent. Note that he will have to pay much more to fend off competition when he is exposed than what he pays to win against all in the benchmark equilibrium of the one-round game in which politicians compete against an unknown rival. Maintaining attention is therefore not worthwhile to the extreme politician who gives up already in the first round.

Proposition 3.3. Above describes the equilibrium behavior in the first stage of the game in which moderates pay more. The corollary below summarizes the predictions of the equilibrium of the game with the two-round campaigns, for a fixed set of two politicians.

Corollary 3.2. *(The dynamics of debate): In equilibrium, (i) with probability* $1 - \alpha$ *the moderate speaks first, and then again in the second period, (ii) with probability* $\alpha(1 - \alpha)$ *the extremist speaks first. With some positive probability the extremist speaks also in the second period and with some probability the moderate responds, (iii) with probability* α^2 *none speaks at the first period, and the extremist speaks in the second period. Thus, the moderate wins with probability of at least* $(1 - \alpha)$ *and the extremist wins with probability of at least* α^2.

Our conclusion from the model so far, is that even when the voter's attention is somewhat limited, and is influenced by either money or effort, then the voter still fares relatively well. He is more likely to make the right choice, as with a higher probability, he has only the moderate option, or both options, on the agenda.

3.5. Extensions and conclusion

We have proposed a simple model to analyze the interaction of election and auction mechanisms in political debates or campaigns. Our future research will focus on the following questions we consider below.

An immediate and obvious question to answer is how the welfare of the voter is affected by the parameters of the model, such as the distribution over politicians' ideal policies and the uncertainty parameter α. In addition, it is a simple model in which we can ask how campaign limits can affect the voter's welfare.

We can extend the model to consider several alternative assumptions. First, we can allow politicians to commit to a different platform other than their own ideal policy. Second, we can extend the number of periods and the number of politicians considered, and extend the uncertainty to other periods other than the first one. Finally, we can consider other, more smooth, election mechanisms, such as probabilistic voting.

On a more general level, an important question is whether, in the presence of time or attention constraints, the mechanism of auction as considered here, is detrimental or helpful to the voter. We have shown here that when there are enough periods of campaigning, and when election is anticipated, the auction mechanism allows for the more moderate politicians to be on the agenda after the first period with a higher probability, and is therefore a good mechanism to aggregate information about politicians' policies. It remains to be seen whether this mechanism is indeed welfare maximizing in more general set ups.

Appendix

Proof of Proposition 3.1.

(i) Uniqueness: it is easy to see that there is no interval with positive measure in which the bidding function is neither increasing nor decreasing. If this is the case, then we can find an infinitesimally small ε so that any x in this interval can increase his bid by ε but increase his probability of winning by a positive probability and hence his utility.

We now show that there cannot be $x' > x > 0$ with $b(x') < b(x)$ (we are dropping the time index from the bidding function as there is only one period). Suppose there is and consider such x, x' where the bidding function decreases for all $z \in x, x']$. Such an interval must exist by continuity. Let $Z = \{z \mid b_z \in b(x'), b(x)]\}$. Consider the incentive compatibility constraint for both x and x'. These satisfy:

$$(1-\alpha)\left(\int_{Z \setminus x, x']} (|v-x|)\, g(v)\, dv + \int_x^{x'} (|v-x|)\, g(v)\, dv\right)$$
$$\geq b(x) - b(x')$$
$$\geq (1-\alpha)\left(\int_{Z \setminus x, x']} (|v-x'|)\, g(v)\, dv + \int_x^{x'} (|v-x'|)\, g(v)\, dv\right)$$

Note that $\int_x^{x'} (|v - x'|)\, g(v)\, dv = \int_x^{x'} (|\, v - x|)\, g(v)\, dv$. Note also that by symmetry of the equilibrium bidding function,

$$-\int_{Z \setminus x,\, x']} (|v - x|)\, g(v)\, dv > -\int_{Z \setminus x,\, x']} (|\, v - x'|)\, g(v)\, dv,$$

a contradiction.

(ii) Existence: The following increasing bidding function

$$b(x) = (1 - \alpha) \int_0^x |2v|\, g(v) dv$$

Satisfies the incentive compatability constraints which are

$$(1 - \alpha) \int_x^{x'} (\,|v - x| + |\, x + v\,|)\, g(v)\, dv \leq b(x') - b(x) \leq$$
$$(1 - \alpha) \int_x^{x'} (\,|\, v - x'| + |\, x' + v\,|)\, g(v)\, dv. \blacksquare$$

Proofs of Lemma 3.1 and Lemma 3.2. Note first that unexposed agents at $[-x,x]$ are the only ones that can potentially bid positive amount as they are the only ones that can win. Suppose that agent x is exposed. Let $b_x(b)$ be the distribution over bids that x plays in equilibrium. Let $\underline{b} = \inf b_{b \setminus b_x(b)>0}$.

We first show that $\underline{b} = 0$. Suppose that $\underline{b} > 0$. Consider the unexposed player: None of his types place bids in $(0, \underline{b})$. Moreover, there cannot be an atom of types that bid $\underline{b}$. If there were, these types should bid zero instead unless $b_x(b)$ places an atom on $\underline{b}$. The latter cannot arise in equilibrium as x loses (with a strictly positive probability) when bidding $\underline{b}$, against players in $[-x,x]$ who bid the same amount. Hence the utility of x from bidding some $\underline{b} + \varepsilon$ has to converge to be strictly lower than bidding ε for $\varepsilon \to 0$.

We now show that $b_x(b)$ does not include any atoms. Suppose it places an atom on some b. Then it cannot be that there is an atom of unexposed types who place bid b (as above). Note that some unexposed types must bid below b and some above it. Moreover, the bidding function of the unexposed agent is continuous in types and that of the exposed has no gaps (proof similar to the above). Thus an unexposed agent who bids b can for a small enough ε bid $b + \varepsilon$ and increase his probability of winning by a strictly positive probability. Hence, $b_x(b)$ is a continuous function.

Now consider the best response of an unexposed agent $y \in -x,x]$. His expected utility from a bid b is (where $\bar{b} = \sup b_{b \setminus b_x(b)>0}$):

$$-\int_b^{\bar{b}} |\, y - x\,|\, b_x(b)\, db - b$$

and the first order condition is

$$b_x(b(y))\,|\,y-x\,|-1$$

Suppose that some agents in $[-x,x]$ bid zero. It can be easily shown that these must be in $[z,\ x]$ for some z. Consider now the exposed agent's expected utility:

$$-\int_{-x}^{b^{-1}(b)}|\,y-x\,|\,g(y)\,dy-b$$

and his first order condition is

$$b'(y)=-\,|\,y-x\,|\,g(y)$$

Consider now condition (1). We know that $1=\int_0^{\bar{b}} b_x(b)\,db=-\int_{-x}^{z} b_x(b(y))\,b'(y)\,dy\leq -\int_{-x}^{x} b_x(b(y))\,b'(y)\,dy=-\int_{-x}^{x}\frac{1}{|x-y|}(-\,|\,y-x\,|\,g(y))\,dy<1$, which implies that there must be an atom in $b_x(b)$, a contradiction as both sides have atoms. Thus, z must equal to x. ■

Proof of Proposition 3.2.

Let $v(x,z^*)$ denote the utility of x facing a player z when z is exposed after the first round, and similarly let $v(x^*,z)$ be the utility of player x facing z when x is exposed. Finally let

$$\Delta_x(z)=v(x,z^*)-v(x^*,z)$$

Note that if $0<x<z$, then $\Delta_x(z)=v(x,z^*)=-\Pr(b^2_{z^*}(x)<b^2(z^*))(z-x)-b^2_{z^*}(x)$ where $b^2_{z^*}(x)$ $(b^2(z^*))$ (denotes the second period bid of $x(z)$ when z is exposed. On the other hand, if $x>z>0$, then $v(x,z^*)=-(x-z)$.

(i) Uniqueness: consider some symmetric equilibrium. We will show that there cannot be an interval $[x,x']$ on which the bidding function is increasing. Consider all $Z=\{z\,|\,b(z)\in(b(x),b(x')),\ z\notin -x',-x]\cup x,x']\}$. If $b(x')>b(x)$ then by the IC constraints:

$$b(x')-b(x)\leq-(1-\alpha)\left(\int_x^{x'}(\Delta_{x'}(z)+\Delta_{x'}(-z))\,g(z)\,dz+\int_Z\Delta_{x'}(z)\,g(z)\,dz\right)$$

$$b(x')-b(x)\geq-(1-\alpha)\left(\int_x^{x'}(\Delta_x(z)+\Delta_x(-z))\,g(z)\,dz+\int_Z\Delta_x(z)\,g(z)\,dz\right)$$

Note that for all $z > x'$ or for $z < -x'$ then by the envelope theorem,

$$\frac{d(\Delta_x(z) + \Delta_x(-z))}{dx} = \Pr(b^2_{z^*}(x) < b^2(z^*)) - \Pr(b^2_{-z^*}(x) < b^2(-z^*)) > 0$$

as in the second stage, both z and $-z$ use the same bidding functions, but x bids lower by Lemma 3.1 when c is exposed than when $-z$ is. Thus, $\Delta_x(z) + \Delta_x(-z)$ increases in x, for these values of z. Recall also that the expected utility from being exposed is like the expected utility of losing to the more moderate players who are not exposed. Thus, the above two constraints cannot simultaneously hold.

It is also easy to see that no equilibrium can have a flat bidding function on any interval as then the most moderate type in this interval would increase his bid by ε.

(ii) Existence: we prove existence by constructing an equilibrium. We show this for general single peaked utilities $V(|x_i - y|)$.

Let

$$b(x) = -\mu(1-\alpha)\int_x^1 (v(z,z^*))\,g(z)\,dz$$

Hence

$$b(x) - b(x') = -\mu(1-\alpha)\int_x^{x'} (v(z,z^*) + (v(z,-z^*))\,g(z)\,dz$$

Recall that the IC constraint is

$$b(x) - b(x') \leq -\mu(1-\alpha)\int_x^{x'} (v(x,z^*) + v(x,-z^*))\,g(z)\,dz$$

The function above satisfies this if $\mu < \inf_{x,x'} \dfrac{\int_{-x'}^{x} (v(x,z^*) + v(x,-z^*))\,g(z)\,dz}{\int_x^{x'} (v(z,z^*) + v(z,-z^*))\,g(z)\,dz}$.

Note that:

(i) $\inf_x \lim_{x' \to x} \dfrac{\int_{-x'}^{-x} (v(x,z^*) + v(x,-z^*))\,g(z)\,dz}{\int_{-x'}^{-x} (v(z,z^*) + v(z,-z^*))\,g(z)\,dz} = 1$, as

$$\lim \frac{\int_{-x'}^{-x} \frac{(v(x,z^*) + v(x,-z^*))}{x' - x}\,g(z)\,d}{\int_{-x'}^{-x} \frac{v(z,z^*) + v(z,-z^*)}{(x' - x)}\,g(z)\,dz} = \frac{(v(x,x^*) + v(x,-x^*))\,g(x)}{(v(x,x^*) + v(x,-x^*))\,g(x)}$$

$$\Big|_{x \to x'}$$

(ii) For any ε, $\inf_{|x-x'|>\varepsilon} \dfrac{-\int_{-x'}^{-x} (v(x,z^*) + v(x,-z^*))\,g(z)\,dz}{-\int_{-x'}^{-x} (v(z,z^*) + v(z,-z^*))\,g(z)\,dz} \geq f(\varepsilon) > 0$.

First, $-\int_{-x'}^{-x} (v(z,z^*) + v(z,-z^*))\,g(z)\,dz \leq k$ for some finite k, as the utility is bounded. Second, the nominator is bounded from below as

$$-\int_{-x'}^{-x} (v(x,z^*) + v(x,-z^*))\,g(z)\,dz$$

$$\geq -\int_{-x'}^{-x} (v(x,z^*)\,g(z)\,dz$$
$$\geq -\int_{-x'}^{-x} b_x(z)\,g(z)\,dz = \int_{-x'}^{-x} \alpha\,(\int_z^x V(|v-z|)\,g(v)dv)g(z)\,dz > 0$$

Where the last equality follows from the second stage characterization that $b_x(z) = \alpha\int_z^x V(|v-z|)\,g(v)\,dv$. The above implies that

$$\inf_{|x-x|>\varepsilon} \int_{-x'}^{-x} \alpha(\int_z^x V(|v-z|)\,g(v)\,dv)\,g(z)\,dz > 0.$$

By (i) for $\delta = \frac{1}{2}$, there exists $\varepsilon(\frac{1}{2})$ such that for any $|x'-x| < \varepsilon(\frac{1}{2})$,

$\dfrac{\int_{-x'}^{-x} (v(x,z^*) + v(x,-z^*))\,g(z)\,dz}{\int_{-x'}^{-x} (v(z,z^*) + v(z,-z^*))\,g(z)\,dz} > 1 - \delta = \frac{1}{2}$. And by (ii), for $|x'-x| \geq \varepsilon(\frac{1}{2})$,

$\dfrac{\int_{-x'}^{-x} (v(x,z^*) + v(x,-z^*))\,g(z)\,dz}{\int_{-x'}^{-x} (v(z,z^*) + v(z,-z^*))\,g(z)\,dz} > f(\varepsilon)$, so we can take $\mu < \min\{f(\varepsilon(0.5)), 0.5\}$. ■

References

ASHWORTH, S. *Campaign Finance and Voter Welfare with Entrenched Incumbents.* Working paper, Harvard, 2003.

BESLEY T. and S. COATE. "An Economic Model of Representative Democracy." *Quarterly Journal of Economics* 108 (1997): 85-114.

COATE, S. "Pareto Improving Campaign Finance Policy." *American Economic Review* 94, no. 3 (2004): 628-655.

GLAZER J. and A. RUBINSTEIN. "Debates and Decisions, On a Rationale of Argumentation Rules." *Games and Economic Behaviour* 36 (2001): 158-173.

GROSSMAN, G. M. and E. HELPMAN. *Special Interest Groups.* MIT Press, 2001.

PRAT, A. "Campaign Advertising and Voter Welfare." *Review of Economic Studies* 69, no. 4 (2002): 997-1017.

OSBORNE, M., J. ROSENTHAL, and M. TURNER. "Meeting with Costly Participation." *American Economic Review* 90, no. 4(2004): 927-943.

OSBORNE M. and A. SLIVINSKI. "A Model of Political Competition with Citizen-Candidates." *Quarterly Journal of Economics* 111 (1996): 65-96.

OTTAVIANI M. and P. SORENSEN. "Information Aggregation in Debate: Who Should Speak First?." *Journal of Public Economics* 81 no. 3 (2001): 393-421.

SCHULTZ, C. "Strategic Campaigns and Redistributive Politics." *The Economic Journal* 117, no. 522 (2007): 936-963.

4

Many Enemies, Much Honor? On the Competitiveness of Elections in Proportional Representation Systems

Matias Iaryczower
Division of Humanities and Social Sciences
California Institute of Technology

Andrea Mattozzi
Division of Humanities and Social Sciences
California Institute of Technology

4.1. Introduction

About one third of all countries and more than thirty percent of all established democracies use a proportional representation (PR) electoral system. In its purest form, a proportional electoral system maps the share of votes obtained by each party in the election into an equal share of seats in the legislature. Since the seminal work of Duverger (1954), PR has been held responsible (at least partially) for the proliferation of political parties in PR democracies. More recently, the political debate shifted its focus to the relation between the number of legislative parties and the quality of the political environment in terms of competence, or corruption of elected politicians. While the existing literature contains numerous studies supporting Duverger's hypothesis, the connection between the number of legislative parties and their investement in *quality* has been overlooked both theoretically and empirically.[1] In this paper, we build on Iaryczower and Mattozzi (2008a) to develop a simple theoretical framework in which the quality and the number of candidates running for office are endogenous equilibrium outcomes,

[1] Regarding the relation between number of candidates/parties and electoral systems see Osborne and Slivinski (1996), Persson, Roland, and Tabellini (2003), Morelli (2004) and Iaryczower and Mattozzi (2008b) among others. Regarding the relation between electoral systems and corruption, see Myerson (1993) and Persson, Tabellini, and Trebbi (2006).

and provide conditions under which elections in PR would result in a positive association between the quality and number of candidates running for office.

The essential features of the model are the following. Potential candidates are horizontally differentiated according to a policy position they represent. In particular, they are endowed with a policy position they can champion in government if they choose to run for office and get elected.[2] With their given policy positions, candidates who choose to run for office then compete along a bounded vertical dimension, which we represent as costly activities (investment of money, time or effort) that increase voters' perception of the quality of a candidate's platform.[3] Politicians derive utility exclusively from rents they can appropriate while in office. We assume that there is a large finite number of risk averse and fully rational voters.

The mapping of votes' shares into seats' shares is given by the electoral system. In this paper we consider the case of a perfect PR system, where vote shares are transformed into seat shares one to one. Regarding the mapping from seats to the distribution of rents, we assume that candidates participate in the distribution of rents proportionally to the share of seats obtained in the election (see for example Lizzeri and Persico (2001)). As for policy outcomes, we adopt the simplifying assumption that the policy outcome is given by a lottery between the policies represented by the candidates participating in the election, with weights equal to their vote shares (or seat share in the assembly). This assumption captures in a stylized fashion the additional uncertainty faced by voters that is introduced by the process of post-election bargaining in PR.[4] An *electoral equilibrium* is a Subgame Perfect Nash Equilibrium in pure strategies of the game of electoral competition, i.e., a strategy profile such that *(i)* voters cannot obtain a preferred policy outcome by voting for a different candidate in any voting game (on and off the equilibrium path), *(ii)* given the location and quality decisions of other

---

[2] For models of differentiation and entry in industrial organization see Anderson and de Palma (1988), Anderson, de Palma, and Thisse (1989), Caplin and Nalebuff (1986), Perloff and Salop (1985), Shaked and Sutton (1982), Shaked and Sutton (1987), and d'Aspremont, Gabszewicz, and Thisse (1979).

[3] This is in the line of Stokes (1963)'s early criticism to the spatial model and the recent literature incorporating vertical differentiation in majoritarian elections. See Groseclose (2001), Aragones and Palfrey (2002), Schofield (2004), Herrera, Levine, and Martinelli (2008), Carrillo and Castanheira (2008), Meirowitz (2007), and Ashworth and Bueno de Mesquita (2007).

[4] Austen-Smith and Banks (1988), Baron and Diermeier (2001), and Persson, Roland, and Tabellini (2003) study strategic voting induced by the process of government and coalition formation among elected representatives in PR for a given number of parties (three for Austen-Smith and Banks (1988), Baron and Diermeier (2001), four for Persson, Roland, and Tabellini (2003)). Iaryczower and Mattozzi (2008b) explore alternative specifications of the policy function mapping elected representatives into policy outcomes.

candidates, and given voters' strategy, candidates cannot increase their expected rents by modifying their investment in quality, *(iii)* candidates running for office collect non-negative rents, and *(iv)* candidates not running for office prefer not to enter: they would make negative rents in an equilibrium of the continuation game.

We start our analysis by focusing on electoral equilibria with two candidates running for office, and we construct an equilibrium where candidates obtain no rents. In electoral equilibria in which two candidates run for office without choosing maximal quality, candidates invest more in quality the less differentiated they are in the policy space and, given differentiation, the weaker is voters' *ideological focus* (Stokes 1963). In an equilibrium with no rents, however, a heightened responsiveness of voters to candidates' quality must result in a larger ideological differentiation between candidates running for office, without (directly) influencing the equilibrium investment in quality. We then extend the analysis to symmetric electoral equilibria with three or more candidates running for office. Within symmetric equilibria, a larger number of candidates leads to less differentiation in the ideological dimension, and thus to candidates being closer substitutes for each other. For the same reason as in two-candidate equilibria then, in symmetric equilibria candidates invest more in quality the larger the number of candidates running for office.

Within the class of symmetric electoral equilibria, we also explore how changes in the "supply side" parameters of the model can affect the number and quality of candidates running for office. In particular, we show that changes in the fixed cost of running for office, or shifts in the cost function of quality induce a positive correlation between the equilibrium number of candidates running for office and their quality. We also explore the role of "demand-side" factors, such as the responsiveness of voters to quality differentiation among candidates. We show that in symmetric equilibria in which no candidate obtains positive rents, a less ideologically focused electorate leads to more differentiation in the policy positions represented in the election, and to a smaller number of candidates. If instead candidates obtain positive rents in equilibrium, the impact of demand-side factors can be absorbed by the expected level of rents without affecting the number of candidates running for office.

Finally, we also show that the positive relation between quality and number of parties extends to the case of limited asymmetry among equilibrium candidates. In particular, we show that in this case there is a positive equilibrium relation between the quality of candidates and the *effective number of parties* (Laakso and

Taagepera (1979)). This is consistent with anecdotal evidence linking (perceived) corruption among public officials and politicians and the effective number of parties.

4.2. The basic model

Let $X = [0,1]$ be the ideological policy space. In any $x \in [0,1]$ there is a potential candidate who can perfectly represent policy x if elected. There are three stages. In the first stage, all potential candidates simultaneously decide whether or not to run in the election. In order to run, a candidate must pay a fixed cost $F > 0$. We denote the set of candidates running for office at the end of the first stage by K={1,..,K}. In the second stage, all candidates running for office simultaneously choose a level of quality $\theta_k \in [0,1]$ at a cost $C(\theta_k)$. We assume that $C(\cdot)$ is increasing and convex, and let $C(1) \equiv \bar{c}$. In the third stage, a large finite number n of fully strategic voters vote in an election.

A voter i with ideal point $z^i \in X$ ranks candidates according to utility function $u(\cdot; z^i)$, which assigns the payoff $u(\theta_k, x_k; z^i) \equiv 2\alpha v(\theta_k) - (x_k - z^i)^2$ to candidate k with characteristics (θ_k, x_k). We assume that $v(\cdot)$ is increasing and concave, and define the function $\Psi(\cdot) \equiv \frac{v'(\cdot)}{C'(\cdot)}$. The parameter $\alpha > 0$ measures voters' responsiveness to candidates' quality. Voters' ideal policies are uniformly distributed in X. Letting s_k denote the proportion of voters voting for k, and m_k denote k's proportion of seats in government after the election, we assume that $m_k = s_k$ (perfect PR). The final policy outcome is a lottery among the candidates participating in the election, with weights equal to their vote shares in the election (or seat share in the assembly). The expected share of rents captured by each candidate is proportional to his vote share in the election. Letting θ_K and x_K denote the vector of quality and ideological positions of candidates running for office, and normalizing the payoff of potential candidates not running for office to zero, the payoff of candidate $k \in K$ is given by

$$\Pi_k (\theta_K, x_K, K) = m_k (\theta_K, x_K) - C(\theta_\kappa) - F. \tag{4.1}$$

A strategy for candidate k is a decision of whether to run ($e_k = 1$) or not for office, and a plan of investment in quality $\theta_k(K, x_K) \in [0,1]$. A strategy for a voter i is a function $\sigma_i(K, x_K, \theta_K) \in K$, where $\sigma_i(K, x_K, \theta_K) = k$ indicates that the choice of voting for candidate k, and $\sigma = \{\sigma_1(\cdot),..., \sigma_n(\cdot)\}$ denotes a voting strategy profile. An *electoral equilibrium* is a Subgame Perfect Nash Equilibrium of the game of electoral competition; i.e., a set of candidates running for office K^*, policy positions $x^*_{K^*}$, quality choices $\theta^*_{K^*}$, and a voting profile σ^* such that:

(i) θ^*_k is optimal for k given $\{\theta^*_{K^*\setminus k}(K^*, x^*_{K^*}), x^*_{K^*}, \sigma(K^*, x^*_{K^*}, \theta^*_{K^*\setminus k}, \theta^*_{K^*})\}$; i.e., $\theta^*_{K^*}$ is a (pure Nash) equilibrium of the continuation game Γ_{K^*};

(ii) if $k \in K^*$, then $\Pi_k (K^*, x^*_{K^*}, \theta^*_{K^*}, \sigma^*(K^*, x^*_{K^*}, \theta^*_{K^*})) \geq 0$ (no exit condition);

(iii) if $k \notin K^*$, then $\Pi_k (K^*, \cup k, x_k, x^*_{K^*}, \theta^*_k, \theta^*_{K^*}, \sigma^* (K^*, \cup k, x_k, x^*_{K^*}, \theta^*_k, \theta^*_{K^*})) < 0$, in an equilibrium of the continuation game (non-profitable entry). An *outcome* of the game is a set of candidates running for office K, policy positions x_K, and quality choices θ_K. A *polity* is a triplet $(\alpha, \bar{c}, F) \in \Re^3_+$. We say that Proportional Representation *admits* an electoral equilibrium with outcome (K, x_K, θ_K) if there exist a set of polities $P \subseteq \Re^3_+$ with positive measure such that if a polity $p \in P$ then there exists an electoral equilibrium with outcome (K, x_K, θ_K).

4.3. Results

We start by characterizing the properties of electoral equilibria with two candidates running for office. First note that, in the absence of investement in quality, equilibrium imposes only relatively weak constraints on the composition of the field of candidates. In particular, the equilibrium requirement of non-negative rents for candidates running for office implies a lower bound on ideological differentiation, while the no-entry condition imposes an upper bound on ideological differentiation. Consider next two candidates 1 and 2 representing policy positions $x_1 = \Delta_0$ and $x_2 = x_1 + \Delta$, with quality θ_1 and θ_2, and let $\tilde{x}_{12} \in R$ denote the (unique) value of x such that $u(\theta_1, x_1; x) = u(\theta_2, x_2; x)$, so that $u(\theta_1, x_1; z^i) > u(\theta_2, x_2; z^i)$ if and only if $z^i > \tilde{x}_{12}$

$$\tilde{x}_{12} = \frac{x_1 + x_2}{2} + \alpha \frac{v(\theta_1) - [v(\theta_2)]}{\Delta}. \quad (4.2)$$

Note next that in our model strategic voting is in fact equivalent to sincere voting on and off the equilibrium path. Since the probability that each candidate running for office is elected and implements his ideology is proportional to the share of votes received in the election, voting for a candidate who is not the most preferred one is always a strictly dominated strategy. In fact, by switching her vote to her most preferred candidate, a voter only affects the lottery's weights of exactly two candidates and, with two alternatives, strategic voting and sincere voting coincide.[5] Thus candidate 1's vote share given (x, θ) is $m_1(\theta, x) = \min\{0, \tilde{x}_{12}\}$. Note that if $\theta_1 \geq \underline{\theta}_1(\theta_2, x)$, where $m_1(\underline{\theta}_1(\theta_2, x); \theta_2, x) \equiv 0$, the vote share mapping $m_\kappa(\theta_k; \theta_{-k}, x)$ is differentiable and the marginal vote share is given by

$$\frac{\partial m_1}{\partial \theta_1} = \frac{\partial v'(\theta_1)}{\Delta}, \tag{4.3}$$

that is, the marginal impact of quality on vote share given the identity of k's relevant competitors is well-defined, increases with α, and decreases with Δ. In the next proposition we focus on equilibria in which exactly two candidates run for office.

Proposition 4.1. *Proportional Representation admits an electoral equilibrium in which exactly two candidates run for office. In any two-candidates equilibrium, candidates choose the same quality,*

$$\theta_1^* = \theta_2^* = \theta^* = \Psi^{-1}\left(\frac{\Delta}{\alpha}\right) = \leq C^{-1}\left(\frac{1}{2} - F\right). \tag{4.4}$$

Furthermore, the more responsive are voters to differences in quality between candidates (the higher is α), the higher is candidates' investment in quality and, if candidates do not capture positive rents, also the higher is the degree of ideological polarization between candidates (Δ).

Proof. To prove this result, we show that if $\bar{c} \leq \frac{1}{4}$, $\bar{c} + F > \frac{1}{2}$, and, $\frac{2\bar{c}}{\Psi(C^{-1}(\frac{1}{2} - F))} \leq \alpha \leq \frac{1 - 2\bar{c}}{\Psi(C^{-1}(\frac{1}{2} - F))}$, there exists an electoral equilibrium in which two symmetrically located candidates run for office with non-maximal quality, and capture zero rents (showing that PR admits an equilibrium with two candidates collecting positive rents follows a similar logic and is therefore omitted).

Suppose that candidates 1 and 2 run for office, and that $\max\{\theta_1^*, \theta_2^*\} < 1$. This implies that the FOCs must be satisfied with equality and, in particular, that $\frac{\alpha}{\Delta} v'(\theta_k) = C'(\theta_k)$ for $k = 1,2$, and hence that $\Delta \geq \alpha\Psi(1)$. Then,

$$\theta_1^* = \theta_2^* = \theta^* = \Psi^{-1}\left(\frac{\Delta}{\alpha}\right). \tag{4.5}$$

[5] See Iaryczower and Mattozzi (2008) for a formal argument.

Note that when $\theta_2^* = \theta^* = \Psi^{-1}\left(\frac{\Delta}{\alpha}\right)$, 1's marginal profit is well-defined, continuous and decreasing at all points $\theta_1 > \underline{\theta}(\theta^*)$. Since the condition for non-negative rents is part of the equilibrium definition, it follows that $\theta_1^* = \theta^*$ is indeed a best response. Furthermore, since $\theta_1^* = \theta_2^*$, we have that $\tilde{x}_{12} = \Delta_0 + \frac{\Delta}{2}$. Given that in equilibrium candidates must collect nonnegative rents, then it must be true that $\Pi_1^* = \Delta_0 + \frac{\Delta}{2} - C(\theta_1^*) - F \geq 0$ and $\Pi_2^* = 1 - \Delta_0 - \frac{\Delta}{2} - C(\theta_1^*) - F \geq 0$, or equivalently,

$$F + C(\theta^*) - \frac{\Delta}{2} \leq \Delta_0 \leq 1 - \frac{\Delta}{2} - C(\theta^*) - F. \tag{4.6}$$

There exists Δ_0 satisfying (5) if and only if $\theta^* \leq C^{-1}(\frac{1}{2} - F)$ or, substituting from (4.5), if and only if $\Delta \geq \alpha\Psi(C^{-1}(\frac{1}{2} - F))$. Since $\bar{c} + F > \frac{1}{2}$, it follows that $\alpha\Psi(C^{-1}(\frac{1}{2} - F)) \geq \alpha\Psi(1)$. Therefore, if $\Delta \geq \alpha\Psi(C^{-1}(\frac{1}{2} - F))$ also $\Delta \geq \alpha\Psi(1)$. Choose then $\Delta = \alpha\Psi(C^{-1}(\frac{1}{2} - F))$. From (4.5), $\theta^* = C^{-1}(\frac{1}{2} - F)$ or $C(\theta^*) = \frac{1}{2} - F$. Also, since inequalities in (4.6) hold as equalities, $\Delta_0 = F + C(\theta^*) - \frac{\Delta}{2} = \frac{1 - \Delta}{2}$, and therefore $\Delta_0 = \frac{1}{2} - \frac{\alpha}{2}\Psi(C^{-1}(\frac{1}{2} - F))$. Hence, $\Pi_1^* = \Pi_2^* = 0$.

Consider next entry of a third candidate j with $x_j \in (x_1, x_2)$ and assume the following continuation play: $\hat{\theta}_1 = \hat{\theta}_2 = \hat{\theta}_j = 1$. The optimality conditions for $k = 1$ and $k = 2$ are $\frac{\alpha}{(1 - \delta_j)\Delta}\Psi(1) \geq 1$ and $\frac{\alpha}{\delta_j\Delta}\Psi(1) \geq 1$, where $\delta_j = \frac{x_2 - x_j}{\Delta}$. The necessary first order condition for j is $\frac{\alpha}{\delta_j(1 - \delta_j)\Delta}\Psi(1) \geq 1$ which is implied by the previous inequalities. These conditions are satisfied if and only if

$$\max\{\delta_j, 1 - \delta_j\}\,\Delta \leq \alpha\Psi(1). \tag{4.7}$$

Now suppose that $\delta_j \leq \frac{1}{2}$. Then (4.7) is $(1 - \delta_j)\,\Delta \leq \alpha\Psi(1)$, and thus we need $\alpha\Psi(C^{-1}(\frac{1}{2} - F)) \leq \Delta \leq \frac{\alpha}{1 - \delta_j}\Psi(1)$. This is feasible if

$$\frac{\Psi(C^{-1}(\frac{1}{2} - F)) - \Psi(1)}{\Psi(C^{-1}(\frac{1}{2} - F))} \leq \delta_j \leq \frac{1}{2}. \tag{4.8}$$

Suppose instead that $\delta_j \geq \frac{1}{2}$. Then (4.7) is $\delta_j\,\Delta \leq \alpha\Psi(1)$, and we need $\alpha\Psi(C^{-1}(\frac{1}{2} - F)) \leq \Delta \leq \frac{\alpha}{\delta_j}\Psi(1)$. This is feasible if:

$$\frac{1}{2} \le \delta_j \le \frac{\Psi(1)}{\Psi(C^{-1}(\frac{1}{2} - F))}. \tag{4.9}$$

Combining (4.8) and (4.9) we obtain

$$\frac{\Psi(C^{-1}(\frac{1}{2} - F)) - \Psi(1)}{\Psi(C^{-1}(\frac{1}{2} - F))} \le \delta_j \le \frac{\Psi(1)}{\Psi(C^{-1}(\frac{1}{2} - F))}. \tag{4.10}$$

When (4.10) holds, i.e., following the entry of a centrist candidate, and $\Delta = \alpha\Psi(C^{-1}(\frac{1}{2} - F))$, then $\hat{\theta}_1 = \hat{\theta}_2 = \hat{\theta}_j = 1$ is a joint best response provided that the incumbent candidates choose not to drop from the race.[6] A sufficient condition for the latter statement to be true (when (4.10) holds) is $\alpha \le \frac{1 - 2\bar{c}}{\Psi(1)}$. When $\hat{\theta}_1 = \hat{\theta}_2 = \hat{\theta}_j = 1$ we have that $\hat{\Pi}_j = \frac{\Delta_1}{2} - \bar{c} - F < 0$, since $\bar{c} - F > \frac{1}{2}$ and $\Delta<1$. Now consider entries such that $\delta_j > \frac{\Psi(1)}{\Psi(C^{-1}(\frac{1}{2} - F))} = \frac{\Psi(1)}{\Delta}$. In this case j enters relatively close to $k = 1$, and a strategy profile such that all three candidates choose maximal quality cannot be an equilibrium of the continuation game. Consider instead $\hat{\theta}_2 \in (0,1)$, and $\hat{\theta}_1 = \hat{\theta}_j = 1$. The FOC for $k = 2$ is $\frac{\alpha}{\delta_j \Delta}\Psi(\hat{\theta}_2) = \Psi^{-1} = 1$, or equivalently $\hat{\theta}_2 = \Psi^{-1}\left(\frac{\delta_j \Delta}{\alpha}\right) = \Psi^{-1}\left(\frac{x^2 - x_j}{\alpha}\right)$. The FOC for $k = 1$ is, as before, $(1 - \delta_j)\Delta \le \alpha\Psi(1)$, and the FOC for j is not relevant. Therefore, we need $\alpha\Psi(C^{-1}(\frac{1}{2} - F)) \le \Delta \le \frac{\alpha}{1-\delta_j}\Psi(1)$, which is feasible if

$$\frac{\Psi(C^{-1}(\frac{1}{2} - F)) - \Psi(1)}{\Psi(C^{-1}(\frac{1}{2} - F))} \le \delta_j, \tag{4.11}$$

and this always holds with $\delta_j > \frac{\Psi(1)}{\Psi(C^{-1}(\frac{1}{2} - F))}$.[7] We need to show now that $\hat{\Pi}_j = \tilde{x}_{j2}(1, \hat{\theta}_2) - \frac{x_1 + x_1}{2} - \bar{c} - F < 0$. If $\tilde{x}_{1j}$ were fixed, j would be better off by choos-

[6] Note that when $\delta_j \le (\ge)\frac{1}{2}$, we need $\Delta \le \frac{\alpha}{1-\delta_j}\Psi(1)(\Delta \le \frac{\alpha}{\delta_j}\Psi(1))$. From (8) this holds for all "feasible" δ_j if and only if $\Delta \le \alpha\Psi(\frac{1}{2} - F))$. But then, since we also need $\Delta \ge \alpha\Psi(C^{-1}(\frac{1}{2} - F))$, this must hold with equality. It is not surprising that, given zero profit in equilibrium, it must be the case that a unique Δ is the one that covers all possible δ_j in (4.10).

[7] Note that before we were satisfying (4.8) with $\delta_j < 1/2$, and now we are satisfying (4.11), which is the first part of (4.8), with $\delta_j > 1/2$. The reason is that before we were forcing k to keep choosing maximal quality even when j was entering relatively far away from him.

ing $\tilde{\theta}_j = \Psi^{-1}\left(\frac{\delta_j \Delta}{\alpha}\right) = \hat{\theta}_2$. But then, $\hat{\Pi}_j < \frac{\Delta}{2} - C(\hat{\theta}_2) - F < \frac{\Delta}{2} - C(\theta^*) - F < 0$. Again, we need to make sure that the incumbent candidates choose not to drop from the electoral race in the continuation game. A sufficient condition for this is $\alpha \le \frac{1-2\bar{c}}{\Psi(C^{-1}(\frac{1}{2}-F))}$. For no entry at the extremes it is sufficient that $\max\{\Delta_0, 1 - \Delta - \Delta_0\} < F$ and $\frac{\Delta}{2} > \bar{c}$ or $\alpha \ge \frac{2\bar{c}}{\Psi(C^{-1}(\frac{1}{2}-F))}$. Hence, if $F + \bar{c} \ge \frac{1}{2}$, $\bar{c} \le \frac{1}{4}$, and $\alpha \in \left(\frac{2\bar{c}}{\Psi(C^{-1}(\frac{1}{2}-F))}, \frac{1-2\bar{c}}{\Psi(C^{-1}(\frac{1}{2}-F))}\right)$ then all the previous conditions hold and an equilibrium exists. The second part of the proposition follows from simple inspection of (4.5) and from noticing that, when candidates are collecting zero rents, $\theta^* = C^{-1}\left(\frac{1}{2} - F\right)$. ■

Note that, since in *any* electoral equilibrium with two candidates running for office and positive rents $\theta_1^* = \theta_2^* = \theta^* = \Psi^{-1}\left(\frac{\Delta}{\alpha}\right)$, candidates become more aggressive in quality competition the less differentiated they are in the policy space and, given Δ, the weaker is voters' ideological focus (the larger is α). To achieve zero rents, however, it must be the case that $\Delta = \alpha\Psi(C^{-1}(\frac{1}{2} - F))$, and thus quality choice is invariant to α. A heightened responsiveness of voters to candidates' quality results entirely in a larger ideological differentiation between candidates running for office. In other words, if we think of the equilibrium with zero rents as a plausible long run political configuration, candidates will be more centrist (less polarized) the *stronger* is voters' ideological focus. Note also that the no-rents condition uniquely pins down observable behavior on the equilibrium path. If instead some candidates are allowed to collect positive rents in equilibrium, other electoral equilibria with some limited asymmetry (in centrality and payoffs) can emerge.

The technique we used to construct an equilibrium with two candidates easily extends to symmetric equilibria with an arbitrary number K of candidates running for office. In fact, even in the latter more general case, "local" changes in the quality choice by one candidate only lead to changes in "local" competition. This is due to the fact that small changes in k's quality choice only lead to changes in the distribution of votes between k and its closest competitors, one on each side of the policy spectrum. On the other hand, dealing with more than two candidates raises some technical issues. Indeed, the identity of the relevant competitors of each candidate will not generically remain fixed: since closer candidates in the issue space are better substitutes for each other, changes in candidate *k*'s quality choice will have a stronger impact on how voters rank *k* relative to its closest competitors than to more differentiated candidates in the policy space. As a result, changes in candidates *k*'s quality choice can in principle lead to changes in the identity of its relevant competitors, and thus to non differentia-bilities in the mapping from quality choice to vote shares. A simple way to get around this problem is to focus on a particular class of symmetric equilibria, in which all candidates running for office are located at the same distance to their closest neighbors. We call equilibria of this class *location-symmetric electoral equilibria* (LSE), and we refer the interested reader to Iaryczower and Mattozzi (2008a) for an exhaustive analysis.[8] Within the class of LSE, best responses are accurately represented by first order conditions. Hence, in a LSE with $K \geq 3$ candidates running for office such that $\theta_k^* < 1$ for all $k = 2,\ldots, K-1$, we have that

$$\theta_k^* = \Psi^{-1}\left(\frac{\Delta}{2\alpha}\right) \forall\ k = 2,\ldots, K-1 \text{ and } \theta_1^* = \theta_K^* = \Psi^{-1}\left(\frac{\Delta}{\alpha}\right) \tag{4.12}$$

In Iaryczower and Mattozzi (2008) we establish sufficient conditions for the existence of LSE with *K* parties. These conditions are entirely *supply side* requirements involving $\bar{c}$ and the entry costs *F*. In particular, if $\bar{c} \leq F$ and $F \in \left(\frac{1}{2K}, \frac{1}{K} - \bar{c}\right)$ we can construct a LSE with *K* candidates running for office. Moreover, if α is relatively small, all candidates will choose interior quality in equilibrium. These sufficient conditions are rather intuitive. In fact, the upper bound on *F* is meant to assure that running for office is profitable for each candidate or, stated differently, it captures the obvious fact that for a given level of entry costs there is a maximal number of candidates running for office that can be supported in a LSE. The lower bound instead, coupled with the assumption that $\bar{c} \leq F$, makes it possible to deter the potential entry of additional candidates.

We can use these conditions and the FOCs to investigate the effect of changes in the budget constraint on the equilibrium number of candidates running for office and their quality. First, notice that as *F* decreases, the upper bound defined above becomes less binding. As a consequence, it will be possible to support LSE with more candidates (of higher quality) running for office. To see why this is the case, note that a necessary condition for existence of a LSE with interior quality is $\Delta \geq C(\Psi^{-1}(\frac{\Delta}{2\alpha})) + F$, i.e., candidates collect non-negative rents in

[8] Formally, an electoral equilibrium is location-symmetric (LSE) if the distance between any two neighboring candidates *k* and $k+1$ for $k = 1,\ldots, K-1$ is $x_{k+1} - x_k = \Delta$, and $x_1 = 1 - x_K = \Delta_0$.

equilibrium. If we consider a decrease in F, the latter inequality becomes less binding and can hold for a smaller value of Δ, which implies a higher quality and a larger number of candidates running for office. A similar logic applies in the case of downward parallel or proportional shifts of the cost function. Hence, changes in the "supply side" of the political environment induce a positive correlation between the number of candidates running for office and their equilibrium quality. We summarize this conclusion informally in the following remark:

Remark 4.1. *Everything else constant, reductions in the fixed cost of running for office F, and/or downward (parallel or proportional) shifts in the cost function C (·), increase both the number of candidates running for office and their quality.*

If we focus on changes on the *demand side* of the political environment, however, the comparison is less clear. Consider changes in the responsiveness of voters to candidates' quality (α). Increasing α has the direct effect of making a given field of candidates "more aggressive" in quality competition. This has the effect of reducing the expected rents of all participants in the election. In a LSE where candidates running for office collect positive rents, the system has enough *flexibility* so that as voters become more responsive to the candidates' quality, quality competition can become tighter without affecting the equilibrium number of candidates. As candidates "compete away" their rents, however, increased voters' responsiveness to candidates' quality *must* lead to changes in the level of ideological differentiation and, eventually, in the number of candidates deciding to run for office. In fact, when candidates collect no rents in equilibrium, optimal quality depends on α only indirectly, through the equilibrium level of differentiation Δ, which is increasing in α.[9] In this case it follows that a less ideologically focused electorate must lead to a smaller number of candidates running for office. The overall effect on quality, however, is ambiguous.

So far we focused on location-symmetric equilibria. Note that in the class of LSE, it follows immediately that the number of candidates (inversely related to the degree of ideological differentiation between candidates) is directly related to the level of quality competition; i.e. the larger the number of candidates, the closer substitutes candidates are to each other, and therefore the more intense quality competition is. This result generalizes with some caveats to configurations of candidates with limited asymmetry.[10] Our first objective is to find a pro-

[9] When equilibrium rents are equal to zero, it follows that Δ is the unique solution to, $\Delta = C(\Psi^{-1}(\frac{\Delta}{2\alpha})) + F$, which is increasing in α.

[10] Note that we can easily extend our previous analysis and results to accommodate *some* limited asymmetry in location. While a full characterization of asymmetric equilibria is beyond the scope of this

per way to measure the number of candidates in an asymmetric environment. Consider, for example, comparing an outcome with four minority candidates each obtaining one percent of the vote and a fifth one capturing the remaining ninety six percent, with a second outcome where three candidates each obtains a third of the votes. As this example suggests, looking at the number of candidates in the context of asymmetric political configurations can be misleading, since the number of *relevant* candidates can be said to be larger in the latter outcome than in the former. One measure that overcomes this problem, and it is largely used in the political science literature, is the *effective number of parties* introduced by Laakso and Taagepera (1979). The Laakso-Taagepera effective number of parties (or candidates for our purposes) is defined as $e = 1/H$, where in turn $H = \sum_{k=1}^{K} m_K^2$ is the Herfindahl index, which is commonly employed to measure concentration of industries in industrial organization. The popularity of the effective number of candidates is due to a number of attractive properties (see Encaousa and Jacquemin (1980)). First, it is symmetric, or invariant to permutations of vote shares, between candidates. Second, it satisfies the *transfer principle*: the transfer of a part of a candidate's vote share to a candidate with a bigger vote share must not increase the effective number of candidates. For a given number of candidates, this condition implies that e attains its maximum value when the candidates have equal vote shares, and its minimum value when a single candidate captures (almost) the entire electorate. Third, the value of e for symmetric candidates must increase when the number of candidates grows from K to $K+1$. In particular, the effective number of candidates (weakly) decreases when we transfer vote share from one candidate to another one with a higher initial vote share. Given the definition of effective number of candidates, we can show the following result:

Proposition 4.2. *Consider an electoral equilibrium with three candidates running for office such that* $\Delta_1 > \Delta_2$. *Then* $\theta_2^* \geq \theta_3^* \geq \theta_1^*$, *with the inequalities strict if quality is non-maximal in equilibrium. Consider an alternative electoral equilibrium with* $x_2' > x_2$. *Then* $\theta_2^{**} > \theta_2^*$ $\theta_3^{**} > \theta_3^*$, *and* $\theta_1^{**} < \theta_1^*$. *Furthermore, if* Ψ *is convex, then the new equilibrium has both a smaller effective number of candidates and a lower average quality of candidates. Similarly, consider a LSE with K parties, and an alternative electoral equilibrium with* $x_{-k}' = x_{-k}$ *and* $x_k' \in (x_k, x_{k+1})$. *Then if* Ψ *is convex, the new equilibrium has both a smaller effective number of candidates and a lower average quality of candidates.*

---

paper, note however that electoral equilibria in PR can never be too asymmetric within our framework, as the joint equilibrium requirements of non-negative rents and no profitable entry imply that the amount of asymmetry which is possible to support in an electoral equilibrium must be rather limited.

Proof. In order to prove this result, first we need to introduce some additional notation, which will prove useful to handle non-symmetryc configurations of candidates. Provided that $\theta_k \geq \underline{\theta}_k$, (θ_{-k}, x), k's vote share $m_k\,(\theta_k\,;\theta_{-k}, x)$ can be expressed as

$$m_k\,(\theta_k\,;\theta_{-k}, x) = \frac{\Delta_k^T}{2} + \alpha\left[\frac{v(\theta_k) - v(\theta_{r(k)})}{\Delta_k^r} + \frac{v(\theta_k) - v(\theta_{l(k)})}{\Delta_k^l}\right], \qquad (4.13)$$

where Δ_k^r and Δ_k^l denote the distance between the policy represented by k and that of its neighbors, $\theta_{r(k)}$ and $\theta_{l(k)}$ denote the campaign effort of k's neighbors, and $\Delta_k^T \equiv \Delta_k^l + \Delta_k^r$. Letting $\delta_k \equiv \frac{\Delta_k^r}{\Delta_k^T}$, it follows that k's FOC is given by

$$\theta_k = \Psi^{-1}\left(\frac{\delta_k\,(1-\delta_k)\,\Delta_k^T}{\alpha}\right), \qquad (4.14)$$

for $k \in (2, K-1)$. In the case of three candidates running for office with $\Delta_1 > \Delta_2$, FOCs deliver $\theta_2^* = \Psi^{-1}\left(\frac{\delta_2\,(1-\delta_2)\,\Delta_k^T}{\alpha}\right)$, $\theta_3^* = \Psi^{-1}\left(\frac{\delta_2\,\Delta_k^T}{\alpha}\right)$, and $\theta_1^* = \Psi^{-1}\left(\frac{\delta_2\,(1-\delta_2)\,\Delta_2^T}{\alpha}\right)$. It follows immediately that $\theta_2^* \geq \theta_3^* \geq \theta_1^*$ since $\Psi(\cdot)$ is decreasing and $\delta_2 < \frac{1}{2}$. Next, note that since $\delta_2\,(1-\delta_2)$ is monotonically increasing in δ_2 for $\delta_2 < \frac{1}{2}$, then $\delta_2' < \delta_2$ implies that $\theta_2^{**} > \theta_2^* > \theta_3^{**}$, and $\theta_1^{**} < \theta_1^*$. Now $\theta_m^* = \frac{1}{3}\sum_k \theta_k^*$, and since $\theta_2^{**} > \theta_2^*$ it is enough to show that $\theta_3^{**} + \theta_1^{**} > \theta_3^*\ \ \theta_1^*$. This can be written as

$$\Psi^{-1}\left(\frac{\delta'_2\,\Delta_2^T}{\alpha}\right) + \Psi^{-1}\left(\frac{(1-\delta'_2)\,\Delta_2^T}{\alpha}\right) > \Psi^{-1}\left(\frac{\delta_2\,\Delta_2^T}{\alpha}\right) + \Psi^{-1}\left(\frac{(1-\delta'_2)\,\Delta_2^T}{\alpha}\right) \qquad (4.15)$$

which follows from convexity of Ψ. In fact, if Ψ is decreasing and convex then Ψ^{-1} is also convex. The last part of the proposition can be proved in a similar way. ■

Note that the result of Proposition 4.2. holds when Ψ is convex, which is not implied by the assumptions of v concave and C convex. Convexity of Ψ, however, is satisfied in the case of many commonly used parametric specifications. For example, when $C(\theta) = A\theta^B$, with $A > 0$ and $B > 1$, and v belongs to the class of hyperbolic absolute risk aversion (HARA) utility functions, i.e., $v(\theta) = \frac{1-d}{d}\left(\frac{a\theta}{1-d}\right)^d$ with $a > 0$ and $d < 1$. The HARA class includes as special cases the constant absolute risk aversion (with $b = 1$ and $d \to -\infty$), constant relative risk aversion (with

$b \to 0$ and $a = 1 - d$), logarithmic (with $b \to 0$ and $d \to 0$), as well as power and exponential utility functions.

References

ANDERSON, S. P. and A. DE PALMA. "Spatial Price Discrimination with Heterogeneous Products." *Review of Economic Studies* 55, no. 4 (1988): 573-592.

ANDERSON, S. P., A. DE PALMA and J-F. THISSE. "Demand for Differentiated Products. Discrete Choice Models and the Characteristics Approach." *Review of Economic Studies* 56, no. 1 (1989): 21-35.

ARAGONES, E. and T. R. PALFREY. "Mixed Equilibrium in a Downsian Model with a Favored Candidate." *Journal of Economic Theory* 103 (2002): 131-161.

ASHWORTH, S. and E. BUENODEMESQUITA. *Valence Competition and Platform Divergence.* Typescript, 2007.

D'ASPREMONT, C., J. GABSZEWICZ and J. F. THISSE. "On Hotelling's Stability in Competition." *Econometrica* 47, no. 5 (1979): 1145-1150.

AUSTEN-SMITH, D. and J. S. BANKS. "Elections, Coalitions, and Legislative Outcomes." *American Political Science Review* 82 (1988): 405-422.

BARON, D. P. and D. DIERMEIER. "Elections, Governmens and Parliaments in Proportional Representation Systems." *The Quarterly Journal of Economics* 116, no. 3 (2001): 933-967.

CAPLIN, A. S. and B. J. NALEBUFF. "MultiDimensional Product Differentiation and Price Competition." *Oxford Economic Papers*, New Series, Vol. 38, Supplement: *Strategic Behaviour and Industrial Competition* 38 (1986): 129-145.

CARRILLO, J. D. and M. CASTANHEIRA. "Information and Strategic Political Polarization." *Economic Journal* (2008). forthcoming.

DUVERGER, M. *Political Parties, their Organization and Activity in the Modern State.* New York: Wiley, 1954.

ENCAOUA, D. and A. JACQUEMIN. "Degree of Monopoly, Indices of Concentration, and Threat of Entry." *International Economic Review* 21 (1980): 87-105.

GROSECLOSE, T. "A Model of Candidate Location when one Candidate has a Valence Advantage." *American Journal of Political Science* 45 (2001): 862-886.

HERRERA, H., D. K. LEVINE and C. MARTINELLI. "Policy Platforms, Campaign Spending and Voter Participation." *Journal of Public Economics* 92 (2008): 501-513.

IARYCZOWER, M. and A. MATTOZZI. *Competition – Enhancing Institutions in PR Elections.* Working Paper, HSS, California Institute of Technology, 2008a.

—. *On the Nature of Competition in Alternative Electoral Systems.* Working Paper, HSS, California Institute of Technology, 2008b.

LAAKSO, M. and R. TAAGEPERA. "Effective Number of Parties: A Measure with Application to Western Europe." *Comparative Political Studies* 12 (1979): 3-27.

LIZZERI, A. and N. PERSICO. "The Provision of Public Goods under Alternative Electoral Incentives." *American Economic Review* 91 (2001): 225-239.

MEIROWITZ, A. *Electoral Contests, Incumbency Advantages and Campaign Finance.* Princeton University, 2007.

MORELLI, M. "Party Formation and Policy Outcomes under Different Electoral Systems." *Review of Economic Studies* 71 (2004): 829-853.

MYERSON, R. B. "Effectiveness of Electoral Systems for Reducing Government Corruption: A Game-Theoretic Analysis." *Games and Economic Behavior* 5 (1993): 118-132.

OSBORNE, M. J. and A. SLIVINSKI. "A Model of Political Competition with Citizen-Candidates." *The Quarterly Journal of Economics* 111, no. 1 (1996): 65-96.

PERLOFF, J. M. and S. C. SALOP. "Equilibrium with Product Differentiation." *Review of Economic Studies* 52, no. 1 (1985): 107-120.

PERSSON, T., G. ROLAND and G. TABELLINI. *How do Electoral Rules Shape Party Structures, Government Coalitions and Economic Policies.* Department of Economics, UC Berkeley, 2003.

PERSSON, T. G. TABELLINI and F. TREBBI. "Electoral Rules and Corruption." *Journal of the European Economic Association* 1 (2006): 958-989.

SCHOFIELD, N. "Equilibrium in the Spatial Valence Model of Politics." *Journal of Theoretical Politics* 16 (2004): 447-481.

SHAKED, A. and J. SUTTON. "Product Differentiation and Industrial Structure." *The Journal of Industrial Economics* 36, no. 2 (1987): 131-146.

—. "Relaxing Price Competition Through Product Differentiation." *Review of Economic Studies* 49, no. 1 (1982): 3-13.

STOKES, D. E. "Spatial Models of Party Competition." *American Political Science Review* 57 (1963): 368-377.

5

An Activist Model of Democracy

Norman Schofield
Department of Economics and Political Science, and Center in Political Economy
Washington University

5.1. Introduction: a stochastic model of elections

The focus of this paper is that actual political systems do not appear to satisfy the property of *convergence to an electoral center* that is often predicted by formal vote models. The key theoretical idea is that the convergence result need not hold if there is an asymmetry in the electoral perception of the "quality" of party leaders (Stokes 1992). The average weight given to the perceived quality of the leader of a party is called the party's *valence.* In empirical models, a party's valence is usually assumed to be *exogenous,* and independent of the party's position. In general, valence reflects the overall degree to which the party is perceived to have shown itself able to govern effectively in the past, or is likely to be able to govern well in the future (Penn 2003).

The motivation for the development of the *activist stochastic electoral model,* which is presented in this paper, is based on a set of empirical results from multinomial logit electoral estimation for the Netherlands, Britain and the United States.[1] These empirical analyses, coupled with theoretical results, indicate that the stochastic model with exogenous valence cannot fully account for the divergence observed in a number of elections in these polities.

Theorem 5.2, presented below, gives the necessary and sufficient conditions for convergence to the electoral mean in the stochastic model with exogenous valence. The necessary condition is that a convergence coefficient, c, is bounded above by the dimension, w, of the policy space, while a sufficient condition is that the coefficient is bounded above by 1. This coefficient is defined in terms of the difference in exogenous valences, the "spatial coefficient" and the electoral variance. The empirical work on Britain and the Netherlands indicates that the necessary condition was

[1] The details of the estimations are presented in Schofield and Sened (2006).

satisfied. Indeed the *mean voter theorem* should have been valid, even though there was empirical evidence that the parties did not converge to an electoral mean.

Section 5.2 of this paper presents the formal results on the extension of the standard stochastic model based on exogenous valence by adopting the assumption that there are two kinds of valence. The first kind is the usual *exogenous valence*, which for a party j is denoted λ_j. As in empirical work, this formal model assumes that λ_j is held constant at the time of an election, and so is independent of the party's position. The second kind of valence is known as *activist valence.* When party j adopts a policy position z_j, we denote the activist valence of the party by $\mu_j(z_j)$. Implicitly the model is an extension of one originally due to Aldrich (1983a,b). In this model, activists provide crucial resources of time and money to their chosen party. The party then uses these resources to enhance its image before the electorate, thus affecting its valence. Although activist valence is affected by party position, it does not operate in the usual way by influencing voter choice through the distance between a voter's preferred policy position, say x_i, and the party position. Rather, as party j's activist support, $\mu_j(z_j)$, increases due to increased contributions to the party in contrast to the support $\mu_k(z_k)$ received by party k, then (in the model) all voters become more likely to support party j over party k. The problem for each party is that activists are likely to be more extreme than the typical voter. By choosing a policy position to maximize activist support, the party will lose centrist voters. The party must therefore calculate the optimal marginal condition to maximize vote share.The main result, Theorem 5.1, gives this as a (first order) *balance condition.* Moreover, because activist support is denominated in terms of time and money, it is reasonable to suppose that the activist function will exhibit decreasing returns, so that the functions themselves are concave, and their Hessians are everywhere negative-definite. Theorem 5.1 asserts that when these functions are sufficiently concave, then the activist vote maximizing model will exhibit a *Pure Strategy Nash Equilibrium.* Theorem 5.2 presents the results when each party attempts to maximize an expected vote share, where this is defined in terms of a weighted sum of the voter probabilities. In principle, these voter weights can be deduced from the elctoral model utilized in the polity. Theorem 5.2 specializes to the egalitarian case where all voter weights are identical, and obtains the necessary and sufficient conditions for the validity of the mean voter theorem. This model is applicable to electoral systems based on proportional representation.[2]

---

[2] The proof of Theorem 5.1 is given in the working paper version and is available at http://polisci.wustl.edu/sub_page.php?s=3&m=0&d=24.

Section 5.3 presents the empirical work on elections in the Netherlands in 1977 and in Britain in 1997. A brief illustration is provided of the application of the model to recent elections in the United States. The concluding section 5.4 argues that there is, in general, no centripetal tendency towards an electoral center. It is consistent with this analysis that activist groups will tend to pull the parties away from the center. Indeed, we can follow Duverger (1954) and Riker (1953) and note that under proportional electoral methods, there is very little motivation for interest groups to coalesce. Another way of expressing, in simplified form, the difference between proportional representation and plurality rule is this: under proportional electoral methods, bargaining to create winning coalitions occurs *after* the election. Under plurality rule, if interest groups do not form a coalition *before* the election, then they can be obliterated. This obviously creates a pressure for activist groups to coalesce. Other work (Schofield and Ozdemir 2008) uses this idea to explore the difference between plurality rule and proportional representation that has been pointed out by Duverger (1954).

5.2. A political economy model of leader support

The model presented here is an extension of the standard multiparty stochastic model, modified by inducing asymmetries in terms of valence.

The key idea underlying the formal model is that political leaders attempt to estimate the effects of their policy positions on the support they receive. Each leader, whether autocrat or opposition, chooses the policy position as best response to opposing position(s), in order to obtain sufficient support either to retain power or to gain power. The stochastic model essentially assumes that a leader cannot predict support precisely, but can estimate an expected support. In the model with valence, the stochastic aspect of the model is associated with the weight given by each citizen, *i,* to the average perceived quality or valence of the party leader.

Definition 5.1. The *Stochastic Model* $E(\lambda, \mu, \beta; \Psi)$ *with Activist Valence.*

The data of the spatial model is a distribution, $\{x_i, \in W: i \in P\}$, of voter ideal points for the members of the *selectorate*, P, of size p. By the selectorate we mean those citizens who have some potential to influence political choice. We assume that W is an open, convex subset of Euclidean space, $\mathbb{R}^w$, with w finite. Each of the leaders in the set $N = \{1,..., j,..., n\}$ chooses a policy, $z_j \in W$, to declare. Let $\mathbf{z} = (z_1,..., z_n) \in W^n$ be a typical vector of leader positions.

Given **z**, each citizen, i, is described by a vector

$$\mathbf{u}_i\,(x_i, \mathbf{z}) = (u_{i1}\,(x_i, z_1),\ldots, u_{ip}\,(x_i, z_n))$$

where

$$u_{i1}\,(x_i, z_1) = \lambda_j + \mu_j\,(z_j) - \beta\,||\,x_i - z_j\,||^2 + \varepsilon_j = u^*_{ij}\,(x_i, z_1) + \varepsilon_j. \qquad (5.1)$$

Here $u^*_{ij}\,(x_i, z_1)$ is the observable component of utility. The term, λ_j, is the fixed or *exogenous valence* of leader j, while the function $\mu_j\,(z_j)$ is the component of valence generated by activist contributions to leader j. The term β is a positive constant, called the *spatial parameter*, giving the importance of policy difference defined in terms of the Euclidean metric, $||a - b\,||$, on W. The vector $\varepsilon = (\varepsilon_1,\ldots, \varepsilon_j,\ldots, \varepsilon_n)$ is the stochastic error, whose multivariate cumulative distribution will be denoted by Ψ.

It is assumed that the exogenous valence vector

$$(\lambda_1, \lambda_2,\ldots, \lambda_n) \text{ satisfies } \lambda_n \geq \lambda_{n-1} \geq\ldots \geq \lambda_2 \geq \lambda_1$$

Citizen behavior is modelled by a probability vector. The probability that a citizen i chooses leader j at the vector **z** is

$$\rho_{ij}\,(\mathbf{z}) = \Pr[\,[u_{ij}(x_i, z_j) > u_{i1}\,(x_i, z_l)\,], \textit{for all } l \neq j] \qquad (5.2)$$

$$= \Pr[\varepsilon_l - \varepsilon_j < u^*_{ij}\,(x_i, z_j) - u^*_{il}\,(x_i, z_j), \textit{for all } l \neq j] \qquad (5.3)$$

Here Pr stands for the probability operator generated by the distribution assumption on ε.

The *expected support* of leader j is.

$$V_j\,(\mathbf{z}) = \frac{\sum_{i\in P} s_{ij}\rho_{ij}\,(\mathbf{z})}{\sum_{i\in P} s_{ij}} \qquad (5.4)$$

The weights $\{s_{ij}\}$ allow for the possibility that individuals belong to different constituencies and have differing political power. Without loss of generality, we normalize and assume for each j that $\sum_{i\in P} s_{ij} = 1$.

In democratic polities based on proportional representaion we can assume that each $s_{ij} = \frac{1}{p}$ for all i, j. We call this the *egalitarian* case. In non-democratic polities the weights s_{ij} may differ widely. The differentiable function $V: W^n \to \mathrm{R}^n$ is called the *leader profile function.*

In the following it is assumed that the stochastic errors have the Type I extreme value (or Gumbel) distribution, Ψ (Train, 2003). The formal model based on Ψ parallels the empirical models based on multinomial logit (MNL) estimation.

Definition 5.2. *The Extreme Value Distribution,* Ψ.

The cumulative distribution, Ψ, has the closed form

$$\Psi(x) = \exp\left[-\exp[-x]\right]$$

The difference between the Gumbel and normal (or Gaussian) distributions is that the latter is perfectly symmetric about zero.

With this distribution assumption, it follows, for each voter i and leader j, that

$$\rho_{ij}(\mathbf{z}) = \frac{\exp[u^*_{ij}(x_i, z_j)]}{\sum_{k=1}^{n} \exp u^*_{ik}(x_i, z_k)} \tag{5.5}$$

In this stochastic electoral model it is assumed that each leader j chooses z_j to maximize V_j, conditional on $\mathbf{z}_{-j} = (z_1,\ldots, z_{j-1},\ldots, z_{j+1},\ldots, z_n)$.

Definition 5.3. *Equilibrium Concepts.*

(i) A strategy vector $\mathbf{z}^* = (z^*_1,\ldots, z^*_{j-1}, z^*_j, z^*_{j+1},\ldots, z^*_n) \in W^n$ *is a local strict Nash equilibrium (LSNE) for the profile function* $V: W^n \to \mathrm{R}^n$ *iff, for each leader* $j \in N$, *there exists a neighborhood* W_j *of* z^*_j *in* W *such that*

$$V_j(z^*_1,\ldots, z^*_{j-1}, z^*_j, z^*_{j+1},\ldots, z^*_p) > V_j(z^*_1,\ldots z^*_{j-1}, z_j, z^*_{j+1}\ldots, z^*_n)$$

for all $z_j \in W_j - \{z^*_j\}$.

(ii) A strategy vector $\mathbf{z}^* = (z^*_1,\ldots, z^*_{j-1}, z^*_j, z^*_{j+1},\ldots, z^*_n)$ *is a local weak Nash equilibrium (LNE) iff, for each agent* j, *there exists a neighborhood* W_j *of* z^*_j *in* W *such that*

$$V_j(z^*_1,\ldots, z^*_{j-1}, z^*_j, z^*_{j+1},\ldots, z^*_n) \geq V_j(z^*_1,\ldots z^*_{j-1}, z_j, z^*_{j+1}\ldots, z^*_n)$$

for all $z_j \in W_j$.

(iii) A strategy vector $\mathbf{z}^* = (z_1^*, \ldots, z_{j-1}^*, z_j^*, z_{j+1}^*, \ldots, z_n^*)$ *is a srict or weak, pure strategy Nash equilibrium (PSNE or PNE) iff* W_j *can be replaced by W in (i),(ii) respectively.*

(iv) The strategy z_j^* *is termed a "local strict best response", a "local weak best response", a "global weak best response", a "global strict best response", respectively to* $\mathbf{z}_{-j}^* = (z_1^*, \ldots, z_{j-1}^*, z_j^*, z_{j+1}^*, \ldots, z_n^*)$. ■

Obviously if $\mathbf{z}^*$ is an LSNE or a PNE it must be an LNE, while if it is a PSNE then it must be an LSNE. We use the notion of LSNE to avoid problems with the degenerate situation when there is a zero eigenvalue to the Hessian. The weaker requirement of LNE allows us to obtain a necessary condition for $\mathbf{z}^*$ to be a LNE and thus a PNE, without having to invoke concavity. Of particular interest is the vector

$$x_j^* = \frac{\sum_{i \in P} s_{ij} x_i}{\sum_{i \in P} s_{ij}} = \sum_{i \in P} s_{ij} x_i \tag{5.6}$$

In the *egalitarian* case, all $s_{ij} = 1/p$, and we can transform coordinates so that in the new coordinate system, $x^* = \Sigma_{i \in P}\, x_i = 0$. We shall refer to $\mathbf{z}_0 = (0, \ldots, 0)$ as the joint *electorate origin*.

Theorem 5.1 shows, even in the egalitarian case, that $\mathbf{z}_0 = (0, \ldots, 0)$ will generally not satisfy the first order condition for a LSNE, namely that the differential of V_j, with respect to z_j be zero. However, if the activist valence function is identically zero, so that only exogenous valence is relevant, then the first order condition at $\mathbf{z}_0$ will be satisfied.

It follows the definition of the Gumbel distribution, that for voter i, with ideal point, x_i, from the probability, $\rho_{ij}(\mathbf{z})$, that i picks j at $\mathbf{z}$ is given by

$$\rho_{ij}(\mathbf{z}) = \left[1 + \Sigma_{k \neq j}[\exp(f_{jk})]\right]^{-1} \tag{5.7}$$

$$\text{where } f_{jk} = \lambda_k + \mu_k(z_k) - \lambda_j - \mu_j(z_j) + \beta\, ||\, x_i - z_j\, ||^2 - \beta\, ||\, x_i - z_k\, ||^2.$$

Schofield (2006a) shows that the first order condition for $\mathbf{z}^*$ to be a LSNE is that it be a *balance solution.*

Definition 5.4. *The balance solution for the model* $E(\lambda, \mu, \beta; \Psi)$.

Let $[\rho_{ij}(\mathbf{z})] = [\rho_{ij}]$ *be the matrix of voter probabilities at the vector* $\mathbf{z}$, *and let*

$$\alpha_{ij} = \frac{s_{ij}\,[\rho_{ij} - \rho_{ij}^2]}{\sum_{k\in P} s_{kj}\,[\rho_{kj} - \rho_{kj}^2]} \tag{5.8}$$

be the matrix of coefficients. The balance equation for z_j^* *is given by expression*

$$z_j^* = \frac{1}{2\beta}\frac{d\mu_j}{dz_j} + \sum_{i=1}^{p} \alpha_{ij}\, x_i \tag{5.9}$$

The vector $\sum_i \alpha_{ij}\, x_i$ *is called the weighted electoral mean for leader j, and can be written*

$$\sum_{i=1}^{p} \alpha_{ij}\, x_i = \frac{d\mathrm{E}_j^*}{dz_j} \tag{5.10}$$

Notice first that the weight α_{ij} shows how the citizen i influences leader j in his choice of policy position. Moreover, the weights for leader j depend on the vector of positions $\{\mathbf{z}_{-j}\}$ of leaders other than j. The balance equation can be rewritten as

$$\left[\frac{d\mathrm{E}_j^*}{dz_j} - z_j^*\right] + \frac{1}{2\beta}\frac{d\mu_j}{dz_j} = 0 \tag{5.11}$$

The bracketed term on the left of this expression is termed the *marginal electoral pull of leader j* and is a gradient vector pointing towards this leader's weighted electoral mean. This position is that point where the electoral pull is zero. The vector $\frac{d\mu_j}{dz_j}$ is called *the marginal activist pull for leader j.*

If $\mathbf{z}^* = (z_1^*, \ldots z_j^*, \ldots z_n^*)$ is such that each z_j^* satisfies the balance equation then call $\mathbf{z}^*$ the *balance solution.*

Theorem 5.1.[3] *Consider the electoral model* $E(\lambda, \mu, \beta; \Psi)$ *based on the Type I extreme value distribution, and including both exogenous and activist valences. The first order condition for* $\mathbf{z}^*$ *to be an LSNE is that it is a balance solution. If all activist valence*

[3] The proof of Theorem 5.1 can be found in Schofield (2006a).

functions are highly concave, in the sense of having negative eigenvalues of sufficiently great magnitude, then the balance solution will be a PNE.

We emphasize that the *marginal electoral pull of leader j* is a gradient vector pointing towards the weighted electoral mean of the leader, and represents the *centripetal* pull to the center. *The marginal activist pull for leader j* represents the *centrifugal force* generated by the resources made available by activists.

In principle, this model can be used to examine the equilibrium position of a political leader, responding to activist demands, and balancing the pull of the selectorate, in order to gain resources that can be used to compete with political opponents. Even without activists, convergence to a centrist position, as in the Downsian model, is impossible if the population is sufficiently heterogenous in its beliefs or preferences.

In the case $\mu_j = 0$ for all j, the balance condition becomes

$$z_j = \sum_{i \in P} s_{ij} x_i \tag{5.12}$$

In the egalitarian case with all weights $\{s_{ij}\}$ identical, then first order balance condition becomes

$$z_j^* = \frac{1}{p} \sum_{i=1}^{p} x_i \tag{5.13}$$

By a change of coordinates we choose $\frac{1}{p}\Sigma x_i = 0$. In this case, the marginal electoral pull is zero at the origin and the joint origin $\mathbf{z}_0 = (0,\dots, 0)$ satisfies the first order condition. However, since $\mu = \mathbf{0}$, we cannot use the concavity of μ to assert the existence of equilibrium. Schofield (2007) shows that if $\mu = \mathbf{0}$, then there is a coefficient, c, defined in terms of all model parameters and the electoral covariance matrix of the voter preferred points such that $c < w$ is a necessary condition for $\mathbf{z}_0$ to be a LSNE in the egalitarian stochastic vote model.

Definition 5.5. *The Electoral Covariance Matrix,* ∇_0.

Let $W = R^w$ *be endowed with a system of coordinate axes* $r = 1,\dots, w$. *For each coordinate axis let* $\xi_r = (x_{1r}, x_{2r},\dots, x_{pr})$ *be the vector of the* r^{th} *coordinates of the set of* p *voter bliss points. The scalar product of* ξ_r *and* ξ_s *is denoted* (ξ_r, ξ_s).

The symmetric $w \times w$ *electoral covariance matrix about the origin is denoted* ∇_0 *and is defined by*

$$\nabla_0 = \frac{1}{p}\left[(\xi_r, \xi_s)\right]_{s=1,\ldots,w}^{r=1,\ldots,w}$$

Let $(\sigma_r, \sigma_s) = \frac{1}{p}(\xi_r, \xi_s)$ *be the electoral covariance between the* r^{th} *and* s^{th} *axes, and* $\sigma_s^2 = \frac{1}{p}(\xi_s, \xi_s)$ *be the electoral variance on the* s^{th} *axis, with*

$$\sigma^2 = \sum_{s=1}^{w} \sigma_s^2 = \frac{1}{p}\sum_{s=1}^{w}(\xi_s, \xi_s) = trace\,(\nabla_0)$$

the total electoral variance.

Theorem 5.2. *(i) The Hessian of the egalitarian vote share function of party j at* $\mathbf{z}_0$ *is a positive multiple of the w by w characteristic matrix.*[4]

$$C_j = 2\beta\,(1 - 2\rho_j)\,\nabla_0 - I \tag{5.14}$$

where I is the w by w identity matrix.

(ii) The necessary and sufficient condition for $\mathbf{z}_0$ *to be an LSNE is that all* C_j *have negative eigenvalues. Since* C_1 *must also have negative eigenvalues, it follows that a necessary condition for* $\mathbf{z}_0$ *to be an LNE is that a convergence coefficient, c, defined by*

$$c = 2\beta\,(1 - 2\rho_1)\,\sigma^2$$

is bounded above by the dimension, w.

(iii) In two dimensions, a sufficient condition is that c is bounded above by 1. *In higher dimensions a sufficient condition can be expressed by appropriate bounds on the cofactors of* C_1.

While maximization of vote share is an appropriate maximand under proportional egalitarian rule, a more appropriate maximand under plurality rule would be a seat share function

$$S_j(\mathbf{z}) = S_j(V_1(\mathbf{z}),\ldots, V_j(\mathbf{z}),\ldots V_n(\mathbf{z}))$$

which might very well be a logistic function of $V_j(\mathbf{z})$. The techniques of the proof of Theorem 5.1 and Theorem 5.2 can be extended to this more general case.

---

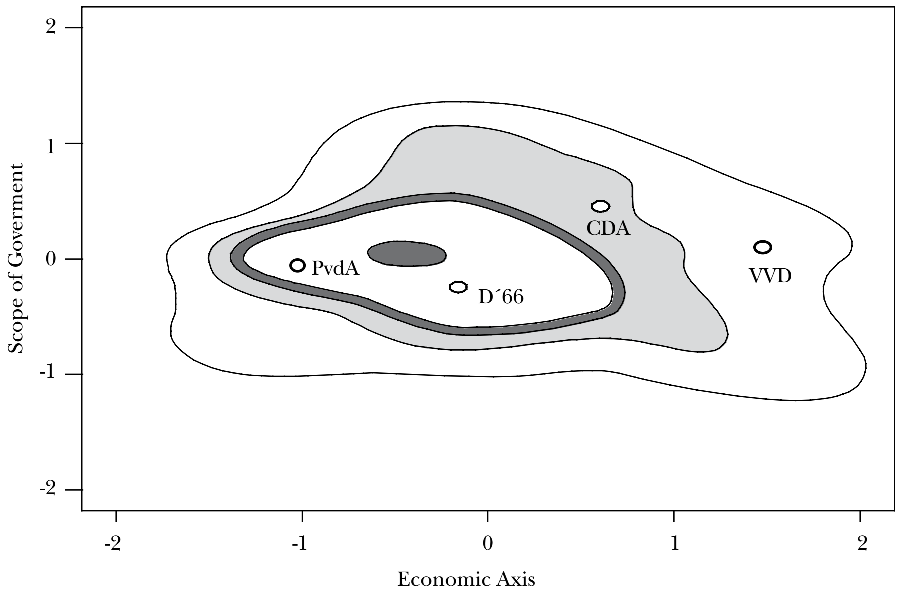
4 The proof of Theorem 5.2 can be found in Schofield (2007).

5.3. Empirical models

5.3.1. Netherlands 1977 and 1981

Next we consider a multinomial logit (MNL) model for the elections of 1977 and 1981 in the Netherlands (Schofield, Martin, Quinn and Whitford 1998; Quinn, Martin and Whitford 1999) using data from the middle level Elites Study (ISEIUM 1983).There are four main parties: Labor (PvdA), Christian Democratic Appeal (CDA), Liberals (VVD) and Democrats (D'66), with approximately 38 percent, 36 percent, 20 percent and 6 percent of the popular vote in 1977. Figure 5.1 gives the estimate of the density contours of the electoral distribution of voter bliss points based on the Rabier Inglehart (1981) Euro-barometer survey.

FIGURE 5.1: Party positions in the Netherlands in 1977



The estimated exogenous valences were normalized, by choosing the D'66 to have exogenous valence $\lambda_{D66} = 0$. The other valences are $\lambda_{VVD} = 1.015$, $\lambda_{CDA} = 1.403$

and $\lambda_{PvdA} = 1.596$. To compute the D'66 Hessian, we note that the electoral variance on the first axis is $\sigma_1^2 = 0.658$, while on the second it is $\sigma_2^2 = 0.289$. The covariance (σ_1, σ_2) is negligible.

The spatial coefficient $\beta = 0.737$ for the model with exogenous valence. Thus the probability of voting for each of the parties, as well as the Hessians when all parties are at the origin, can be calculated as follows:

$$\rho_{D66} = \frac{1}{1 + e^{1.015} + e^{1.043} + e^{1.596}} = 0.078.$$

$$2\beta\ (1 - 2\rho_{D66}) = 2 \times 0.737 \times 0.844 = 1.244$$

$$\text{Hence } C_{D66} = (1.244)\ C_{D66} = (1.244) \begin{bmatrix} 0.658 & 0 \\ 0 & 0.289 \end{bmatrix} - I$$

$$= \begin{pmatrix} -0.18 & 0 \\ 0 & -0.64 \end{pmatrix},$$

$$\text{so} c = 2 \times 0.622 \times 0.947) = 1.178$$

Although the convergence coefficient exceeds 1.0, so the sufficient condition, given by Theorem 5.2 is not satisfied, the necessary condition of the Theorem is satisfied, and the eigenvalues for the characteristic matrix for D'66 can be seen to be negative. Thus the joint origin is an LSNE for the stochastic model with exogenous valence.

In a similar way, we can compute the other probabilities, giving

$$(\rho_{D66}\,, \rho_{VVD}\,, \rho_{CDA}\,, \rho_{PvdA}) = (0.078, 0.217, 0.319, 0.386)$$

This vector can be identified as the expected vote shares of the parties when all occupy the electoral origin. Note also that these expected vote shares are very similar to the sample vote shares

$$(S^*_{D66}\,, S^*_{VVD}, S^*_{CDA}, S^*_{PvdA}) = (0.104, 0.189, 0.338, 0.369),$$

as well as the average of the national vote shares in the two elections.

$$(E^*_{D66}\,, E^*_{VVD}, E^*_{CDA}, E^*_{PvdA}) = (0.094, 0.199, 0.356, 0.352)$$

These national vote shares can be regarded as approximations of the expected vote shares. Quinn and Martin (2002) performed a simulation of the empirical model and showed that the joint origin was indeed a PSNE for the vote-maximizing model with the exogenous valence values estimated by the MNL model. Moreover, the positions given in figure 5.1 could not be an LSNE of the stochastic model with exogenous valence alone. This conflict between the predicted equilibrium positions of the model and the estimated positions suggest that the activists for the parties played an important role in determining the party positions. Although we do not have data available on the activist valences for the parties, these empirical results indicate that Theorem 5.1 is compatible with the following two hypotheses:

(i) the party positions given in figure 5.1 are a close approximation to the actual positions of the parties;
(ii) each party was at a Nash equilibrium position in an electoral contest involving a balance for each party between the centripetal electoral pull for the party and the centrifugal activist pull on the party.

5.3.2. The election in the United Kingdom in 1997

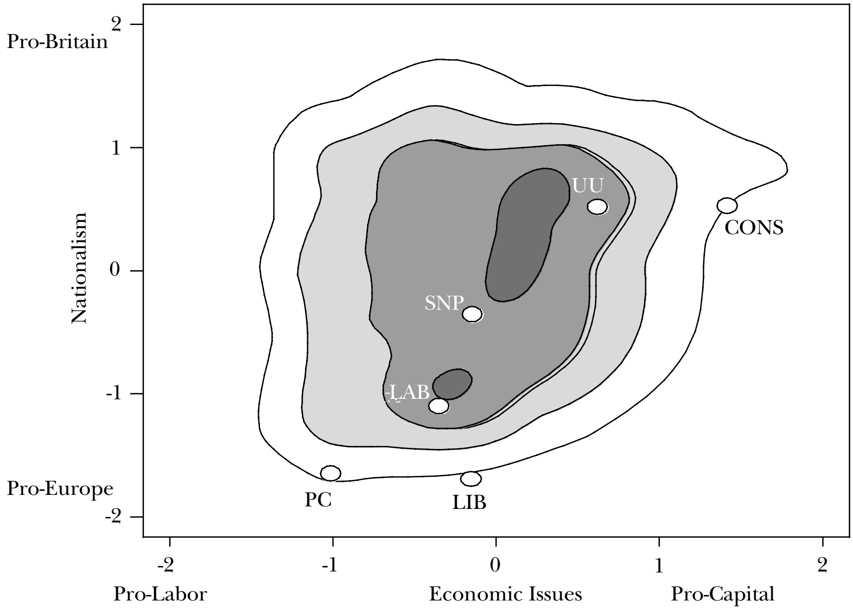
Figure 5.2 shows the estimated positions of the parties, based on a survey of Party MPs in 1997 (Schofield 2005a,b). In addition to the Conservative Party (CONS), Labor[5] Party (LAB) and Liberal Democrat Party (LIB) responses were obtained from Ulster Unionists (UU), Scottish Nationalists (SNP) and Plaid Cymru (PC). The first axis is economic, the second axis concerned attitudes to the European Union (pro-Europe to the "south" of the vertical axis, and pro-Britain to the "North"). The electoral model with exogenous valence was estimated for the election in 1997.

For 1997 $(\lambda_{con}, \lambda_{lab}, \lambda_{lib}, \beta)_{1997} = (+1.24, 0.97, 0.0, 0.5)$ so

$$\rho_{lib} = \frac{e^0}{e^0 + e^{1.24} + e^{0.97}} = \frac{1}{7.08} = 0.14$$

[5] We use the U.S. spelling for this party.

FIGURE 5.2: Party positions in the United Kingdom



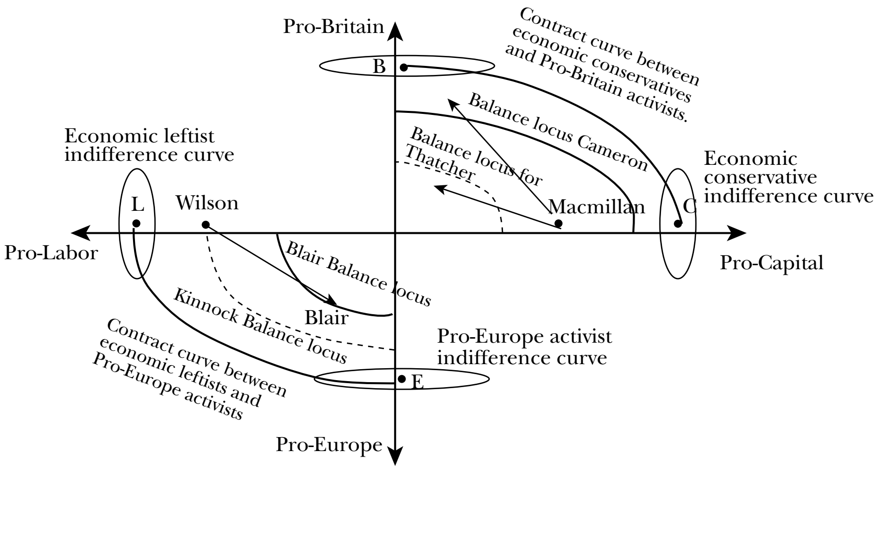
Since the electoral variance is 1.0 on the first economic axis and 1.5 on the European axis, we obtain

$$A_{lib} = \beta\ (1 - 2\rho_{\text{lib}}) = 0.36 \text{ and}$$

$$C_{lib} = \left[(0.72) \begin{pmatrix} 1.0 & 0 \\ 0 & 1.5 \end{pmatrix} \right] - I = \begin{bmatrix} -0.28 & 0 \\ 0 & +0.08 \end{bmatrix}$$

The convergence coefficient can be calculated to be 1.8, so the sufficient condition fails. Although the necessary condition is satisfied, the origin is clearly a saddlepoint for the Liberal Democrat Party. Note that the second "European" axis is a "principal electoral axis" exhibiting greater electoral variance. This axis is the eigenvector associated with the positive eigenvalue. Because the covariance between the two electoral axes is negligible, we can infer that, for each party, the eigenvalue of the Hessian at the origin is negative on the first or minor "economic" axis. According to the formal model with exogenous valence, all parties should have converged to the origin on this minor axis. Because the eigenvalue for the Liberal Democrat Party is positive on the second axis, we have an explanation for its position away from the origin on the Europe axis in figure 5.2. However there is no explanation for the location of the Conservative Party so far from the origin on both axes. Figure 5.3 gives an illustration taken from Schofield (2005) based on the empirical model for Britain for recent elections. The Labor Party benefits from resources from two potential activist groups, with preferred policy positions at L and E. The contract curve is the curve connecting these preferred positions of an activist group (L) on the economic left and an activist group (E), supporting a membership of a strong European Union. At the same time, the falling exogenous valence of the Conservative Party leader increased the marginal importance of two opposed activist groups in the party: one group "pro-capital" (at C) and one group "pro-Britain" (at B). Figure 5.3 suggests that the Labor Party position has moved from a location denoted "Wilson" along the balance locus to "Blair" , while the Conservative party has shifted postion from "Macmillan" along different balance loci to "Thatcher" in the 1980's and more recently along the "Cameron" balance locus.

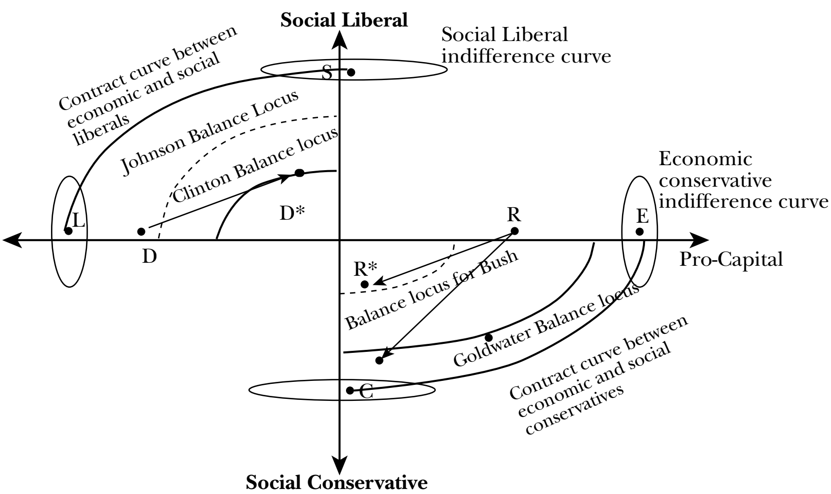
FIGURE 5.3: Balance loci in the United Kingdom



5.3.3. Elections in the United States

Miller and Schofield (2003, 2008) and Schofield and Miller (2007) have used this model (based on an economic axis and a social axis). For example, suppose that the critical condition of Theorem 5.2 fails. As suggested by the notion of a balance locus, candidates for office in a two party system must balance the centripetal electoral gradient against a centrifugal activist gradient. Figure 5.4 illustrates these formal results, by showing the contract curve between E and C for a Republican candidate, and the contract curve between L and S for a Democrat candidate. The equilibrium position for a Republican candidate will depend on the Republican exogenous valence and the position adopted by the opposition candidate. When there is a single economic dimension, then the valence difference between the contenders will separate them on left and right. Potential activist concerns can then bring the second, social dimension into existence. Optimal, or vote maximizing, candidate positions will lie on the two balance loci. In general the optimal position for a low valence candidate like Goldwater will lie on a balance locus farther from the electoral center than that of a candidate like Bush whose valence is relatively higher. As figure 5.4 suggests, the changing configuration of centripetal and centrifugal forces appears to lead to a slow rotation in the configuration of the parties. Schofield, Miller and Martin (2003) argue that a political realignment (Sundquist 1973) occurs when the two party configuration is changed suddenly (as the result of a constitutional quandary). The historical analysis suggests that this has tended to occur in a clockwise direction since the election of McKinley in 1896 (See Schofield 2006b).

FIGURE 5.4: Balance loci in the U.S.



To provide a quick test of whether the convergence condition holds in the United States, consider table 5.1, which presents a one dimensional multinomial logit (MNL) model of the 1992 presidential contest between Clinton, Perot and G. H. W. Bush.[6]

TABLE 5.1: MNL model of the 1992 presidential election in the U.S.
(normalized w.r.t Perot)

		Coef.	Std. dev.	z	prob	95%	Conf. Interval
	β coeff.	0.12	0.02	–5.34	0.00	0.08	0.16
Bush	λ_{BUSH}	–1.16	1.02	–1.13	0.26	–3.16	0.85
Clinton	$\lambda_{CLINTON}$	–0.48	0.96	–0.51	0.61	–2.36	1.39
Bush	worsefinan	–0.481	0.259	–1.86	0.063	–0.987	0.026
Clinton		0.122	0.23	0.53	0.596	–0.329	0.573
Bush	worseecon	–0.381	0.244	–1.56	0.118	–0.86	0.097
Clinton		0.669	0.27	2.48	0.013	0.14	1.198
Bush	govjobs	0.117	0.086	1.37	0.172	–0.051	0.285
Clinton		0.067	0.075	0.89	0.372	–0.08	0.215
Bush	govhealth	0.22	0.066	3.34	0.001	0.091	0.35
Clinton		0.069	0.067	1.02	0.306	–0.063	0.2
Bush	black	–0.002	0.084	–0.03	0.979	–0.166	0.162
Clinton		–0.21	0.074	–2.85	0.004	–0.354	–0.065
Bush	abortion	–0.451	0.113	–4.01	0	–0.672	–0.231
Clinton		–0.021	0.117	–0.18	0.857	–0.25	0.208
Bush	term	0.272	0.321	0.85	0.397	–0.357	0.901
Clinton		0.177	0.27	0.65	0.513	–0.352	0.705
Bush	deficit	–1.003	0.268	–3.74	0	–1.528	–0.478
Clinton		–0.418	0.275	–1.52	0.129	–0.958	0.121
Bush	east	–0.277	0.32	–0.86	0.388	–0.905	0.352
Clinton		0.407	0.293	1.39	0.165	–0.168	0.981
Bush	south	0.406	0.302	1.34	0.179	–0.186	0.999
Clinton		0.65	0.3	2.17	0.03	0.062	1.239

[6] The survey was the National Election Survey for 1992. The socio-demographic terms in table 5.1 are self explanatory. The table is based on research by Guido Cataife.

TABLE 5.1 *(cont.)*: **MNL model of the 1992 presidential election in the U.S.** (normalized w.r.t Perot)

		Coef.	Std. dev.	z	prob	95%	Conf. Interval
Bush	west	–0.307	0.304	–1.01	0.313	–0.904	0.289
Clinton		0.239	0.301	0.79	0.427	–0.35	0.828
Bush	newvoter	0.497	0.325	1.53	0.127	–0.141	1.134
Clinton		–0.283	0.294	–0.96	0.335	–0.858	0.292
Bush	dem	–0.527	0.448	–1.18	0.24	–1.404	0.351
Clinton		1.651	0.319	5.17	0	1.025	2.277
Bush	rep	1.366	0.387	3.53	0	0.608	2.124
Clinton		–0.83	0.365	–2.28	0.023	–1.545	–0.115
Bush	female	0.563	0.231	2.43	0.015	0.11	1.017
Clinton		0.191	0.22	0.87	0.387	–0.241	0.622
Bush	educyrs	0.101	0.055	1.81	0.07	–0.008	0.209
Clinton		0.032	0.052	0.62	0.534	–0.069	0.134
Bush	age 18-29	–1.18	0.39	–3.03	0.002	–1.944	–0.417
Clinton		–0.83	0.377	–2.2	0.028	–1.568	–0.092
Bush	age 30-44	–0.731	0.32	–2.28	0.022	–1.358	–0.103
Clinton		–0.729	0.323	–2.26	0.024	–1.362	–0.095
Bush	age 45-59	–0.453	0.352	–1.28	0.199	–1.143	0.238
Clinton		–0.14	0.346	–0.41	0.685	–0.818	0.538

Log likelihood = –565

$p = 905$

TABLE 5.2: **Explanation of variables**

Variable	Explanation
Worsefinan	Whether the voter thinks the national economy got worse.
Worseecon	Whether the voter thinks his personal finances got worse.
Govjobs	1: The government should see people have jobs. 7: The government should let each person get his own job without intervention.
Govhealth	1: The government should provide health plan. 7: Private plans.
Govblack	1: The government should help blacks. 7: Blacks should help themselves.

TABLE 5.2 *(cont.)*: **Explanation of variables**

Variable	Explanation
Abortion	1: Always be permited; 4: Never be permitted (2 & 3 intermediate cases).
Term	0: Does not favor term limits. 1: Favors.
Deficit	The respondent thinks the size of the budget deficit is one of the most important problems.
East	Whether the respondent is from East.
South	Whether the respondent is from South.
West	Whether the respondent is from West.
Newvoter	Whether the respondent is a new voter.
Dem	Whether the respondent is a democrat.
Rep	Whether the respondent is a republican.
Female	Whether the respondent is female.
Educyrs	Years of education.

Instead of (5.1) we use the expression

$$u_{ij}(x_i, z_j) = \lambda_j - \beta|| x_i - z_j ||^2 + \theta_j^T \eta_i + \varepsilon_j. \tag{5.15}$$

where the k-vector θ_j represents the effect of the k different sociodemographic parameters (class, domicile, education, income, etc.) on voting for the party j while η_i is a k-vector denoting the i^{th} individual's relevant "sociodemographic" characteristics. We use θ_j^T to denote the transpose of θ_j so $\theta_j^T \eta_i$ is a scalar. The terms $\{\lambda_j\}$ are the intrinsic valences, and assumed constant at each election, as in Section 5.2. Using (5.5) we find that the low valence candidate, Bush, has $\lambda_{BUSH} = -1.158$, while $\lambda_{CLINTON} = -0.482$ and $\lambda_{PEROT} = 0$. Thus

$$\rho_{BUSH} = \frac{e^0}{e^0 + e^{1.158-0.482} + e^{1.158}}$$

$$= \frac{1}{1 + e^{.678} + e^{1.158}} = 0.16$$

In the same way, $\rho_{CLINTON} = 0.32$.

The spatial coefficient is $\beta = 0.120$, and the electoral variance is $\sigma^2 = 6.22$. Thus

$$c = 2\beta\ (1 - 2\rho_{BUSH})\ \sigma^2$$

$$= 2\ (0.120)\ (0.68)\ (6.22) = 1.015.$$

Since $w = 1$, the necessary condition fails.The eigenvalue for Bush is + 0.015, which, though small, is positive, signifying that the electoral origin is a minimum for the vote share function of Bush. In contrast,

$$C_{CLINTON} = 2\beta\ (1 - 2\rho_{CLINTON})\ \sigma^2 - 1$$

$$= 2\ (0.120)\ (0.36)\ (6.22) - 1 = -0.91.$$

Thus the electoral origin is a maximum for Clinton's vote share function. In fact, using the [−2.0, +2.0] scale as in figure 5.1 and figure 5.2 gives the electorally perceived positions of the candidates as

$$(z_{CLINTON}, z_{PEROT}, z_{BUSH}) = (-0.31\ +0.57,\ 1.07).$$

On this economic scale, Clinton is just left of center, Perot moderately right of center, and G. H. W. Bush fairly far right of center. These positions are compatible with local Nash equilibrium positions for the vote maximizing model, since Clinton's best response to a Bush position on the right must be to the left. This provides some justification for the validity of the model.

5.4. Conclusion

This paper has discussed elections in the Netherlands, Britain and the United States. It is evident that they all display complex and distinct characteristic features. The main empirical point that emerges is that any centripetal tendency towards an electoral center is very weak. It is consistent with this analysis that activist groups will tend to pull the parties away from the center. Indeed, we can follow Duverger (1954) and note that under proportional electoral methods, there is very little motivation for interest groups to coalesce. Another way of expressing, in simplified form, the difference between proportional representation and plurality rule is this: under proportional electoral methods, bargaining to create winning coalitions occurs *after* the election. Under plurality rule, if interest groups do not form a coalition *before* the election, then they can be obliterated. This obviously creates a pressure for activist groups to coalesce.

References

ALDRICH, J. H. "A Spatial Model with Party Activists: Implications for Electoral Dynamics." *Public Choice* 41 (1983a): 63-100.

—. "A Downsian Spatial Model with Party Activists." *American Political Science Review* 77 (1983b): 974-990.

BANKS, J. S. and J. DUGGAN. "The Theory of Probabilistic Voting in the Spatial Model of Elections." In D. Austen-Smith and J. Duggan (Eds.),. *Social Choice and Strategic Decisions:* Heidelberg: Springer, 2005.

DOW, J. K. and J. ENDERSBY. "Multinomial Probit and Multinomial Logit: A Comparison of Choice Models for Voting Research." *Electoral Studies* 23 (2004): 107-122.

DUVERGER, M. *Political Parties: Their Organization and Activity in the Modern State.* New York: Wiley,1954.

HINICH, M. "Equilibrium in Spatial Voting: The Median Voter Result is an Artifact." *Journal of Economic Theory* 16 (1977): 208-219.

HOTELLING, H. "Stability in competition." *Economic Journal* 39 (1929): 41-57.

ISEIUM. *European Elections Study: European Political Parties' Middle Level Elites.* Mannheim: Europa Institut, 1983.

LIN, T., M. J. ENELOW and H. DORUSSEN. "Equilibrium in Multicandidate Probabilistic Spatial Voting." *Public Choice* 98 (1999): 59-82.

MCKELVEY, R. D. and J. W. PATTY. "A Theory of Voting in Large Elections." *Games and Economic Behavior* 57 (2006): 155-180.

MILLER, G. and N. SCHOFIELD. "Activists and Partisan Realignment in the U.S." *American Political Science Review* 97 (2003): 245-260.

—. *The Transformation of the Republican and Democratic Coalitions in the US.* Perspectives on Politics, 2008 (in press).

PENN, E. *A model of far-sighted voting.* Typescript: Harvard University, 2003.

POOLE, K. and H. ROSENTHAL. "U.S. Presidential Elections 1968-1980: A Spatial Analysis." *American Journal of Political Science* 28 (1984): 283-312.

QUINN, K., A. MARTIN and A. WHITFORD. "Voter Choice in Multiparty Democracies." *American Journal of Political Science* 43 (1999): 1231-1247.

QUINN, K. and A. MARTIN. "An Integrated Computational Model of Multiparty Electoral Competition." *Statistical Science* 17 (2002): 405-419.

RABIER, J. and R. INGLEHART. *Eurobarometer II April 1979. The Year of the Child in Europe.* Ann Arbor: Inter-University Consortium for Political and Social Research, 1981.

RIKER, W. H. and P. C. ORDESHOOK. *An Introduction to Positive Political Theory.* Englewood Cliffs, NJ: Prentice-Hall, 1973.

SCHOFIELD, N. "A Valence Model of Political Competition in Britain: 1992-1997." *Electoral Studies* 24 (2005): 347-370.

—. "Equilibria in the Spatial Stochastic Model with Party Activists." *The Review of Economic Design* 10 (2006a): 183-203.

—. *Architects of Political Change: Constitutional Quandaries and Social Choice Theory.* Cambridge: Cambridge University Press, 2006b.

—. "The Mean Voter Theorem: Necessary and Sufficient Conditions for Convergent Equilibrium." *The Review of Economic Studies* 74 (2007): 965-980.

Schofield, N. and G. Milller. "Elections and Activist Coalitions in the U.S." *The American Journal of Political Science* 51 (2007): 518-531.

Schofield, N., G. Miller and A. Martin. "Critical Elections and Political Realignments in the U.S.: 1860-2000." *Political Studies* 51 (2003): 217-240.

Schofield, N., A. Martin, K. Quinn and A. Whitford. "Multiparty Electoral Competition in the Netherlands and Germany: A Model based on Multinomial Probit." *Public Choice* 97 (1998): 257-293.

Schofield, N., and U. Ozdemir. *A Spatial Model of Party Positioning under Proportional Representation and Plurality Rule.* Typescript: Washington University in Saint Louis, 2008.

Schofield, N. and I. Sened. *Multiparty Democracy: Elections and Legislative Politics.* Cambridge: Cambridge University Press, 2006.

Stokes, D. Valence Politics. In D. Kavanagh [Ed.], *Electoral Politics.* Oxford: Clarendon Press, 1992.

Sundquist, J. L. *Dynamics of the Party System: Alignment and Realignment of Political Parties in the United States.* Washington D.C.: The Brookings Institution, 1973.

Train, K. *Discrete Choice Methods for Simulation.* Cambridge: Cambridge University Press. United States National Election Study, 2003.

6

Authoritarian Regimes and Political Institutions

Carles Boix
Department of Politics and Woodrow Wilson School of Public and International Affairs

...every form of government or administration... must contain a supreme power over the whole state, and this supreme power must necessarily be in the hands of one person, or few, or many...
(ARISTOTLE, *Politics,* Book 3, Chapter 7)

6.1. Introduction

Contemporary research on political regimes and institutions suffers from an acute and somewhat paradoxical imbalance. Democracy has always constituted a very exceptional form of government in human history. Until the last two hundred years, republican polities were confined to a few cities in the classical world and in medieval and modern Europe—and even then their democratic institutions were of the most imperfect sort. In fact, close to ninety nine percent of mankind has been governed by authoritarian rulers—tyrants, monarchs, princes and warlords of all venues—since it was born 100,000 years ago. Yet the overwhelming majority of the academic literature has focused on the causes, nature and performance of democratic regimes. By contrast, the theoretical examination of nondemocratic political regimes still remains at its infancy.

The underdeveloped study of authoritarianism has several causes. Here I will very briefly consider two of them. First, in contrast to the literature on elections and democratic institutions, which has been gradually transformed by the reception of analytical tools and the development of causal models, the examination of authoritarian systems is still wedded to a sociological approach committed to the construction and description of ideal types. Employing some general principles, such as ideology, organization of the dictatorship, participation of the military, the existence of particular political institutions, and even the objective function of the autocrat, various authors have proposed specific classifications or types of authoritarian regimes (Linz 1975; Geddes 1999; Wintrobe 1998). Some of these classifications are rather simple, distinguishing between military and civil autocrats or between authoritarian and totalitarian regimes. But others, responding to the fact that no single principle can accommodate all the variety of autocracies in place, have resulted in a sprawling and mostly ad hoc list of types, such as military dictatorships, traditional absolutist monarchies, one-party states, totalitarian and post-totalitarian systems, parliamentary democracies, city-state oligarchies, "sultanistic" principalities and so on. These ideal types have turned out to be scarcely informative about the mechanisms through which autocracies work. In this scientific tradition, researchers describe the traits of each type—in other words, they engage in the process of tallying the most frequent elements of each ideal model. But they hardly explain the mechanisms through which power is maintained and the consequences those different institutional structures may have on outcomes such as political stability, citizen compliance and economic development.

Second, a substantial part of the literature on dictatorships (in fact, the analytically most perceptive part) has treated tyranny or the unconstrained rule of a polity by one person as the standard, almost stereotypical type of dictatorships. The classical literature on dictatorships mainly investigated personal autocracies and the mechanisms employed by dictators to govern and secure the acquiescence of their subjects (Xenophon and Strauss 1961; Machiavelli 1513/1985). Similarly, the post-war literature on dictatorships focused on the phenomenon of totalitarianism and on the means by which the totalitarian leader and his party exercised absolute control over society (Arendt 1973; Friedrich and Brzezinski 1965; Linz 1975, 2000; Neumann 1957).[1] The formal literature which has replaced that descriptive body of work has not abandoned the basic point of departure of the traditional research on autocracies: dictatorships have continued to be modelled as regimes in which a single tyrant governs and is not subject to any external constraint or influence (Haber 2007; Kuran 1991; Tullock 1987; Wintrobe 1998). The problem is that, for all their historical and theoretical importance, single-ruler dictatorships constitute a minority of the universe of authoritarian regimes. Less than a fourth of all dictatorships since the end of World War II and only about a tenth of all currently existing countries have been governed by a single ruler. Furthermore, roughly three-fourths of all dictatorships in the last sixty years have had a legislature, while more than 60 percent have

[1] Linz (1975, 2000) is an exception in that he also examines non-totalitarian regimes.

relied on a political party to organize their base of support.[2] Even in regimes without those institutions, the leadership often maintains a smaller institutionalized body, such as a ruling council or a politburo, which sustains regularized political interaction that may serve to restrain the tyrannical tendencies of any single ruler.

With those shortcomings of the current literature in mind, this chapter is written to accomplish two things. First, it sketches a theory of dictatorships that departs from analytical fundamentals—that is, the goals, incentives and constraints which characterize rulers in dictatorial setting. Second, it tries to go beyond tyrannical rule to describe a broad range of outcomes in the universe of authoritarian regimes. Accordingly, it is organized as follows. Section 6.2 presents data on the variation in institutions in dictatorships over time. After a general conceptual discussion of the ways in which power is organized in dictatorships (in Section 6.3), the following three sections examine the sources of power and the working mechanisms of dictatorial regimes by sequentially considering the rule of a dictator without any formal allies (Section 6.4), the rule of a dictator with allies (Section 6.5) and, finally, the underpinning structures of an authoritarian system with a class of notables that govern together (Section 6.6).

6.2. Some statistics on dictatorships

For the purposes of this chapter, I define as a *dictatorship* any regime where at least one of the two following conditions is not met: free and competitive legislative elections and an executive that is accountable to citizens, either directly via elections in presidential systems or indirectly via the legislature in parliamentary systems.[3]

As a first step to give a sense of the variety of political arrangements that emerge within the universe of dictatorial regimes, graph 6.1 displays the number and proportion of dictatorships with and without legislatures in the world from 1951 to 1999.[4] Graph 6.2 reports the proportion of both types of dictatorships. I employ two data sets to determine the distribution of these two types of dictatorships. I use the data collected by Przeworski et al. (2000) (ACPL) to track the trends in dictatorships with and without legislatures from 1951 to 1990. I rely on

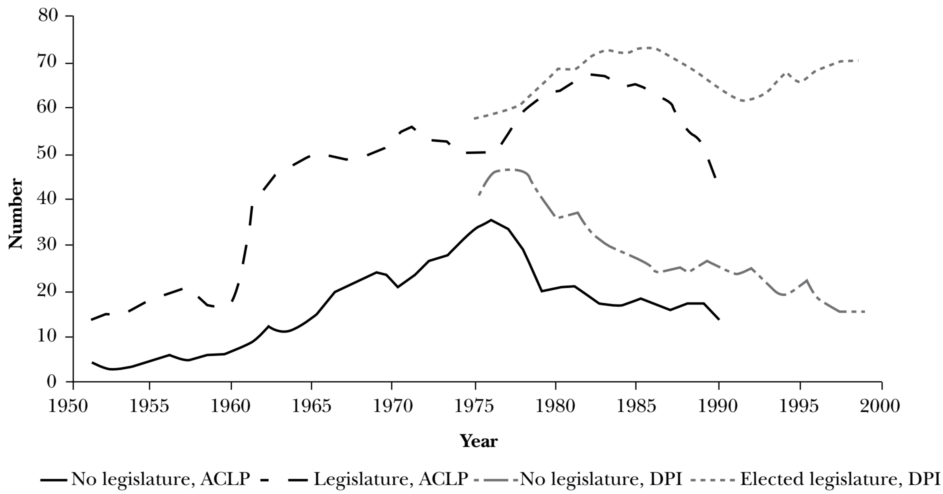
[2] The data on legislatures in dictatorships are from Przeworski et al. (2000) and Keefer (2002). The party data are based on Geddes (1999).

[3] The definition and the coding is taken from Boix and Rosato (2001).

[4] In our data, dictatorships with advisory or legislative bodies appointed by the dictator are classified as dictatorships *without* legislatures.

Keefer's *Database of Political Institutions* (Keefer 2002) (DPI) to construct the same two series from 1975 to 1999. Because the Przeworski et al.'s dataset does not report oil-exporting countries, Keefer's dataset provides an alternative count of dictatorships without legislatures.

GRAPH 6.1: Number of dictatorships with and without legislature, 1951-1999



The total number of dictatorships grew steadily from about 40 in 1951 to a peak of 108 in 1978. The number of dictatorships then declined to about 90 by the turn of the century. The number of dictatorships without legislatures grew until the mid 1970s: in 1976 there were 47 dictatorships without legislative bodies. Dictatorships with legislatures also multiplied sharply in the early 1960s, following the process of decolonization. Their number remained steady for about fifteen years before growing again after the late 1970s. By the early 1980s, there were around 70 authoritarian regimes with legislatures.

In spite of the growth in the overall number of dictatorships, the ratio of dictatorships with legislatures to dictatorships without legislatures has been remarkably stable throughout the second half of the 20th century. As shown in graph 6.2, between 70 and 80 percent of all authoritarian regimes have had an elected legislature. Only during the seventies did this proportion fell to less than 60 percent, mirroring a dramatic increase in the number of dictatorships without legislatures.

GRAPH 6.2: Proportion of dictatorships with and without legislature, 1951-1999

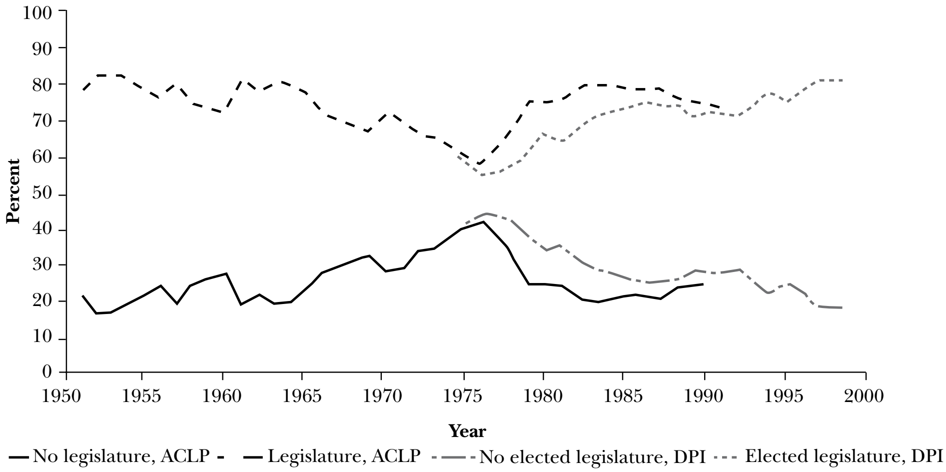


Table 6.1 reports the number of transitions to dictatorships with and without legislatures at the time of independence and from already sovereign democracies.

TABLE 6.1: Transitions to dictatorial regimes, 1955-99

	Become independent and autocratic		From democracy to autocracy	
	legislature	**no legisl.**	**legislature**	**no legisl.**
1955-59	3	1	1	2
1960-64	20	1	3	6
1965-69	7	0	1	7
1970-74	2	1	0	4
Total 1955-74	**32**	**3**	**5**	**19**
1975-79	1	0	2	0
1980-84	0	0	6	0
1985-89	0	1	2	0
1990-94	2	0	4	0
1995-99	0	0	1	0
Total 1975-99	**3**	**1**	**15**	**0**

Source: Przeworski et al. (2000) for 1955-74. Keefer (2002) for 1975-99.

Table 6.2 reports the number of transitions between both types of dictatorships as well as their overall rate of regime breakdown (to a different type of dictatorship and to democracy).[5]

Most regime transitions occurred before 1975. At the time of independence, 32 countries became dictatorships with legislatures while another three became dictatorships without legislatures. Furthermore 19 democracies became dictatorships without legislatures and five turned into dictatorships with legislatures. The number of transitions between the two types of dictatorships were substantial: 26 introduced legislatures and 37 lost them. After 1975, however, the rate of transitions was more subdued. From that year until 1999, there was no transitions between the two types of dictatorships, only a handful of countries introduced a legislature at the time of independence, and most regime transitions were democratic breakdowns resulting in dictatorships with legislatures (15 cases).

TABLE 6.2: Regime breakdowns among dictatorships, 1955-99
Probability of Dictatorship Breakdown. Own estimation of number of regimes *R* that transition to different type (including democracy) over number of regimes *R* in the previous year

	Dictatorship introduces legislature	Dictatorship loses legislature	Probability of breakdown without legislature	Dictatorship (in percent) with legislature
1955-59	2	4	19.7	7.6
1960-64	6	5	24	6.7
1965-69	5	12	9.5	5
1970-74	13	16	13	7.3
Total 1955-74	**26**	**37**	**16.5**	**6.6**
1975-79	0	0	2.7	0.4
1980-84	0	0	4.1	0.9
1985-89	0	0	3	1.4
1990-94	0	0	1.8	4.7
1995-99	0	0	0	0
Total 1975-99	**0**	**0**	**2.3**	**1.5**

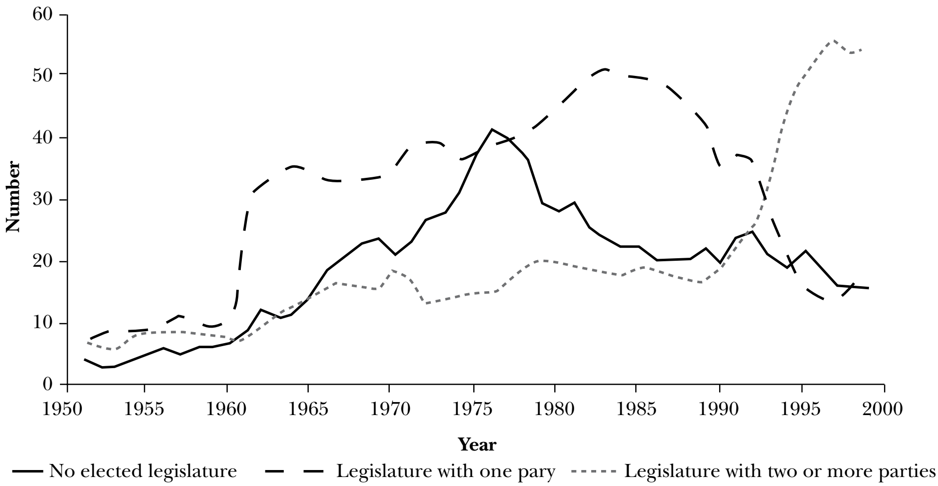
Source: Przeworski et al. (2000) for 1955-74. Keefer (2002) for 1975-99.

[5] The rate of regime breakdown is calculated as the number of transitions away from a particular type over the number of countries with that regime type in the previous year.

Table 6.2 also shows that dictatorships without legislatures are less stable than dictatorships with legislatures. Before 1975, one out of six dictatorships without legislatures broke down every year—that is three times the rate of breakdown among dictatorships with legislatures. After 1975, this rate of breakdown declined for all authoritarian regimes but it still remained higher among dictatorships without legislatures.

Graph 6.3 and graph 6.4 show the number and proportion of dictatorships classified into three categories: those without legislatures, those with a legislature and at most one political party, and those with a legislature and more than one party.[6] Both figures indicate that the number and proportion of dictatorships with more than one party tripled in the early 1990s.

GRAPH 6.3: Number of dictatorships as a function of number of parties, 1951-1999

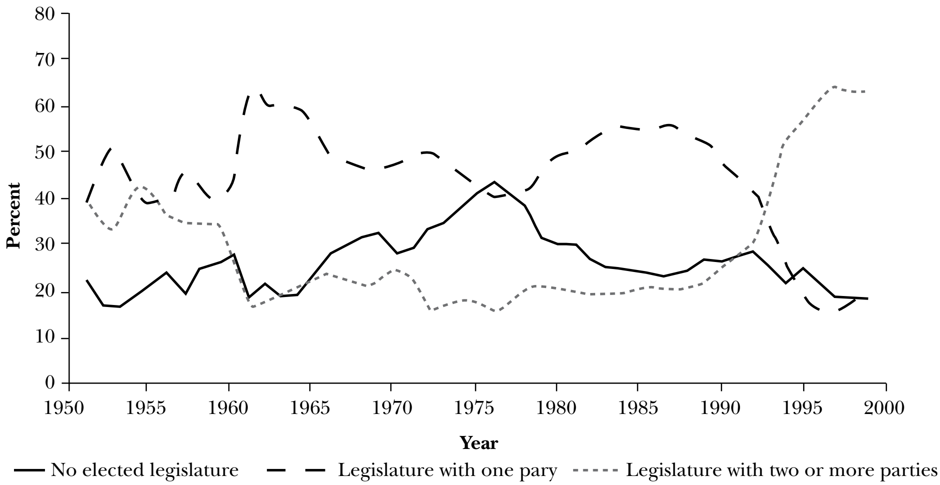


This brief overview of the data on authoritarian regimes and their institutions reveals several interesting patterns. First, most dictatorships have some type of institutions in the form of legislatures or parties (with their own committee structures) and thus poorly approximate the ideal type of tyranny, so commonly ana-

[6] In order to construct Figures 6.3 and 6.4, we have averaged the data from Przeworski et al. (2000) and Keefer (2002). For those years in which they overlap, the difference is marginal.

lyzed in the literature. Second, although the ratio of dictatorships with legislatures to those without has been remarkably stable throughout the period under study, there is a significant amount of fluidity between these two types of authoritarian regimes. However, all of this change was limited to the period before 1975. Third, about four fifths of all authoritarian regimes have had on one or more political parties. More interestingly, a substantial number of dictatorships opened up to having more than one party in the early nineties. Finally, breakdowns turned out to be more frequent among dictatorships without legislatures than among those with legislatures—with a particularly high level of instability before 1975.

GRAPH 6.4: Proportion of dictatorships as a function of number of parties, 1951-1999



6.3. The ruler and his allies

To get some leverage on the way dictatorships work, how decisions are made and how authority is structured and sustained, a plausible starting point may consist in analyzing the extent to which the dictator shares power with other political agents. In other words, it may be worth thinking of authoritarian regimes as distributed along a continuum that extends from those cases in which a single ruler governs alone (and does so without relying on any institutional structure where his agents or his allies may have some representation as a collective body) to those instances in which the dictator governs in conjunction with allies who are included in or represented through some regular institutions or political bodies. For purely functional or presentational reasons, that is, without trying to build a new typology, we may wish to refer to those cases closest to the first extreme as *tyrannies*. We may instead call those cases where there is some form of power sharing as *autocracies*. Let us now try to make these concepts more precise.

In tyrannical regimes the dictator decides alone. The tyrant may (indeed, must) have political agents on which to rely to implement his decisions. But ultimately he is the sole bearer of power. As such his exercise of power is not subject to any institutionalized limits or conditions except for those that he himself has imposed and that he can likewise abolish or redraw at will.

In autocracies the ruler (o rulers, since there may be more than one) also retains all the power of the state vis-à-vis its subjects (that is, that part of the population excluded from making political decisions). In this sphere, that is, in the relationship between the ruling clique (institutionalized through an assembly, committee or junta) and its subjects, the ruler or ruling class remains as unconstrained as the sole tyrant is in relation to everyone else but himself.

But tyrannies and autocracies differ markedly otherwise, that is, in the decision making process that takes place within the sovereign entity or body. In tyrannies the state itself and its rules embody the will of the tyrant: no one else decides jointly with the dictator or has mechanisms to hold him accountable to his promises. By contrast, in all the other cases (which I refer to as autocracies), the dictator has no direct control over enough resources to govern alone and, as a result, the dictator seeks the support of a set of allies to "share" power with them. This distinction is crucial because the incentives and behavior of the ruler differ considerably in each instance. The tyrant must make sure everybody complies with his orders in a political set-up that mirrors a coordination game where obedience to the ruler finally rests on the fact that everybody else obeys. In an autocracy the problem of coordination may not disappear. But the central problem of governance is of a different nature. The power-sharing agreement in dictatorships is beset by a credible commitment problem. Because there is no independent authority (from a third party) to guarantee that the spoils of joint rule will be divided as the dictator and his allies have agreed, the power-sharing agreement constitutes the only foundation of political authority within the authoritarian regime. Hence the central preoccupation (of ruler and allies) consists in designing a mechanism that allows all parties involved in the pact to commit in a credible manner to rule jointly. Such a mechanism will take the form of some institutional structure that, one, reduces the informational asymmetries that exist between ruler and allies and, two, eases the processes through which the allies can sanction the ruler.

6.4. The lonely ruler

In a tyrannical dictatorship, the authority of the ruler ultimately rests upon the threat of force. But it does embedded in a coordination game in which what matters are the beliefs that the rulers have about the ability of the ruler to hold power.

At the individual level the ability to use force (to get someone else's compliance) rests upon the presence of some kind of physical or intellectual inequality. The threat to inflict some pain can only be uttered in a credible way by an individual stronger than the rest of persons he is trying to coerce into some particular behavior (or, perhaps, by someone who is smart enough to outwit all the others he is trying to subject to his interests). It is immediately obvious, however, that the direct (individualized) application of force (from the ruler to his subjects) cannot suffice to secure compliance (at least from a large group of individuals). The capacity of any single individual to project power and to coerce others is rather limited because, at least on average, the distribution of physical force across individuals is relatively similar. The use of (or the threat to use) individual force may be enough to hold together and shape the behavior of a small group of persons. But it cannot sustain on its own any significant or viable political structure.

To govern the ruler needs the backing of others, that is, he needs to rely on the support of a set of individuals whom he can in turn employ to force the general population to follow his orders. Let us call these supporters, which are different from the general population subject to the dictator, the *agents* of the ruler (and, for the purposes of this chapter, the ruler qua dictator). To a good extent, the political agents of the dictatorship, such as army officers, members of the police or even top state bureaucrats, support the dictatorship in exchange for some payoffs or *rents*, that is, for receiving some privileged treatment vis-à-vis the general population (in the form of higher salaries, higher status, etc.).

But this mechanism is in itself insufficient to account for the ultimate support those agents grant to the dictator hiring or enlisting them. The reason is the following. It is true that the relationship between the dictator and his agents is one in which each agent gains from collaborating with the tyrant (as opposed to being part of the general population). But it is also true that each agent may have the temptation either of collaborating with a challenger to get rid of the current ruler (and improve his payoffs) or of proclaiming himself the next dictator.

The fact that the political agent does not challenge the dictator simply derives from the fact that she believes that no one else (or, more precisely, that only an insufficient number of agents) will challenge the ruler. If she believed or knew that a sufficiently large number of the dictator's current agents would shift to a different ruler, he would shift as well (to avoid being penalized by the winning coalition). In other words, the position of the tyrant rests upon a 'coordination game' in which everybody accepts the ruler because everybody else accepts him as a ruler and believes that everybody else does. In the context of coordination games, with a solution of multiple equilibria, the current dictator is simply one possible outcome among many. As also happens in most social choice models of preference aggregation in committees, there are many possible majorities available to govern. In a world where cyclical majorities are possible, the current outcome remains in place as long as there is no perturbation that unsettles the status quo. In the particular case of dictatorships, the ruler's power is sustained by some reputational element that makes support for the dictator some type of focal point around which everybody coordinates.

Knowing that, the ruler behaves in such a way as to make sure that everybody thinks that everybody respects him. Hence, tyrants invest on making the process of coordination among potential challengers more expensive. Among other things, they rotate personnel periodically to sap the creation of strong bonds among a subset of agents. They create different forces (such as army, standard police and different secret policing bodies) that watch and check each other. They liquidate any agents that may emerge or appear to emerge as 'rising stars' and that could become a focal point around which others may end up coordinating. They structure their personal interactions to convey to everyone that all the others are favorably disposed to support the dictator. For example, the dictator may decide to treat different people in a privileged way at different times, in a rotation manner, to ask for advice from everyone without committing himself in advance to anyone.[7]

Dictators systematically invest on the construction of public practices and a public discourse that present them as unassailable. They construct elaborate formal ceremonies that underlie their political centrality and power, that convey in

[7] For a good even though informal discussion on the strategies pursued by tyrants, see Kapuściński (1983), part one.

a very public form their capacity to order and punish, and that make visible to everyone the undisputed allegiance of all their courtiers, bureaucrats and subjects (Wedeen 1999). They also spend on propaganda and indoctrination to make it impossible for any potential challenger to gauge the extent of resistance to the dictator and to spread any seeds of doubt among their co-agents. The purpose of deploying what Kuran (1991) has called a 'public discourse' of lies is not to indoctrinate the population or even to change their preferences (even though dictators also welcome that effect when it happens). The goal is to construct a set of messages and signals that force everyone to hide their true (private) preferences (since revealing them may simply result in them being punished) and therefore reinforce the pro-dictatorship equilibrium.

6.5. Power-sharing agreements and institutions in authoritarian regimes

The purpose of any dictator (and most probably of any politician) is to maximize power and hence to govern alone. But, due to various reasons (which I do not discuss here), in most instances the dictator does not have the capacity, resources or *fortuna* to govern as a tyrant. That is, it is impossible for him to forge and sustain the set of beliefs and behaviors that characterize the type of coordination game upon which tyrannies are based. Accordingly, he must then rely on the support of an elite or set of allies or notables to hold power.

As pointed out earlier, this tacit or sometimes explicit "deal" among the members of the autocratic or ruling elite to govern together according to some predetermined rule is beset by a credible commitment problem. To maximize his power, to secure higher rents, to lower the probability of rebellions among ambitious notables or to build up a more cohesive country, the dictator strategizes to get rid of or at least subject with a much firmer grasp the class of allies who support him. The leader may try to check, imprison or kill all his supporters at the same time and then proclaim himself absolute ruler or monarch. Alternatively, he may use more gradual tactics. He may get rid of a few notables at a time without the rest taking notice, proceed to absorb the fortunes and power of those he just eliminated and use his growing power to further purge more notables. A few sequential rounds of such type of "salami" tactics will transform him into a tyrant.[8]

[8] In turn, one or several allies may as well try to upset the existing balance of power. They may attempt to accumulate resources, organize particular networks of supporters or reshape the class of

A verbal pact among autocrats, i.e. an orally given promise to respect the position or status of everyone and to consult everyone informally to decide over any issue is not sufficient to sustain an autocratic deal over the medium run. A mere written agreement—in the form of a contract accepted and signed by everyone is not enough either. As in any pact, the autocratic deal needs to rely on some external guarantees, that is, on some guarantees that go beyond the strict promise to behave well and keep the agreement.

The autocratic "pact" will only last over a relatively prolonged period of time if some public body or institution confirms the nature of the deal made among autocrats and guarantees its maintenance. This body (or organization) cannot be a third party independent from the autocratic elite to whom the latter entrusts the enforcement of the pact since in that instance the sovereignty of the state would be actually lying in its hands and not under the control of the governing elite. The institution (or a set of institutions) which embodies (and preserves) the pact must be some structure where the members of the elite are represented or in which they participate and which reflects the nature of the pact among the members of the elite. Through this institutional structure, the ruling elite receives the right type of information about the resources of the dictator and the internal flows of income within the elite and, therefore, more generally, about the current balance of power and the possible attempts made to alter the latter. The allies of the dictator make sure that the leader does not develop strategies to shift the distribution of power, assets and status. Similarly, every member of the ruling coalition observes the nature, size and stability of the existing factions in the country. In doing so, they verify that no section of the ruling class is too loyal to the leader or, in other words, too "monarchical". The very routine of meeting in a committee, party congress or assembly may serve as a yardstick to measure the intentions of the leader. Any attempt by the national leader to block or not convene his allies is a signal that he is indeed intent on disrupting the old balance of power and should therefore trigger an immediate backlash from them. This reduction in the level of secrecy of dictatorships and therefore on the ease with which a dictatorship can renege from the pact makes autocracies more stable (than tyrannies).

To see in a more precise way how institutionalized power-sharing agreements work, let us start first by looking at a power-sharing agreement in which no institutions are in place. We will then introduce institutions, in a sense that will be-

---

notables itself to constrain the dictator or to fire him. Conversely, one or several allies may as well try to upset the existing balance of power. They may attempt to accumulate resources, organize particular networks of supporters or reshape the coalition of allies to constrain the dictator or get rid of him.

come clearer later, and explore how the regime dynamics will change. (The rest of this section summarizes formal and empirical work developed in Boix and Svolik 2009).

Assume that at an initial time the ruler and his allies strike a power-sharing agreement in which the ruler pays allies some share of government revenue during good economic times and no revenue in bad times. The economy behaves in such way that good economic times happen with some probability p and bad times with probability 1-p. After ruler and allies have agreed to cooperate with each other, nature determines the state of the economy. The ruler privately observes revenue. He then reports it to allies and pays them. Crucially, the allies observe the ruler's report and their compensation but not revenue. Allies solve their coordination problem with some probability. If they do, they may rebel conditional on the revenue they observe.[9]

Notice, first of all, that staging a rebellion is the only punishment available to the allies to discipline the ruler into complying with the pact. Observe as well that the (threat of a) rebellion serves two purposes. On the one hand, it is used to discourage the ruler from lying about the state of the economy he obtains in each period. To avoid paying his allies, the ruler can choose to lie about the state of the treasury. He may claim that there is a fiscal crisis and that revenue is 0 (even though this is not true) simply because he does not want to pay each ally the revenue they agreed to share to start with. Hence, to discipline the ruler, i.e. to make sure he lies less often, the allies may threaten to rebel in any period in which the ruler claims revenue is 0.

On the other hand, the rebellion is also employed to discourage the ruler from not acting on his promise to transfer some fraction of his revenue to his allies. Threatening the ruler with the possibility of rebellion if they receive a payoff other than the promised amount is done independently of the dictator's claim about the state of the economy. That is, the dictator may decide not to lie about the economic cycle—but, still, after acknowledging that it is good, he may proceed not to transfer any revenue to his allies.

Either type of defection (lying or not complying) hurts the allies equally and yields the same benefit to the ruler. But, crucially, they are different. This difference can be exploited to explain why certain types of institutions may lessen the dictator's temptation and therefore reduce the incentive to rebel.

To see how an institutional solution sustains a power-sharing agreement, notice that, from the point of view of the ruler, the dictator complies with the agree-

[9] The way in which they solve their coordination problem is modelled in Boix and Svolik (2009).

ment only if complying over time is better that reneging. Now, in a power-sharing agreement as described, that is, in a dictatorship where no (credible) information is given to the allies, we have already indicated that the latter need to stage a rebellion every time the ruler claims there is a fiscal crisis (to discipline the ruler).

However, suppose that those allies knew (i.e. were informed in a credible way) about the true state of the economy. Under that circumstance, their need to resort to the threat of rebellion would necessarily decline. They would only need to rebel when the ruler did not comply during a good economic spell. (Remember that, given the nature of the agreement, the ruler makes no transfers when the economic cycle is bad.) In short, the frequency of rebellions declines with transparency vis-à-vis those cases with no information.

Under that circumstance, that is, with no asymmetries of information among ruler and allies, and hence with a lower need to threaten with rebellions, the ruler would have a higher expected payoff (provided he decides to comply with the agreement) than in power-sharing agreements where the true evolution of the economy and the treasury was opaque to allies. In other words, with informational mechanisms in place that would make the state of the world transparent to everyone, the ruler would become less tempted to renege on his promises. As a result, the power-sharing agreement would become more stable.

To sum up, given the type of problems that come from having a power-sharing agreement, both the ruler and the allies benefit from establishing an institutional mechanism that reveals government revenue to all parties in each period. This mechanism could be, for instance, a periodic review of government spending and revenue or the consultation of major policies by a council of allies or their representatives—where lying about revenue by the ruler will be, in turn, observable to the allies. In short, the introduction of institutions in an authoritarian setting is the counterpart of some degree of joint rule between dictator and allies.

6.6. "Horizontal" autocracies

In our analysis of how power is allocated in autocracies, we have progressed from tyrannical models, where the tyrant rules unbound by any institutional form, to autocratic regimes in which there are power-sharing agreements with institutions that reduce informational asymmetries and therefore stabilize the pact of ruler and allies. In the latter solution, the ruler still is hierarchically positioned vis-à-vis his allies: the allies share in information but, except for the terms of the power-sharing agreement, do not make policy and, importantly, do not choose the ruler.

We can think, however, of nondemocratic regimes, that is, regimes where only a minority governs to the exclusion of the rest, where institutions perform a much stronger role. That would be the case of autocracies with institutions employed to make policy and choose the ruler. The type of autocracies which were described in the preceding section were of a vertical nature, that is, their dictators are unelected. But many other authoritarian regimes are structured in a much more 'horizontal' manner, that is, they have several notables governing together with similar status and power. These autocratic regimes are the result of a pact among individuals who are roughly equal among themselves. The governing class (a landholding aristocracy, a urban oligarchy, the upper ranks of the national army or a set of guerrilla leaders or party comrades who have somehow managed to retain equal status among themselves) is formed by members that are similar to all the other members of his class in status and resources and that accept a system of government in which each of them has roughly the same weight.

These "horizontal" autocracies are generally governed by an autocratic class that meets and decides in a *committee structure* (one or many committees): an assembly, a parliament, a military junta, a municipal council. As in any committee, decisions are taken by aggregating the votes or opinions of the members of the assembly, council or junta. The particular rules to decide vary with the autocracy in place: they extend from a unanimity system (such as the one that prevailed in Poland in the modern era, where each representative to the *Sejm* had a "liberum veto" or the right to veto the decisions of the assembly) to a principle of majority vote.

A government by committee meets the requirements of the autocratic pact: it can monitor the behavior of the components of the autocratic clique (since it is in the committee that any ruling decisions are taken and therefore everyone can observe the actions of everyone else) and it has (in principle) the means to enforce the pact (since it is the committee itself that embodies the balance of power and that governs the state).

At the same time, although the committee system may reflect a pact of equals and has the tools to enforce such a pact, it is not exempt of any dangers (that is, it may not avoid the collapse of the autocratic arrangement it aims to preserve). For an autocratic arrangement (of a committee type) to work properly, that is, to be accepted as a stable arrangement by all, everyone has to have a reasonable chance of being part of the majority regularly—that is, everyone should be voting willingly with the majority with some frequency. Conversely, the autocratic regime normally collapses (that is, it converges toward a tyranny or breaks down as the result of internal infighting) if a fixed majority (i.e. one composed by the same individuals) forms permanently in the assembly or council and excludes the minority from government. Under such circumstances, the majority will be very likely to exploit the minority. Taking advantage of their unassailable position, the members of the majority will decide to approve policies that reduce the status and resources of the minority—directly either through expropriation or by barring the latter's access to state offices or indirectly by changing the relative prices of the assets of the minority. Once the minority is sufficiently weakened, the majority will be able to rewrite the nature of the autocratic pact without facing any serious opposition, that is, it will be able to expel the minority from the committee forever. If this process of establishing a permanent majority which then proceeds to shrink the committee is repeated sequentially (so that the new majority that emerges from the new committee purges the rest of the committee and establishes a new council that is again remodeled by its majority and so on), autocracies unravel until they become a tyranny.

Democracies face, in principle, the same threat: the formation of a majority that redistributes to itself until the excluded minority is too weak to participate in politics. However, the sheer size of democracies makes this outcome (their transformation into autocracies or even tyrannies) very unlikely (although not implausible). Autocratic cliques contain few individuals by definition and hence the construction of permanent coalitions and the exclusion and/or physical elimination of the minority may be much easier to accomplish.

The instability of committee rule in autocracies may be lessened by at least three mechanisms. First, the members of the autocratic clique may impose particularly demanding rules to make any decision, such as a principle of qualified majorities or of unanimity. Such a method reduces (or simply eliminates in the case of unanimity) the possibility of having a majority excluding the minority systematically. Naturally that solution comes at a considerable cost—the danger of complete paralysis in government. A partial way to protect minorities while avoiding stagnation would go as follows: elect by a unanimity or quasi-unanimity principle a smaller committee which would then be in charge of all executive functions and would decide on the basis of a simple majority principle. Since even this solution may not succeed in hampering the efforts of a well-organized and obstinate clique to control the state (or, for that matter, of the elected small committee), a second method to reduce an internal coup may consist in the multiplication of the number of committees, with slightly different rules of election for each one and with partially different political responsibilities. This would make sure that the committees' internal composition would differ partially and there would be no majority that could control all the committees and exclude the rest. Finally, the autocratic elite may employ some lottery system to select parts of the government. Electing committees by lot would secure the rotation of all interests in government and decrease the extension of vote-buying and the formation of patronage networks.

The Venetian Republic, controlled by a class of commercial patricians, employed a combination of all these strategies to minimize the chances of an internal coup d'etat. Formally since the late 13th century (although informally from earlier dates), the government of Venice was in the hands of a tiny fraction of its population—about 1,200-1,500 men (out of a population of about 100,000 and hence about 5 percent of the eligible male inhabitants). This patriciate, that formed the Grand Council, elected several committees (the Collegio and the Senate, among others) as well as the doge or executive officer of the Republic (in what was an appointment for life). Moreover, in the nomination process of many officers, Venetians employed extremely long and complex procedures. For example, they employed about ten committees (sequentially elected, either through lottery or votes) to choose their doge (Finer 1997: 998 ff.).

Venice's complicated structure of government, with several committees and a convoluted process of nominations, was in all likelihood a prudent response to the perils that besiege horizontal autocracies. Venice's system was extremely successful. The city lasted as an independent country for about a thousand years and enjoyed enormous political stability. There were hardly any riots or elite coups and the constitutional reforms implemented in 1297 remained unchanged until Venice was conquered by the Napoleonic armies five hundred years later. The success of Venice becomes even more striking when compared with the story of all the other Italian republics of the time such as Genoa, Venice or Milan, permanently shaken by internal strife and eventually falling into the hands of tyrants or foreign monarchs.

The Venetian example may be less infrequent that we tend to think. Consider the case of Mexico under the PRI (as well as many other large contemporary autocracies). The distribution of elite positions followed a complicated procedure, with multiple layers of government (at the federal and state level), topped by a particular system to appoint the president. That system was also underpinned by regular (even if fraudulent) elections where the local elites structured a network of clients that allow them to deter potential entrants (Magalone 2007) and to give enough information to other politicians about the relative weight of the different factions that constitute the ruling elite.

6.7. Conclusions

In contrast to most of the current literature on authoritarian regimes, which has focused on the construction of typologies and which has tended to model dictators as unconstrained tyrants, this chapter explores how power is structured and exercised in different authoritarian arrangements. The chapter suggests that the extent to which power is accumulated in an authoritarian regime admits of different solutions. In some (in fact the least) cases the dictator operates in an almost unconstraint manner. In other instances the ruler needs the explicit support of some allies to whom he offers some portion of the spoils of government. In yet other regimes a coalition of individuals govern together (as relative equals) to exclude the majority of the population. In each instance the ruler or rulers have to solve a key political problem respectively. In tyrannical systems the ruler must make sure everybody coordinates in the belief of his invincibility. In vertical autocracies the ruler and his allies establish some institutional mechanism to solve the credible commitment problem that beset their initial agreement. In horizontal autocracies the ruling clique must try to organize the decision-making and selection processes to minimize the possibility of exclusion of part of the initial coalition.

References

Arendt, H. *The Origins of Totalitarianism.* Harvest Books, 1973.

Boix, C. and S. Rosato. "A complete data set of political regimes, 1800-1999." Department of Political Science, University of Chicago, 2001.

Boix, C. and M. Svolic. *The Foundations of Limited Authoritarian Government: Institutions and Power – Sharing in Dictatorships.* Typescript: Princenton University, 2009.

Friedrich, K. and Z. Brzezinski. *Totalitarian Dictatorship and Autocracy.* Cambridge, Mass. Harvard University Press, 1965.

Finer, Samuel E. *The History of Government.* New York: Oxford University Press. Volume 2: 1997.

Geddes, B. *Authoritarian Breakdown: Empirical Test of a Game Theoretic Argument.* Unpublished manuscript, UCLA, 1999.

Haber, S. *Authoritarian Government.* In Barry R. Weingast and Donald A. Wittman. The Oxford Handbook of Political Economy. New York: Oxford University Press, 2007: 693-707.

Huntington, S. *Political Order in Changing Societies.* New Haven: Yale University Press, 1968.

Kapuściński, R. *The Emperor.* New York: Harcourt Brace Jovanovich, 1983.

KEEFER, P. *DPI2000: Database of Political Institutions*. Development Research Group. The World Bank, 2002.

KURAN, T. "Now Out of Never: The Element of Surprise in the East European Revolution of 1989 (in Liberalization and Democratization in the Soviet Union and Eastern Europe)." *World Politics* 44 (1991): 7-48.

LINZ, J. "Totalitarian and Authoritarian Regimes." In Fred Greenstein and Nelson Polsby, eds. *Handbook of Political Science*. Reading, Mass.: Addison-Wesley Publishing Company. Volume 3 (1975): 191-357,

—. *Totalitarian and authoritarian regimes*. Boulder, Col.: Lynne Rienner Publishers, 2000.

MACHIAVELLI, N. *The Prince*. Chicago: The University of Chicago Press, 1513/1985.

MAGALONE, B. *Voting for Autocracy: Hegemonic Party Survival and its Demise in Mexico*. New York: Cambridge University Press, 2007.

NEUMANN, F. *The Democratic and Authoritarian State*. New York: Free Press, 1957.

PRZEWORSKI, A., M. E. ÁLVAREZ, J. A. CHEIBUB and F. LIMONGI. *Democracy and Development: Political Institutions and Well-Being in the World, 1950-1990*. New York: Cambridge University Press, 2000.

TULLOCK, G. *Autocracy*. Boston: Kluwer, 1987.

WEDEEN, L. *Ambiguities of Domination: Politics, Rhetoric, and Symbols In Contemporary Syria*. University of Chicago Press, 1999.

WINTROBE, R. *The Political Economy of Dictatorship*. New York: Cambridge University Press, 1998.

XENOPHON and L. STRAUSS. *On Tyranny*. Chicago: The University of Chicago Press, 1961.

7

The Condorcet-Duverger Trade-off: Swing Voters and Voting Equilibria

Laurent Bouton
ECARES
Free University of Brussels

Micael Castanheira
ECARES
Free University of Brussels

7.1. Introduction

There is a striking discrepancy between the way elections operate and the way we model them. During elections, candidates campaign to advertise their ideas and platforms, and voters use this information to learn which candidate would best represent them. The information being difficult to process, many "swing" voters remain undecided or uncertain.[1] A substantial part of the divisions among the voters also stem from the different interpretations and beliefs about what are just policies.[2]

The models studying the properties of electoral systems overlook this learning process and the presence of "swing" voters. Typically, the modeler's first assumption is to endow voters with a preference ordering over candidates. This ordering is fixed by assumption: there is no learning; preferences do not change. We show that this assumption is far from innocuous: it hides the presence of some equilibria and affects the properties of others.

We focus our attention on plurality (aka *first-past-the-post*) elections and model voter preferences as in the Condorcet Jury Theorem literature (Austen-Smith

---

[1] Probabilistic voting models implicitly recognize this by letting a part of the voting behavior be determined by a random component; see e.g. Dixit and Londregan (1998). Persson and Tabellini (2000) provide an enlightening review of these models.

[2] The dynamics of beliefs may then generate substantially different economic equilibria. See Piketty (1995), Alesina and Angeletos (2005) and Benabou and Tirole (2006) among others.

and Banks 1996; Feddersen and Pesendorfer 1996, 1997, 1998, 1999; Myerson 1998a). This approach to voter preferences reflects a world in which voters may have common goals, such as improving their economic condition, their personal security, etc. but disagree about the best way to achieve this: voters agree about the ends but not necessarily about the means. Depending on their information, they form beliefs about which candidate would be best, but this information is imperfect. The difference with the standard approach is thus that the relationship between voter preferences and their ranking of candidates is fuzzier; it depends on an unobservable state of nature that determines which candidate is "truly" best for the swing voters.

We consider three types of voters who support three different candidates. Types t_A and t_B support candidates A and B respectively, and rank candidate C as their worst candidate. The division among t_A and t_B voters is information-based: they hold opposite beliefs about the true state of nature, but have identical underlying preferences. In other words, they would agree on which candidate is truly the best if they held identical beliefs. Those will be the *swing voters*: if the election revealed sufficient information, some would change their mind. Types t_C are *stalwart voters*. They always support C. Those are a minority: the election of C is thus the worst possible outcome.

In section 7.3, we begin by reviewing two-candidate elections to briefly summarize the literature on the Condorcet Jury Theorem. In section 7.4, we study three-candidate elections. We highlight an important trade-off between "vote division", which is a necessary condition for the Condorcet Jury Theorem to be valid, and "Duvergerian" forces that induce swing voters to coordinate all their votes on a single candidate, to beat C. This is what we call the *Condorcet-Duverger trade-off*. Under plurality, the "Duvergerian" forces may be either too strong or too weak. If they are too strong, the election of the best candidate can be jeopardized, because majority voters do not sufficiently divide their voters. If they are too weak, swing voters may coordinate insufficiently and let C win the election.

This Condorcet-Duverger trade-off in plurality elections shows why it is important to take account of swing voters in the analysis of any electoral system. In Bouton and Castanheira (2008), we study other systems: in the spirit of Myerson and Weber (1993), we compare the equilibrium properties of plurality elections with that of run-off elections and of Approval Voting. We show that the Condorcet-Duverger trade-off is still present under run-off but is absent under Approval Voting. Under the latter system, there is a unique voting equilibrium, and therefore no possible coordination failure, and the Condorcet Jury Theorem holds. There is thus *full information and coordination equivalence*.

7.2. The model

We study the equilibrium properties of plurality elections when there are swing voters and compare these properties with that of a model in which the presence of swing voters is overlooked. In plurality elections, the candidate receiving the most votes is elected. Ties are resolved by the toss of a fair coin.

We conduct our analysis under the assumption that the total number of voters is distributed according to a Poisson distribution with some mean n (see Myerson 1998b and 2000 for the properties of Poisson Games). The probability that there are k voters in the population is therefore:

$$\Pr(k \mid n) = \frac{e^{-n} n^k}{k!}$$

There are three types of voters $t \in \{t_A, t_B, t_C\}$ and two states of nature: $\omega \in \{a, b\}$. We are thus analyzing an *extended Poisson Game* as introduced by Myerson (1998a), in which the probability that a given voter has type t depends on the actual state of nature ω. These probabilities are denoted $r(t \mid \omega)$, with $\sum_t r(t \mid \omega) = 1$. The actual state of nature is unknown to the voters. They only know that the probability of each state, $q(\omega) \in (0,1)$, s.t. $q(a) + q(b) = 1$.

There will be up to three candidates, $P = A$, B and C. We denote the utility of the voters by the function $U(P, t, \omega)$, where P is the party winning the election, t is the voter's type, and ω is the state of nature. An *equilibrium* is found when each voter's strategy maximizes her expected utility given the vote share of each party, and these vote shares are coherent with the voters' actions (Myerson and Weber 1993; Myerson 2002). Note that the act of voting is costless; if abstention happens in equilibrium, it thus reveals that some votes would strictly reduce the voters' expected utility.

7.2.1. Voter types: minority and majority blocks

Types t_C represent the *minority block*. They are *stalwart* in the sense that they prefer candidate C independently of the state of nature. For the sake of tractability, we also assume that they are indifferent between the other two candidates:

$$U(P, t_C, \omega) = 1 \text{ if } P = C$$
$$= 0 \text{ if } P \in \{A, B\}$$

The probability that a given voter has type t_C is below one half: we impose $r\,(t_C \mid a) = r\,(t_C \mid b) < 1/2$.

Types t_A and t_B together represent the *majority block*. They are *swing voters* in the sense that their preferences over candidates, as well as their share in the electorate, are state-contingent. There are more types t_A in state a than in state b: $r\,(t_A \mid a) > r\,(t_A \mid b)$, and conversely for types t_B. By Bayesian updating, a type t voter thus infers that:

$$q\,(\omega \mid t) = \frac{q(\omega)\ r(t \mid \omega)}{q(a)\ r(t \mid a) + q(b)\ r(t \mid b)}$$

Since $r\,(t_A \mid a) > r\,(t_A \mid b)$ and $r\,(t_B \mid a) < r\,(t_B \mid b)$, types t_A and t_B hold different beliefs about the likelihood of the two states of nature. To introduce divisions within the majority, we impose that types t_A and t_B support a different candidate:

$$\frac{q(a \mid t_A)}{q(b \mid t_A)} > 1 > \frac{q(a \mid t_B)}{q(b \mid t_B)} \tag{7.1}$$

Conditional on the state of nature, their preferences are aligned: for types $t \in \{t_A, t_B\}$, we have

$$\begin{aligned} U\,(P, t, \omega) &= 1 \text{ if } (P, \omega) = (A, a) \text{ or } (B, b) \\ &= 0 \text{ if } (P, \omega) = (A, b) \text{ or } (B, a) \\ &= -1 \text{ if } P = C \end{aligned}$$

Thus, majority-block voters have common views about the objectives that policy should pursue but they have opposite priors about the means to reach these objectives; they have opposite views regarding the true state of nature. In other words, the mapping between objectives and candidates is blurred; it depends on an unobservable state of nature. This is why we call these voters "swing" . With sufficiently convincing information, they may admit that their priors were wrong, and modify their support for one or the other candidate.

Finally, we make the technical assumption that, on average, there can be more types t_A than types t_B in the population:

$$r\,(t_A \mid a) + r\,(t_A \mid b) \geq r\,(t_B \mid a) + r\,(t_B \mid b)$$

This assumption is only meant to ensure that our results do not hinge on any type of symmetry across types.

7.2.2. Strategy set and action profiles

The voters' action set is denoted by $\Psi = \{\varnothing, A, B, C\}$. That is, voters can either abstain or vote for one of the three candidates. Let $\sigma(\psi \mid t)$ denote the probability that a player plays $\psi \in \Psi$ if he has type t. The usual constraints apply: $\sigma(\psi \mid t) \geq 0$ and $\sum_{\psi} \sigma(\Psi \mid t) = 1 \; \forall t$. For short, $\sigma(t)$ will denote the vector of these probabilities.

Aggregating strategies, the *expected share* of voters playing action ψ in state ω is therefore:

$$\tau(\psi \mid \omega) = \sum_{t} r(t \mid \omega)\, \sigma(\psi \mid t).$$

Remark that these expected fractions can differ across states, but only because the fraction of each type $r(t \mid \omega)$ is state-dependent.

Thus, there is an *informational trap* if t_A and t_B voters adopt the same strategy $\sigma(t)$. In that case, the expected result of the election is the same in both states of nature; observing the election outcome cannot reveal additional information about the actual state of nature. Under an informational trap, voters with different types cannot eventually agree on a candidate. When there is no informational trap, the outcome of the election reveals a lot of information about the actual state of nature. In that case, swing voters expect the election to potentially modify their priors.

Following Myerson (1998b, 2000), if the expected size of the population is n and if *expected shares* are $\tau(\psi \mid \omega)$, then the *realized number of votes* for ψ, $x(\psi)$, is a Poisson variable with mean $n\tau(\psi \mid \omega)$:

$$\Pr\left(x(\psi) \mid \tau(\cdot)\right) = \frac{e^{-n\tau(\psi \mid \omega)}\left[n\tau(\psi \mid \omega)\right]^{x(\psi)}}{x(\psi)!}$$

This distribution depends on the voters' strategy and on the state of nature ω.

Using Theorem 1 of Myerson (2000) we can characterize the limiting probability that the number of votes for each action is some vector $\vec{x} = \left(x(A), x(B), x(C), x(\varnothing)\right)$. This probability centrally depends on the *magnitude*, denoted *mag*, of the considered event $\vec{x}$:

Property 7.1 *(Myerson 2000, Theorem 1) Subject to* $\sum_{\psi \in \{\varnothing, A, B, C\}} \tau(\psi \mid \omega) = 1, \omega \in \{a, b\}$ *and given expected shares* $\tau(\omega)$*, the probability that the actual number of votes is* $\vec{x} = (x(A), x(B), x(C), x(\varnothing))$ *converges to*

$$\Pr(\vec{x} \mid \tau(\omega)) \underset{n\to\infty}{\to} \max_{\vec{x}} \frac{\exp[mag[\vec{x}]\, n]}{\prod_{\psi} \sqrt{2\pi x(\psi) + \frac{\pi}{3}}}$$

$$\text{where: } mag[\vec{x}] = \sum_{\psi} \frac{x(\psi)}{n} \left(1 - \log\left(\frac{x(\psi)}{n\tau(\psi \mid \omega)}\right)\right) - 1 \; (\leq 0)$$

Our results only exploit this magnitude theorem and are thus valid for basically any distribution of voters that generate the same comparative statics. The comparative statics implication of the magnitude theorem is that the probability of an event is exponentially decreasing in population size n. Therefore, if we compare two events, call them 1 and 2, with magnitudes $mag_1 > mag_2$, then, as $n \to \infty$, event 1 will become infinitely more likely to happen than event 2.

Formally, $\lim_{n\to\infty} \log[\Pr(\vec{x} \mid \tau(\omega))]/n = mag[x]$. Hence, the most likely events are those with magnitude 0. The event with magnitude 0 is $\vec{x} = \tau(\omega) \cdot n$, i.e. the event that actual vote shares are arbitrarily close to expected vote shares. The magnitude of any other event is strictly negative.

7.3. Two-candidate elections

This section reviews some of the main results in the Condorcet Jury Theorem literature. This literature primarily focused on two-candidate elections and showed how the presence of swing voters affects the equilibrium properties of the election. The fundamental change is that voters may prefer to vote against their *a priori* preferred alternative to enhance election efficiency. That is, two-candidate elections generate *full information equivalence*: the winning alternative is the one that would have been chosen under full information (Austen-Smith and Banks 1996; Feddersen and Pesendorfer 1997 and Myerson 1998a). Full information equivalence requires that A ranks first in state a and B ranks first in state b.

To summarize these findings, we can shrink the fraction of t_C-voters to zero in our model: this produces a two-candidate setup that is almost identical to the one of Myerson (1998a). The difference between our setup and Myerson's is that we allow for abstention. We show that voters never vote against their *a priori* preferred alternative, because abstention dominates such cross-voting. In the next section, we extend the analysis to a three-candidate setup.

The *expected* value of a ballot is $G(\psi \mid t)$. It depends on the probability of being pivotal against the other candidate. The value of a vote for candidate A is:

$$G(A \mid t) = q(a \mid t) \Pr(piv_{AB} \mid a) - q(b \mid t) \Pr(piv_{AB} \mid b) \qquad (7.2)$$

which reads as follows: a type t expects that the state of nature is a with probability $q(a \mid t)$. In that state, a majority block voter's utility is 1 if A wins and 0 if B wins. Therefore, if the vote is pivotal (in favor of A, against B), utility **increases** by 1. If the actual state is b, utility **decreases** by 1. The value of a vote for B is derived in the same way:

$$G(B \mid t) = q(b \mid t) \Pr(piv_{AB} \mid b) - q(a \mid t) \Pr(piv_{BA} \mid a) \qquad (7.3)$$

Note the difference between a swing voter and a stalwarts voter: the swing voter is trying to elect the best candidate, while the stalwarts voter is only trying to elect her candidate. Stalwarts voters can be represented as voters who assign probability 0 to the "other" state of nature. That is, stalwarts types t_A would assign a probability 1 to a and a probability 0 to b, and the opposite for stalwarts types t_B. These stalwarts types have a simple dominant strategy: vote for their own candidates.

Austen-Smith and Banks (1996) showed that this strategy is generally not an equilibrium for swing voters. In a setup where abstention is not allowed, types t_A develop an incentive to vote with strictly positive probability for the candidate they like least. The idea is that, since t_A's are more abundant, the outcome of the election would be biased in favor of candidate A if everyone voted "sincerely". Swing voters prefer to compensate this bias, in order to maximize the probability that the best candidate is elected.

In our setup, we can show that such a strategy is also dominated by a strategy of mixing between abstention and voting for one's own candidate. The following lemma establishes that:

Lemma 7.1. *In equilibrium, voters never mix between A and B.*

Proof. See Appendix.

This result is a manifestation of the swing voter's curse identified by Feddersen and Pesendorfer (1996, 1999). The difference between their result and ours is that we do not consider voters with different information qualities. Combining their result and ours shows that, if different voters had different information qualities, those with worse information would abstain more.

Proposition 7.1 shows that the equilibrium of the voting game with swing voters is unique:[3]

Proposition 7.1. *The equilibrium of the two-candidate election game is unique and such that types-t_A may only mix between $\psi = A$ and abstention, while types-t_B play $\psi = B$ in pure strategy. This equilibrium features full information equivalence*

Proof. See Appendix.

The literature focused on a population only composed of swing voters.[4] Yet, it would be straightforward to incorporate stalwarts voters who always vote *A* or *B:* swing voters would simply compensate the votes of stalwarts types by "leaning against the wind". That is, if there were more stalwarts t_A, then swing t_A's would have to abstain more often. The only constraint would be that the fraction of swing voters is sufficiently large for full information equivalence to be sustained.

Two corollaries result from this proposition. First, Austen-Smith and Banks (1996) have shown that the only case in which voters vote sincerely is when $r(t_A \mid a) = r(t_B \mid b)$. The same result holds in our setup: abstention is also the result of an imbalance between the proportion of the two types across states of nature:

Corollary 7.1. *In equilibrium, types-t_A abstain with positive probability if and only if $r(t_A \mid a) > r(t_B \mid b)$.*

Second, we have seen that full information equivalence requires that *A* be first in state *a* and *B* be first in state *b*. Hence:

Corollary 7.2. *A necessary condition for full information equivalence is that swing voters split their votes between A and B.*

---

[3] Feddersen and Pesendorfer (1999) have a more general model, in which they provide sufficient conditions for full information equivalence to hold in equilibrium (Proposition 4).

[4] An exception is Castanheira (2003), in which voters have fixed preferences. Yet, voters have an incentive to vote for losers, in order to inform parties that they are sufficiently numerous. This may attract parties closer to their preferred policy in subsequent elections.

7.4. Three-candidate elections

The results of the previous section show that if there are sufficiently many swing voters, elections will select the candidate who is socially preferred. That is, the winning candidate is the same as if there were no information imperfections. This full information equivalence was shown to extend to qualified majorities, as long as unanimity is not required (Feddersen and Pesendorfer 1997, 1998, 1999).

Yet, this literature largely overlooked multicandidate elections (two exceptions are Piketty 2000 and Castanheira 2003). In this section, we show how the properties of two-candidate elections are altered when a third candidate enters the electoral race. With a third candidate, our setup is releted to the one-period case analyzed by Piketty (2000). The difference, again, is that we introduce abstention. Also, we shed new light on the properties of the various equilibria that emerge and prove the existence of new equilibria.

We raise three issues. The first one is the known strategic effect of plurality elections: the existence of a third candidate is sufficient to generate "Duvergerian" equilibria, in which either *A* or *B* receive zero vote. In these equilibria, there is of course no full information equivalence (see also Piketty 2000, Proposition 7.5). Second, we investigate the properties of a "Condorcet-Jury" type of equilibrium, in which the three candidates receive a strictly positive vote share. We show that this equilibrium is stable and produces full information equivalence if the vote share of *C*, is sufficiently low. If the vote share of *C* is too large, information aggregation is impossible. Third, we prove the existence of additional equilibria with *partial information equivalence*, when the vote share of *C* is not too large.

7.4.1. Issue 1. Duvergerian equilibria always exist

The equilibrium properties of plurality elections have been widely analyzed in a setup without swing voters (see for instance Riker 1982; Myerson and Weber 1993). One of the main results of that analysis is the validation of *Duverger's Law* (Duverger 1954): strategic motivations induce some voters to abandon their preferred candidate and to focus their votes on the top-two candidates, with the largest vote shares. In our setup, this implies that there is one equilibrium in which all majority types vote for *A*, and another equilibrium in which they all vote for *B*. These two equilibria are stable. A third, equilibrium is that majority type voters divide their votes equally between *A* and *B*.

Proposition 7.2, which is reminiscent of Piketty (2000, Proposition 7.5), shows that such equilibria also exist in a setup with swing voters:

Proposition 7.2. *In multicandidate elections, Duvergerian equilibria always exist. That is,* $\tau(A|\cdot) = 0$ *and* $\tau(B|\cdot) = 0$ *are (self-fulfilling) equilibria. These equilibria are inefficient, because they prevent learning and the election of the candidate who would be chosen under full information.*

To prove this result, first note that majority-type voters play A (respectively: B) with probability 1 if the following pay-off difference is strictly positive (resp. negative):

$$\begin{aligned} G(A \mid t) - G(B \mid t) = q(a \mid t)\,\{&2 \Pr(piv_{AC} \mid a) - \Pr(piv_{BC} \mid a) \\ &+ \Pr(piv_{AB} \mid a) + \Pr(piv_{BA} \mid a)\} \\ + q(b \mid t)\,\{&\Pr(piv_{AC} \mid b) - 2 \Pr(piv_{BC} \mid b) \\ &- \Pr(piv_{AB} \mid b) - \Pr(piv_{BA} \mid b)\}. \end{aligned} \tag{7.4}$$

To show the existence of Duvergerian equilibria, we thus have to show that the difference is strictly positive when $\tau(B \mid \omega)$ is sufficiently small and conversely for $\tau(A \mid \omega)$ small. To this end, we show that the magnitude of the events $(piv_{BC} \mid \omega)$ and $(piv_{BA} \mid \omega)$ are smaller than the magnitude of $(piv_{AC} \mid \omega)$ when $\tau(B \mid \omega)$ is sufficiently small.

Using the properties of Poisson Games, we actually find that the probability of being pivotal in favor of the candidate with the smallest vote share actually becomes *infinitesimally* smaller than the one in favor of the leading candidate:

Lemma 7.2. *The magnitude of the pivot probability between two parties P and Q is:*

$$mag\,(piv_{PQ} \mid \omega) = -\left(\sqrt{\tau(P \mid \omega)} - \sqrt{\tau(Q \mid \omega)}\right)^2,$$

if these two parties are the top-two candidates, and it is smaller than that value for the bottom-two candidates.

Hence, if the three parties have different vote shares: $\tau(P \mid \omega) > \tau(Q \mid \omega) > \tau(R \mid \omega)$, the following pivot probability ratio converges to infinity as population size increases:

$$\lim_{n \to \infty} \frac{\Pr(piv_{PQ} \mid \omega)}{\max\{\Pr(piv_{PR} \mid \omega), \Pr(piv_{PQ} \mid \omega)\}} = \infty$$

Proof. See Appendix.

This lemma is then largely sufficient to establish that (7.4) is necessarily positive if the vote share of B is sufficiently small, and negative if the vote share of B is sufficiently large.

In other words, a given voter who best responds to the voting patterns of the rest of the electorate should follow the lead of the majority, and abandon trailing candidates. This is why the two Duvergerian equilibria are self-fulfilling: it is the *expectation* that a candidate will be trailing behind that triggers this response.[5]

7.4.2. Issue 2. Full information equivalence can be impossible to attain

When there are only two candidates, full information equivalence only requires that the equilibrium vote share of candidate A be larger than that of candidate B in state a and conversely in state b. Matters become more complex when a strictly positive fraction of types t_C vote for the third candidate, C. In that case, full information equivalence requires that:

$$\begin{Bmatrix} \tau\ (A \mid a) > \max\ [\tau\ (B \mid a), \tau\ (C)] \\ \tau\ (B \mid b) > \max\ [\tau\ (A \mid b), \tau\ (C)] \end{Bmatrix} \tag{7.5}$$

That is, the leader's vote share—A's or B's, depending on the state—must also be above C's. This constraint is far from trivial: as we show below, full information equivalence is actually impossible to reach when $\tau(C)$ is large. By contrast, when $\tau(C)$ is small, full information equivalence is not only feasible; it is actually a stable equilibrium.

Case 1. $\tau(C)$ **is large**

If the fraction of types t_C is large enough, C will necessarily be one of the top-two candidates. By virtue of Lemma 7.2, this implies that the pivotability between A and B, which was central to the results for two-candidate elections, is now a second-order concern for the voters. The largest magnitude is necessarily associated to one of the pivotabilities against C.

[5] Remark that abstention is also a dominated action in these equilibria.

In this case, the only non-Duvergerian equilibrium of the game is "unstable" and requires that A and B have the same vote share in "their" respective state:[6]

Proposition 7.3. *For* $\tau\,(C) > 1/[2 + r\,(t_A \mid b)\,/\,r\,(t_A \mid a)]$, *abstention is a dominated strategy and the only non-Duvergerian equilibrium is such that:*

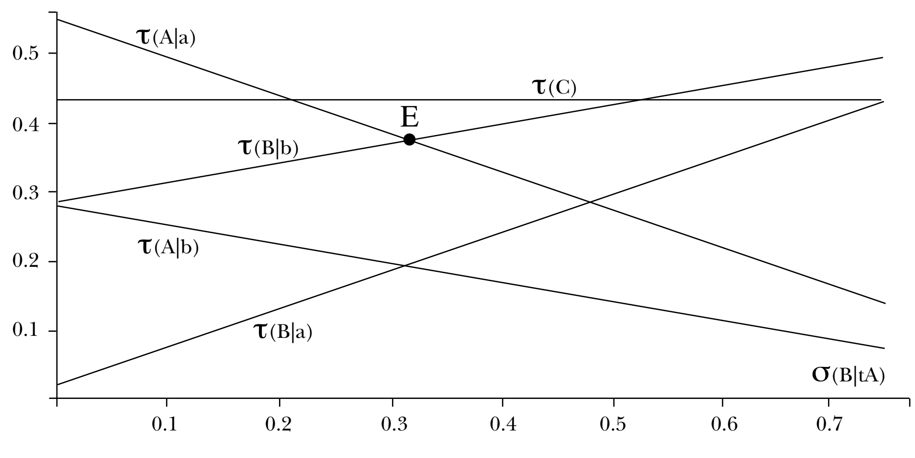
$$\tau\,(C) > \tau\,(A \mid a) \simeq \tau\,(B \mid b) > \tau\,(A \mid b) \simeq \tau\,(B \mid a) > 0.$$

This equilibrium is not "stable" and does not produce full information equivalence, since it induces the election of the dominated candidate C with a probability that converges to 1 as $n \to \infty$.

Proof. See Appendix.

Figure 7.1 illustrates this result: the horizontal axis displays the strategy of types t_A (as seen in the previous section, types t_B vote for B with probability 1). Moving from left to right, they vote for B with increasing probability. The upward sloping lines represent B's vote share in each state of nature. The downward sloping lines represent A's vote share.

FIGURE 7.1: Knife edge equilibrium for τ (C) large



[6] "Stability" here is used in the same way as in a Cournot equilibrium: assume that expected vote shares are $\tau^0\,(P \mid \omega)$. Given $\tau^0\,(P \mid \omega)$, allow a tiny fraction of the electorate to choose their strategy, and then compute the new expected vote shares $\tau^1\,(P \mid \omega)$, and iterate to identify a sequence $\tau^k\,(P \mid \omega)$, $k =$ 1, 2,... We call an equilibrium "stable" if there exists a neighborhood of the equilibrium $\tau^*\,(P \mid \omega)$ such that the sequence $\tau^k\,(P \mid \omega)$ converges to $\tau^*\,(P \mid \omega)$.

The equilibrium, represented by point E on the graph, is such that $\tau\ (A \mid a) \simeq \tau\ (B \mid b)$. To the left of that point, the top two contenders are A and C, and the pivot probability $\Pr\ (piv_{AC} \mid a)$ is—by orders of magnitude—larger than the other pivot probabilities. Hence, majority-group voters would strictly prefer to deviate by voting for A only. Conversely, the same holds for B to the right of E. By a fixed point argument, they must be indifferent between A and B at point E, which proves the existence of the equilibrium. Note that the threshold $1/\ [2 + r\ (t_A \mid b)\ /\ r\ (t_A \mid a)]$ simply identifies the vote share $\tau\ (C)$ such that $\tau\ (C) = \tau\ (A \mid a) = \tau\ (B \mid b)$ at point E.

Case 2. $\tau\ (C)$ is small

When $\tau\ (C)$ is below that threshold, there exists a range of strategies around point E such that both $\tau\ (A \mid a)$ and $\tau\ (B \mid b)$ are larger than $\tau\ (C)$. In that case, the equilibrium at point E is actually stable:

Proposition 7.4. *For* $\tau\ (C) < 1/[2 + r\ (t_A \mid b)\ /\ r\ (t_A \mid a)]$, *there exists a "stable" equilibrium with full information equivalence: vote shares are such that*

$$\tau\ (A \mid a) \simeq \tau\ (B \mid b) > \max\ [\tau\ (C), \tau\ (A \mid b) \simeq \tau\ (B \mid a)] > 0$$

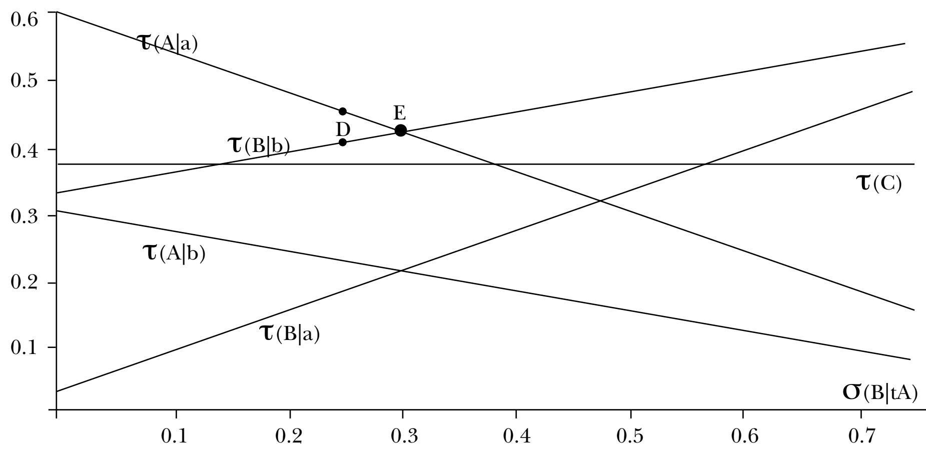
Sketch of the proof:

The proof itself is quite straightforward, although tedious. So, we only develop a graphic argument that illustrates how the proof proceeds.

In Figure 7.2, $\tau\ (C)$ has a value below $1/[2 + r\ (t_A \mid b)\ /\ r\ (t_A \mid a)]$. The important implication of this smaller value is that the ranking of pivot probabilities is now opposite to the ranking we identified in figure 7.1. Indeed, at point D in Figure 7.2, we have $\tau\ (A \mid a) > \tau\ (B \mid b) > \tau\ (C)]$. This means that A beats C by a large margin in state a but B only beats C by a small margin in state b. Hence, the pivot probability in favor of B in state b dominates, by orders of magnitude. This implies that $G\ (A \mid t) - G\ (B \mid t)$ in (7.4) is necessarily negative: if the vote share of B falls slightly below its equilibrium level, all voters wish to vote for B. In other words, a decrease in $\sigma\ (B \mid t_A)$ induces a Nash response that increases this value back to point E: the equilibrium is "stable" in that sense. A similar mechanism holds to the right of E.

Interestingly, if $\tau\ (C)$ is also smaller than $1/[2 + r\ (t_A \mid a)\ /\ r\ (t_A \mid b)]$, then C would have the smallest vote share of the three alternatives. This implies that, at point E on the figures, the voters will behave "as if" C was not present in the electoral race. Concretely, this implies that t_A voters must be abstaining with some probability at the equilibrium.

FIGURE 7.2: Stable equilibrium for τ (C) small



Remark 7.1. *If abstention is not in the action set and if the fraction of types t_C is sufficiently small, t_A -voters use C as a surrogate for abstention. t_A -voters would then vote C with positive probability.*

7.4.3. Issue 3. Existence of additional equilibria

Proposition 7.3 and Proposition 7.4 emphasize little-known or unknown properties of the "Condorcet equilibrium". Here, we prove the existence of additional equilibria when $\tau\ (C)$ is small. In these equilibria, majority-type voters divide their votes between A and B in such a way that C wins in one of the two states of nature:

Proposition 7.5. *For* $1/3 < \tau\ (C) < 1/\ [2 + r\ (t_A \mid b)\ /\ r\ (t_A \mid a)]$, *there exist two "unstable" equilibria with partial information equivalence: vote shares are such that*

$\tau(B \mid b) > \tau(C) > \tau(A \mid a) = \tau(B \mid a) > 0$ in one equilibrium, and
$\tau(A \mid a) > \tau(C) > \tau(A \mid b) = \tau(B \mid b) > 0$ in one equilibrium

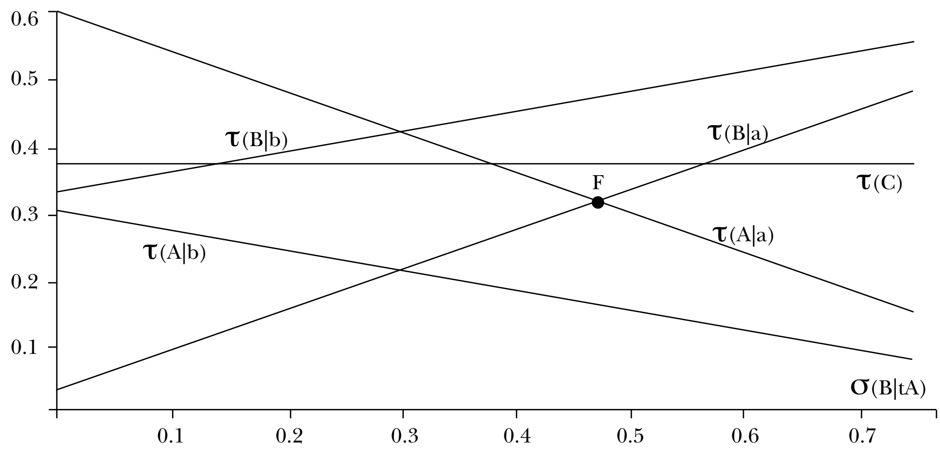
In the former equilibrium, *B* wins in state *b* and *C* wins in state *a*. In the latter, *A* wins in state *a* and *C* wins in state *b*.

The proof is again both straightforward and tedious, which is why we only develop a sketch of the proof.

Sketch of the proof:

Figure 7.3 relies on the same parameter values as figure 7.2. The only difference is thus that it points towards another equilibrium, represented by point *F* on the figure.

FIGURE 7.3: New type of equilibrium for τ (C) small



For a strategy $\sigma(B \mid t_A)$ slightly to the left of *F*, we have: $\tau(C) > \tau(A \mid a) > \tau(B \mid a)$ in state *a*, and $\tau(B \mid b) > \tau(C) > \tau(A \mid b)$ in state *b*. The two pivot probabilities that dominate are therefore $\Pr(piv_{AC} \mid a)$ and $\Pr(piv_{BC} \mid b)$. Yet, the gap between *A* and *C* in state *a* is smaller than the gap between *B* and *C* in state *b*. Therefore: $\Pr(piv_{AC} \mid a) >>> \Pr(piv_{BC} \mid b)$, which implies that a vote for *A* is more valuable than a vote for *B*.

Conversely, for a strategy σ $(B \mid t_A)$ slightly to the right of F, we have: τ (C) > τ $(B \mid a)$ > τ $(A \mid a)$ in state a (nothing changes in state b), which implies that a vote for B is now more valuable than a vote for A. By a fixed point argument, it is immediate that t_A voters must be indifferent between voting A and B at point F. By symmetry, there also exists a similar point F' for σ $(\mathrm{B} \mid t_A) = 0$ and σ $(B \mid t_B) \in (0, 1)$, in which A wins in state a and C wins in state b.

Note also that, if τ (C) is smaller than 1/3, these equilibria still exist but generate a different type of outcome. In the equilibrium similar to point F in figure 7.3, B would win in both states of nature. Thus, like in a Duvergerian equilibrium, the winning alternative is independent of the state of nature but, unlike a Duvergerian equilibrium, the three candidates receive a strictly positive vote share.

7.5. Conclusions

We have studied voting equilibria in plurality elections when swing voters compose a majority of the electorate. With the help of a simple model with two states of nature and three candidates, we have showed that new equilibria arise and that the properties of some of the existing equilibria are modified. We have also showed that swing voters tend to adopt different abstention rates depending on the equilibrium that is played.

The comparison between these equilibria also underlines that some of them are efficient, whereas other equilibria are not. Some candidates may win only due to information imperfections and coordination failures. That is, if voters were fully informed about the state of nature and could decide on which equilibrium to coordinate, they would elect one candidate in some cases, and another candidate in other cases. This is not possible under plurality elections. As underlined by Duverger (1954), voters develop an incentive to coordinate their votes on a "strong" candidate in plurality elections.

This coordination process is not only at the roots of the equilibrium multiplicity, it also prevents collective learning by the electorate. This is what we call the *Duverger-Condorcet trade-off*: ensuring the election of a candidate requires voters to coordinate their votes on that candidate. By contrast, information aggregation requires them to divide their votes across different candidates, so as to reveal the different elements of information scattered in the electorate.

To conclude, the "known" properties of electoral systems appear to actually overlook the presence of swing voters. In future work, it will thus prove important to study the properties of other electoral systems in such a setup. As explained in the introduction, we make a first step in that direction in Bouton and Castanheira (2008). We show that under Approval Voting, there is no Duverger-Condorcet trade-off: there is full information and coordination equivalence. We contend that this is a strong argument in favor of electoral reform.

Appendix

Proof of Lemma 7.1

Proof. We show that when type-t voters prefer to abstain rather than to mix between A and B:

$$G(A|t) = G(B|t) \Rightarrow G(B|t) < 0 \tag{7.6}$$

By Myerson's offset theorem:

$$\Pr(piv_{BA}|\omega) = \Pr(piv_{AB}|\omega)\sqrt{\frac{\tau(A|\omega)}{\tau(B|\omega)}}$$

Then, from (7.2) and (7.3), we have that $G(A|t) = G(B|t)$ boils down to

$$\frac{q(a|t)}{q(b|t)} = \frac{\Pr(piv_{AB}|b)}{\Pr(piv_{AB}|a)}\frac{\sqrt{\tau(B|a)}}{\sqrt{\tau(B|b)}}$$

Substituting for $q(a|t)$ in (7.2) yields:

$$G(A|t) = q(b|t)\Pr(piv_{AB}|b)\left(\frac{\sqrt{\tau(B|a)}}{\sqrt{\tau(B|b)}} - 1\right)$$

Since $\tau(B|b) > \tau(B|a)$, we have that $G(A|t) = G(B|t)$ implies $G(A|t) = G(B|t) < 0$, and hence that abstention is preferred.

Proof of Proposition 7.1

Proof. First, observe that $\sigma(B|t_A) > 0$ implies $\sigma(B|t_B) = 1$. From (7.2) and (7.3) we have that $G(B|t_A) > G(A|t_A)$ if

$$\frac{q\,(a \mid t_A)}{q\,(b \mid t_A)} < \frac{\Pr\,(piv_{AB} \mid b) + \Pr\,(piv_{BA} \mid b)}{\Pr\,(piv_{AB} \mid a) + \Pr\,(piv_{BA} \mid a)}$$

Since $\frac{q\,(a \mid t_A)}{q\,(b \mid t_A)} > \frac{q\,(a \mid t_B)}{q\,(b \mid t_B)}$, we have that $G\,(B \mid t_A) > G\,(A \mid t_A)$ implies $G\,(B \mid t_B) > G\,(A \mid t_B)$. Similarly, we can proof that $G\,(B \mid t_A) > 0$ implies $G\,(B \mid t_B) > 0$. By analogy, we have that $\sigma\,(A \mid t_B) > 0$ implies $\sigma\,(A \mid t_B) = 1$.

Second, we show that $\sigma\,(B \mid t_A) > 0$ cannot be true in equilibrium. Since $\sigma\,(B \mid t_A) > 0$ implies $\sigma\,(B \mid t_B) = 1$, which implies that $G\,(A \mid t) > G\,(B \mid t)$ and 0, $\forall t \in \{t_A, t_B\}$. The proof is similar for types t_B. This show that neither A nor B are deserted by the voters. Hence, in equilibrium we must have:

$$G\,(A \mid t_A) \geq 0, \text{ and } G\,(B \mid t_B) \geq 0. \tag{7.7}$$

Since $G\,(A \mid t) \gtreqless 0$, $\forall\, t \in \{t_A, t_B\}$ when $mag\,(piv_{AB} \mid A) \gtreqless mag\,(piv_{AB} \mid b)$, a necessary condition for $G\,(A \mid t) = 0$ is therefore:

$$mag\,(piv_{AB} \mid a) = mag\,(piv_{AB} \mid b) \tag{7.8}$$

This is satisfied when

$$-\left(\sqrt{\tau\,(A \mid a)} - \sqrt{\tau\,(B \mid a)}\right)^2 = -\left(\sqrt{\tau\,(B \mid b)} - \sqrt{\tau\,(A \mid b)}\right)^2 \tag{7.9}$$

Since $\sigma\,(A \mid t_A) + \sigma\,(\emptyset \mid t_A) = 1$ and $\sigma\,(B \mid t_B) + \sigma\,(\emptyset \mid t_B) = 1$, we have that $\tau\,(A \mid a) > \tau\,(\mathrm{A} \mid b)$ and $\tau\,(B \mid a) < \tau\,(\mathrm{B} \mid b)$. Then (7.9) is satisfied iff

$$\frac{\sigma\,(A \mid t_A)}{\sigma\,(B \mid t_B)} = \left(\frac{\sqrt{r\,(t_B \mid b)} + \sqrt{r\,(t_B \mid a)}}{\sqrt{r\,(t_A \mid a)} + \sqrt{r\,(t_A \mid b)}}\right)^2 \tag{7.10}$$

We still need to show that the equilibrium is unique. To do that, we have to prove that when (7.10) is satisfied, $G\,(B \mid t_B) > 0$. By assumption, $\left(\frac{\sqrt{r\,(t_B \mid b)} + \sqrt{r\,(t_B \mid a)}}{\sqrt{r\,(t_A \mid a)} + \sqrt{r\,(t_A \mid b)}}\right)^2 \leq 1$. Therefore, $\sigma\,(A \mid t_A) \leq 1$ is necessary in equilibrium otherwise (7.10) would imply $\sigma\,(B \mid t_B) > 1$. In equilibrium, we must then have $G\,(A \mid t_A) = 0$. From (7.6) and (7.7), $G\,(A \mid t_A) = 0$ directly implies that $G\,(B \mid t_B) > 0$.

Proof of Lemma 7.2

Proof. First, we use Property 7.1 to compute the magnitude of the probability that P and Q have the same vote share in the state ω:

$$mag\,(piv_{PQ} \mid \omega) = \max_{\vec{x}} \sum_{\psi} \frac{x\,(\psi \mid \omega)}{n} \left(1 - \log \frac{x\,(\psi \mid \omega)}{n\,\tau\,(\psi \mid \omega)}\right) - 1 \qquad (7.11)$$

$$s.t.\ x\,(P \mid \omega) = x\,(Q \mid \omega)$$

If we denote $x\,(P \mid \omega) = x\,(Q \mid \omega) = x\,(\omega)$ we find that this is maximized in:

$$x^{**}_{PQ}\,(\omega) = n\sqrt{\tau\,(P \mid \omega)\,\tau\,(Q \mid \omega)},\ \mathrm{x}^{**}\,(R \mid \omega)_{PQ} = n\tau\,(R \mid \omega),\ x^{**}\,(\varnothing \mid \omega)_{PQ} = n\tau\,(\varnothing \mid \omega)$$

Second, a vote can only be pivotal between P and Q if the third candidate, R, has fewer votes than P and Q. This imposes an additional condition: $x \geq x\,(R \mid \omega)$. Introducing that condition in the maximization problem, we find

$$\text{If } \sqrt{\tau\,(P \mid \omega)\,\tau\,(Q \mid \omega)} \geq \tau\,(R \mid \omega), \text{ then } \begin{cases} x^{*}_{PQ}\,(\omega) = x^{**}_{PQ}\,(\omega) \\ x^{*}\,(R \mid \omega)_{PQ} = x^{**}\,(R \mid \omega)_{PQ} \end{cases}$$

$$\text{If } \sqrt{\tau\,(P \mid \omega)\,\tau\,(Q \mid \omega)} < \tau\,(R \mid \omega), \text{ then } \begin{cases} x^{*}_{PQ}\,(\omega) = 1/3 \\ x^{*}\,(R \mid \omega)_{PQ} = 1/3 \end{cases}$$

where the * refers to the solution of the maximization problem that takes the new condition into account. (Note that $mag\,(piv_{PR} \mid \omega)$ and $mag\,(piv_{QR} \mid \omega)$ can be computed in the same way).

Whenever $\sqrt{\tau\,(P \mid \omega)\,\tau\,(Q \mid \omega)} \geq \tau\,(R \mid \omega)$, $mag\,(piv_{PQ} \mid \omega)$ is:

$$mag\,(piv_{PQ} \mid \omega) = -\left(\sqrt{\tau\,(P \mid \omega)} - \sqrt{\tau\,(Q \mid \omega)}\right)^2$$

while if $\sqrt{\tau\,(P \mid \omega)\,\tau\,(Q \mid \omega)} < \tau\,(R \mid \omega)$, the magnitude is:

$$mag\,(piv_{PQ} \mid \omega) = 3\,[\tau(P \mid \omega)\,\tau(Q \mid \omega)\,\tau(R \mid \omega)]^{1/3} + \tau(\varnothing \mid \omega) - 1$$

Since $\tau(P \mid \omega) > \tau(Q \mid \omega) > \tau(R \mid \omega)$, we have that $x^{**}_{PQ}\,(\omega) > 1/3$, and that $x^{**}_{QR}\,(\omega) < 1/3$. Therefore, $x^{*}_{PQ}\,(\omega) = x^{**}_{PQ}\,(\omega)$, and $x^{*}_{QR}\,(\omega) = 1/3$, which implies that:

$$mag\,(piv_{PQ} \mid \omega) = -\left(\sqrt{\tau\,(P \mid \omega)} - \sqrt{\tau\,(Q \mid \omega)}\right)^2$$

$$mag\,(piv_{QR} \mid \omega) = 3\,[\tau(P \mid \omega)\,\tau(Q \mid \omega)\,\tau(R \mid \omega)]^{1/3} + \tau(\varnothing \mid \omega) - 1$$

Concerning *mag* $(piv_{PR} \mid \omega)$, the situation is a priori unclear since $x^{**}_{PR}(\omega) \in (0,1/2)$. Nonetheless, $\left(\sqrt{\tau(P \mid \omega)} - \sqrt{\tau(Q \mid \omega)}\right)^2 < \left(\sqrt{\tau(P \mid \omega)} - \sqrt{\tau(R \mid \omega)}\right)^2$ implies that *mag* $(piv_{PQ} \mid \omega) >$ *mag* $(piv_{PR} \mid \omega)$ when $x^{*}_{PR}(\omega) = x^{**}_{PR}(\omega)$. Since $x^{*}_{PR}(\omega) = 1/3$ results from an additional constraint, it is obvious that *mag* $(piv_{PR} \mid \omega)$ computed for this value of $x^{*}_{PR}(\omega)$ is smaller or equal to the unrestricted value, that is when computed in $x^{*}_{PR}(\omega) = x^{**}_{PR}(\omega)$. This reinforces the inequality.

Finally, since *mag* $(piv_{QR} \mid \omega)$ is identical to the restricted magnitude of *mag* $(piv_{PR} \mid \omega)$, it follows directly that *mag* $(piv_{PQ} \mid \omega) >$ *mag* $(piv_{PR} \mid \omega) \geq$ *mag* $(piv_{QR} \mid \omega)$.

Proof of Proposition 7.3

Proof. We first prove that such an equilibrium exists and necessarily entails $\tau(A \mid a) \simeq \tau(B \mid b) > \tau(A \mid b) \simeq \tau(B \mid a) > 0$. That is, there is a *unique* non-Duvergerian equilibrium. Second, we verify whether it satisfies the "stability" properties.

Step 1. $\tau(A \mid a) \simeq \tau(B \mid b) > \tau(A \mid b) \simeq \tau(B \mid a) > 0$ is an equilibrium

If $\tau(C) > 1/[2 + r(t_A \mid b) / r(t_A \mid a)]$, all the strategy profiles that leads to $\tau(A \mid a) \simeq \tau(B \mid b) > \tau(A \mid b) \simeq \tau(B \mid a) > 0$, imply that $\tau(C) > \tau(A \mid a) \simeq \tau(B \mid b)$. Indeed, for the strategy profile implying the largest value of $\tau(A \mid a)$ and $\tau(B \mid b)$ conditional on $\tau(A \mid a) \simeq \tau(B \mid b) > \tau(A \mid b) \simeq \tau(B \mid a) > 0$,

i.e. $\sigma(\emptyset \mid t_A) = 0 = \sigma(\emptyset \mid t_B)$, $\sigma(B \mid t_B) = 1$, $\sigma(A \mid t_A) = \dfrac{r(t_B \mid b) + r(t_A \mid b)}{r(t_A \mid a) + r(t_A \mid b)}$, and $\sigma(B \mid t_A) = 1 - \dfrac{r(t_B \mid b) + r(t_A \mid b)}{r(t_A \mid a) + r(t_A \mid b)}$, we have $\tau(C) > \tau(A \mid a); \tau(B \mid b)$. Then, the pivot probabilities between A and C in state a and between B and C in state b become infinitely larger than any other pivot probability, by Property 2. The payoffs $G(A \mid t)$ and $G(B \mid t)$ are both larger than zero. Abstention is therefore a dominated strategy.

Following Theorem 2 of Myerson 1998, note that if a type $t \in \{t_A, t_B\}$ adopts a strictly mixed strategy, then the other type $t' \neq t$, $t' \in \{t_A, t_B\}$ votes for "his" candidate with probability 1. The reason is that the priors $q(a \mid t)$ and $q(b \mid t)$ are different across types, which implies $G(A \mid t_A) - G(B \mid t_A) > G(A \mid t_B) - G(B \mid t_B)$ for any expected voting profile.

Having noted this, we know that the strategy profile leading to $\tau(A \mid a) \simeq \tau(B \mid b)$ is an equilibrium if, for that ranking of the vote shares, we have

$$lG(A \mid t_A) - G(B \mid t_A) \geq 0, \text{ and } G(A \mid t_B) - G(B \mid t_B) \leq 0 \tag{7.12}$$

with at least one strict inequality. That is, types t_A must be willing to support A, and conversely for types t_B. Using (), it is immediate to check that these inequalities hold iff $\tau(A \mid a) \simeq \tau(B \mid b) > \tau(A \mid b) \simeq \tau(B \mid a) > 0$.

Next, remark that: *a)* pivot probabilities are continuous in $\sigma(A \mid t)$ and $\sigma(B \mid t)$, and *b)* payoffs are bounded, which allows us to apply Kakutani's fixed point theorem on $G(A \mid T) - G(B \mid t)$.

If voters marginally increase their propensity to vote A above the point in which $\tau(A \mid a) \simeq \tau(B \mid b)$, we have: $\tau(A \mid a) > \tau(B \mid b) > \tau(A \mid b) > \tau(B \mid a)$ and

$$G(A \mid t) - G(B \mid t) > 0 \text{ for both } t \in \{t_A, t_B\}, \text{ if } \tau(A \mid a) \simeq \tau(B \mid b) > \tau(C)$$
$$G(A \mid t) - G(B \mid t) < 0 \text{ for both } t \in \{t_A, t_B\}, \text{ if } \tau(A \mid a) \simeq \tau(B \mid b) < \tau(C),$$

and the inequalities are reversed if the voters' propensity to vote for B increases. Two conclusions follow: *i) existence:* there must exist a strategy profile in the neighborhood of $\tau(A \mid a) = \tau(B \mid b)$ such that () holds. *ii) uniqueness:* $\tau(A \mid a) \simeq \tau(B \mid b) > \tau(A \mid b) \simeq \tau(B \mid a) > 0$ is a necessary condition for the non-Duvergerian equilibrium.

Step 2. If $\tau(C) > 1/[2 + r(t_A \mid b) / r(t_A \mid a)]$, the equilibrium is neither Condorcet-like nor stable

As stated in step 1, if $\tau(C) > 1/[2 + r(t_A \mid b) / r(t_A \mid a)]$, all the strategy profiles that leads to $\tau(A \mid a) \simeq \tau(B \mid b) > \tau(A \mid b) \simeq \tau(B \mid a) > 0$, imply that $\tau(C) > \tau(A \mid a) \simeq \tau(B \mid b)$. Hence, () is not satisfied and thus the equilibrium is not Condorcet-like.

$\tau(C) > \tau(A \mid a) \simeq \tau(B \mid b)$ also implies that the difference $G(A \mid t) - G(B \mid t)$ converges to:

$$\lim_{n \to \infty} G(A \mid t) - G(B \mid t) = q(a \mid t)\, 2 \Pr(piv_{AC} \mid a) - q(b \mid t)\, 2 \Pr(piv_{BC} \mid b).$$

Then, if the vote share of A increases (the argument is symmetric if it decreases), $\tau(C) - \tau(A \mid a)$ decreases, and $\tau(C) - \tau(B \mid b)$ increases. This increases the magnitude of $\Pr(piv_{AC} \mid a)$ and decreases that of $\Pr(piv_{BC} \mid b)$. Hence, the initial rise in $\tau(A \mid a)$ induces the value of a vote for A to increase, for both types t_A and t_B. The equilibrium is thus not "stable" in the sense that an exogenous increase in the vote share of A induces voters to increase their propensity to vote for A.

References

ALESINA, A. and G-M. ANGELETOS. "Fairness and Redistribution." *American Economic Review* 95, no. 4 (2005).

BENABOU, R. and J. TIROLE. "Belief in a Just World and Redistributive Politics." *The Quarterly Journal of Economics* 121, no. 2 (2006): 699-746.

BOUTON, L. and M. CASTANHEIRA. *One Person, Many Votes: Divided Majorities and Information Aggregation.* CEPR Discussion Paper 6695, 2008.

AUSTEN-SMITH, D. and S. BANKS. "Information Aggregation, Rationality, and the Condorcet's Jury Theorem." *American Political Science Review* 90 (1996): 34-45.

CASTANHEIRA, M. "Why Vote for Losers?." *Journal of the European Economic Association* 1 (2003): 1207-1238.

DIXIT, A. and J. LONDREGAN. "Ideology, Tactics, And Efficiency In Redistributive Politics." *The Quarterly Journal of Economics* 113, no. 2 (1998): 497-529

DUVERGER, M. *Political Parties.* New York: Wiley, 1954.

FEDDERSEN, T. and W. PESENDORFER. "The Swing Voter's Curse." *American Economic Review* 86 (1996): 408-424.

—. "Voting Behavior and Information Aggregation in Elections with Private Information." *Econometrica* 65 (1997): 1029-1058.

—. "Convicting the Innocent: The Inferiority of Unanimous Jury Verdicts under Strategic Voting." *American Political Science Review* 92 (1998): 22-35.

—. "Abstention in Elections with Asymmetric Information and Diverse Preferences." *American Political Science Review* 93 (1999): 381-398

MYERSON, R. "Extended Poisson Games and the Condorcet Jury Theorem." *Games and Economic Behavior* 25 (1998a): 111-131.

—. "Population Uncertainty and Poisson Games." *International Journal of Game Theory* 27 (1998b): 375-392.

—. "Large Poisson Games." *Journal of Economic Theory* 94 (2000): 7-45.

—. "Comparison of Scoring Rules in Poisson Voting Games." *Journal of Economic Theory* 103 (2002): 219-251.

MYERSON, R. and R. WEBER. "A Theory of Voting Equilibria." *American Political Science Review* 87 (1993): 102-114.

PERSSON, T. and G. TABELLINI. *Political Economics: Explaining Economic Policy.* MIT Press, 2000.

PIKETTY, T. "Social Mobility and Redistributive Politics." *Quarterly Journal of Economics* 110. no. 3 (1995): 551-584.

—. " Voting as Communicating." *Review of Economic Studies* 67 (2000): 169-191.

RIKER, W. "The Two-Party System and Duverger's Law: An Essay on the History of Political Science." *American Political Science Review* 76 (1982): 753-766.

WEBER, R. "Comparison of Voting Systems." Cowles Foundation Discussion Paper No 498 (1977). New Haven: Yale University.

8

Why do Parties Exclude Important Issues from their Electoral Campaigns?

Josep M. Colomer
Spanish National Research Council (CSIC)
and Pompeu Fabra University

Humberto Llavador
Department of Economics and Business
Pompeu Fabra University

8.1. Introduction

Several survey polls asked voters to identify the most important issues in the 2004 U.S. presidential election. Observers broadly coincide in identifying 'moral values', 'the economy', 'terrorism', and 'the Iraq war', in this order, followed by others with minor impact such as 'health care', 'taxes', and 'education'. (See National Election Pool 2004; Stroud and Kenski 2007, as well as similar operationalizations for previous elections by RePass 1971; McCombs and Zhu 1995; Burden and Sanberg 2003; and discussion by Wlezien 2005).

Regarding the actual choice of issues during the electoral campaign by the two major candidates, Republican George W. Bush and Democrat John Kerry, there is also high coincidence in the academic literature. Bush chose 'terrorism' as the first issue; Kerry did not focus on 'Iraq', but on 'the economy'. "George W. Bush ran for re-election as a war-time president, emphasizing the importance of staying the course on the 'War on Terrorism'. At the same time, the economy had not fully recovered from the recession, and the job picture remained lackluster; his opponent, John Kerry, stressed the economy and other domestic issues such as health care" (Weisberg 2005, p. 777; see also Weisberg and Christenson 2007).

We can observe that none of the candidates chose to campaign in priority for the issue which could have higher pre-campaign salience among voters, 'moral values'. Numerous observers noted that, in spite of pundits' comments, 'moral values' ranked low in the issues list predicting actual voters' choices at the end of the electoral campaign (for instance, Langer and Cohen 2005). An explanation for this is that Republican candidate George W. Bush chose the issue in which his policy had the highest consensus, even if it was considered less important by the voters, 'terrorism'. In turn, Democrat candidate John Kerry did not choose in priority the issue 'Iraq', which would have been a direct response to the initiative in favor of 'terrorism' taken by Bush. He instead chose the issue in which he could obtain the highest consensus, 'economy/jobs'. Both candidates gave, thus, priority to those issues in which they could expect more favorable consensus among the voters and higher advantage regarding the other candidate, rather than those which were more salient in voters' concerns.

This paper discusses the criteria for party choices of issues and the subsequent campaign outcomes, explaining why highly ranked issues in voters' concerns may be left out of the electoral campaign. We present a formal model of electoral competition focusing on the formation of the public agenda, in which political parties or candidates compete to win an election by choosing an issue and a policy position on that issue to which they try to give salience. Giving salience to an issue implies proposing an innovative policy proposal on the issue as an alternative to the status-quo policy, as well as talking about it, usually with a value or argument, and making it news with some effort investment in order to make it relevant for voters' electoral decisions.

A party will choose a priority issue to campaign for if it is a likely winning issue, that is, it has a likely winning position and it is likely to become decisive in the election. Whether an issue will become a winning issue depends on two variables: (*i*) the ex-ante '*pre-campaign salience*' of the issue in voters' concerns and (*ii*) the voters' support or '*consensus*' in favor of a policy proposal on the issue.

Thus, parties have to trade off the two variables. If one issue is highly salient in the voters' concerns, but voters are highly divided about which one of the possible policy alternatives to the status-quo is better, choosing to campaign on the issue by holding one of the policy alternatives may be risky. If, on the contrary, there is broad social consensus about the best policy alternative to the status-quo on one issue, but the issue is not a priority for voters' electoral decision, running on that issue can attract little attention.

Hence, whether parties compete by raising the same issue and proposing two different policy alternatives on it or by choosing different issues does not depend only on voters' priority concerns, but mainly on each party's likelihood to hold potentially winning policy positions. It is always possible that the issues which are considered the most important ones by a majority of voters are not given political salience by parties during the electoral campaign. As a consequence, mediocre policies broadly rejected by the electorate, as well as incumbent parties with no good performance in government, may survive.

The plan for the rest of the paper is the following. In section 8.2 we present a spatial model of agenda formation in which parties compete on one issue at a time. For each issue there is some probability of victory for the party holding the most popular policy alternative. We introduce the concepts of issue salience and the degree of consensus. Section 8.3 characterizes the space of issues in terms of salience and consensus. Section 8.4 provides two examples showing that parties can compete on issues with either high salience or broad consensus or both. They do not compete, in equilibrium, on issues with both low salience among voters and low consensus regarding the best policy alternative. However, parties may choose not to campaign on those issues with highest salience in voters' concerns, thus postponing solutions to unpopular status-quo policies with considerable social discontent. Section 8.5 concludes.

8.2. The model

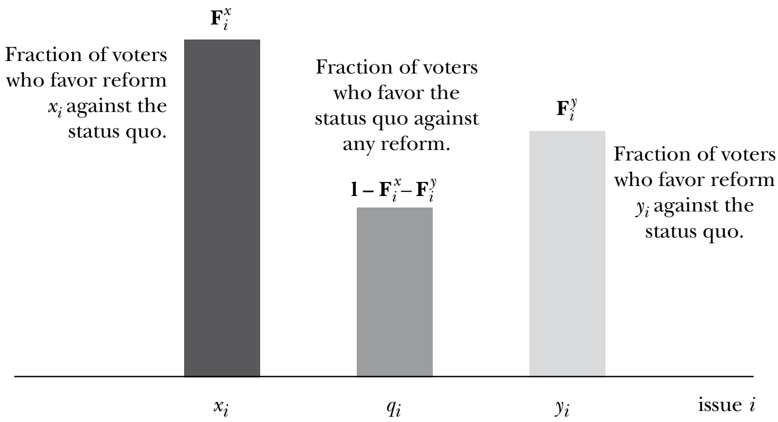
Consider an incumbent party in government (G) and a challenger party in opposition (O) that compete to win an election by choosing an issue and a policy position on that issue. There are N potential issues, and for each issue $i = 1, \ldots, N$, there exists a status-quo policy (q_i) and two innovative policy proposals located *on different sides* of q_i, which can be called x_i and y_i respectively. If the issue is, for example, taxes, one of the alternatives implies higher taxes, and the other lower taxes, than the status-quo, and similarly for any other issue, so that the two alternatives are *on different sides* of the status-quo. Denote the set alternatives by $A_i = \{x_i, q_i, y_i\}$.

8.2.1. Electorate

Voters have preferences over the different alternatives on each issue and vote accordingly. Let $\mathbf{F}_i\ (a_i, a_i')$ be the fraction of citizens who prefer a_i to a_i', for $a_i, a_i' \in A_i$. Denote by $\mathbf{F}_i^x = \mathbf{F}_i\ (x_i, q_i)$ and by $\mathbf{F}_i^y = \mathbf{F}_i\ (y_i, q_i)$ the support for each one of the alternatives against the status-quo. For simplicity, assume that $\mathbf{F}_i^x \neq \mathbf{F}_i^y$ and relabel the alternatives to the status-quo so that $\mathbf{F}_i^x > \mathbf{F}_i^y$. See figure 8.1 for clarification.

FIGURE 8.1: Voters' support for each alternative in issue *i*

Because the aternatives are on *different sides* of the status-quo, voters who favor reform y_i would align with the status-quo if they had to choose between x_i and q_i



The assumption that innovative policy proposals are on different sides of the status-quo implies that voters who favor alternative a_i prefer the status-quo to the other alternative a_i'.[1] That is, intuitively, what this means is that voters who would like lower taxes would vote against a tax increase, by supporting the status-quo.

8.2.2. Issues: salience and consensus

Each issue is characterized by some ex-ante or *pre-campaign salience*, which reflects voters' concerns and some *consensus* on a best policy.

The interest of voters regarding which issues should be more important in the election can be formed as a consequence of personal experiences, as well as media emphases, interest groups' promotions or uncontrolled events. For the rest of the chapter, we will use salience (without qualification) to refer to this ex-ante or pre-campaign salience.

It seems logical that the degree of salience of an issue should be related to the degree of social discontent with the status-quo policy on the issue. In this sense, we measure the salience of issue i as inversely related to the agreement or con-

[1] Formally, this is equivalent to assume that voter's preferences satisfy that if $x_i \succ q_i$, then $q_i \succ y_i$. And similarly, if $y_i \succ q_i$, then $q_i \succ x_i$.

sensus with the status-quo policy. Therefore, high-salience indicates that a large group of voters ($\mathbf{F}_i^x + \mathbf{F}_i^y$) disagree on the prevailing status-quo on that issue.

Definition 8.1. *Define the (pre-campaign) salience of issue i as* $\sigma_i = \mathbf{F}_i^x + \mathbf{F}_i^y$.

However, social discontent with the status-quo, and hence high salience, does not necessarily imply a broad consensus on the best policy alternative. It may be, on the contrary, that voters are highly divided on which alternative would be better than the status-quo.

Definition 8.2. *Define the degree of policy consensus on issue i as* $\zeta_i = \max\{\mathbf{F}_i^x, \mathbf{F}_i^y, 1 - \mathbf{F}_i^x - \mathbf{F}_i^y\}$.

The maximum value of ζ is 1, when there is total consensus with either the status-quo or one of the policy alternatives, and the minimum value of ζ_i is 1/3, when the three alternatives have equal support, $\mathbf{F}_i^x = \mathbf{F}_i^y = 1 - \mathbf{F}_i^x - \mathbf{F}_i^y = 1/3$.

8.2.3. The probability of winning on an issue

The winning alternative on a particular issue i is the one receiving a majority of the vote. However, there is uncertainty on the outcome of the election, and no alternative wins with probability one.

Definition 8.3. *Let* $\pi_i\colon A_i \times A_i \to (0,1)$ *be the probability of victory on issue i function, with* $\pi_i\,(a_i, a_i')$ *representing the probability of victory of* a_i *against* a_i'.

We will assume that the probability of winning is increasing in the fraction of the votes received and that a party cannot win by proposing an alternative already claimed by the other party. The latter point implies that a party proposing a policy alternative on one issue takes the alternative and hence forces the other party to defend something different if it wants to compete on that same issue.

Assumption 8.1. *Suppose that one party has proposed alternative* $a_i \in A_i$ *on issue i, then*

1. *the other party cannot win by proposing the same alternative later on the campaign;*

2. *the probability of winning is an increasing function of the votes received.*

Let $\pi_i^x = \pi_i\ (x_i, q_i)$ and $\pi_i^y = (y_i, q_i)$ represent the probabilities that alternatives x_i and y_i defeat the status-quo on issue *i*. Because $\mathbf{F}_i^x > \mathbf{F}_i^y$, it follows from Assumption 1 that $\pi_i^x > \pi_i^y$, that is x_i is the advantaged alternative in issue *i*. Without loss of generality, we can sort the issues $i = 1, \ldots, N$ according to the probability of victory of the advantaged alternative. We assume for simplicity a strict ordering:

$$\pi_1^x > \pi_2^x > \ldots > \pi_{N-1}^x > \pi_N^x \tag{8.1}$$

8.2.4. Post-electoral or political salience

In order to make an issue decisive in the election, parties try to make it 'salient' in voters' decision by giving it political salience. Parties confer political salience by campaigning on the issue, by either proposing a policy alternative to the status-quo or by defending the status-quo against a proposed alternative. Let us call the political salience or post-campaign salience of issue *k*, the probability that *k* becomes the decisive issue once parties have campaigned on issues *i* and *j*.

Definition 8.4. *Define the post-campaign salience or political salience of issue k when parties have politicized issues i and j,* s_{ij} *(k), as the probability that issue k becomes the decisive issue.*

We impose the following assumptions on the measure of political salience. First, parties can give political salience to an issue only by proposing an alternative to the status-quo. This implies that issues not raised in the electoral campaign do not get salience and hence cannot be decisive. This is a very natural assumption in our context since voters could not tell parties apart on the basis of that issue. Secondly, if both parties decide to campaign on the same issue, then it becomes the decisive issue. For this is the only issue raised during the electoral campaign. Finally, if parties give political salience to two different issues, the probability for each issue to become decisive equals its relative salience.

Assumption 8.2. *Given the pair of issues, i, j,chosen by the government and opposition parties:*

1. $s_{ij}(k) = 0$ *for all* $k \notin \{i, j\}$,

2. $s_{ii}(i) = 1$, *for all* $i = 1, \ldots, N$

3. $s_{ij}(i) = \dfrac{\sigma_i}{\sigma_i + \sigma_j}$ *for all* $i \neq j$

It follows that for any pair of distinct issues $s_{ij}(i) = 1 - s_{ij}(j)$. We will write $s_{ij} = s_{ij}(i)$ whenever there is no ambiguity.

8.2.5. Parties' objective: the expected probability of victory

Parties want to win the election. Observe that parties face uncertainty on the identity of the decisive issue as well as on the winning position on each issue. When parties compete on the same issue, this issue becomes decisive and their probability of victory coincides with their probability of holding the winning policy position on that issue. On the other hand, when parties campaign on different issues, the probability of victory is the expected probability of holding the winning policy position on the decisive issue. Formally, we define the *expected probability of victory* Π for a party proposing an alternative on issue i, a_i, while the other party proposes an alternative on issue j, a'_j, as

$$\Pi(a_i, a'_j) = \begin{cases} \pi_i(a_i, a'_j) & \text{if } i = j \\ s_{ij}\pi_i(a_i, q_i) + (1 - s_{ij})(1 - \pi_j(a'_j, q_j)) & \text{if } i \neq j \end{cases} \tag{8.2}$$

where voters associate to the status quo the position of a party which does not propose a policy alternative on a particular issue.

8.2.6. Timing and equilibrium

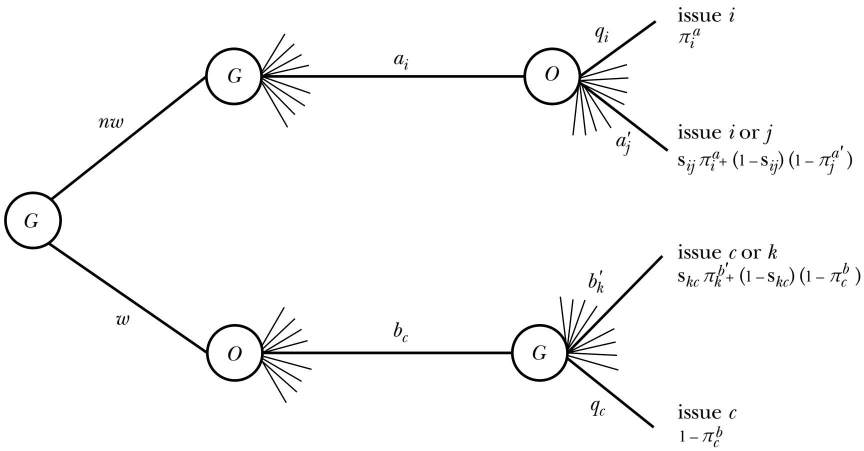
The political game consists of choosing issues on which to compete for the next election. It develops sequentially (see figure 8.2). First, the government party may decide either not to wait and take the initiative (nw) or to wait (w). Taking the initiative means that the government party chooses one issue i on which it proposes a policy alternative to the status-quo $a_i \in A_i$, $a_i \neq q_i$.[2] Then the opposition party can fight the government's proposal either by defending the status quo q_i or

[2] Recall that proposing an alternative to the status quo is the only way to confer political salience to an issue.

the other alternative on the issue, or by devoting its efforts to raising another issue j on which to propose a policy alternative $a_j' \neq q_j$, $j \neq i$. If, on the contrary, the government decides to wait, the opposition can choose one issue c on which to propose a policy alternative to the status-quo $b_c \in A_c$, $b_c \neq q_c$. Then the government party can either compete on the issue or raise a new issue k ($b_k' \neq q_k$, $k \neq c$).

FIGURE 8.2: Game tree showing the timing of the game

The final nodes represent the issues raised during the electoral campaign and the government's expected probabilities of victory



We focus the analysis on subgame perfect equilibria, the standard concept in sequential games with complete information. Because this is a finite, zero-sum game, a subgame perfect equilibrium always exists and parties will have the same probabilities of victory in all equilibria. Moreover, it is easy to see that, for any issue i, policy y_i is a strictly dominated action and will never be chosen by a party at equilibrium.[3]

8.3. The space of issues: salience and policy consensus

It follows from the definitions that salience and consensus are not independent from each other. For example, as suggested before, low salience of an issue indi-

[3] Recall that $\pi_i^x > \pi_i^y$ and $\pi_i(q_i, x_i) > \pi_i(y_i, x_i)$ (see footnote 1 and Assumption 8.1).

cates broad consensus with the status-quo policy on the issue. The following proposition describes the relationship between issue salience and policy consensus.

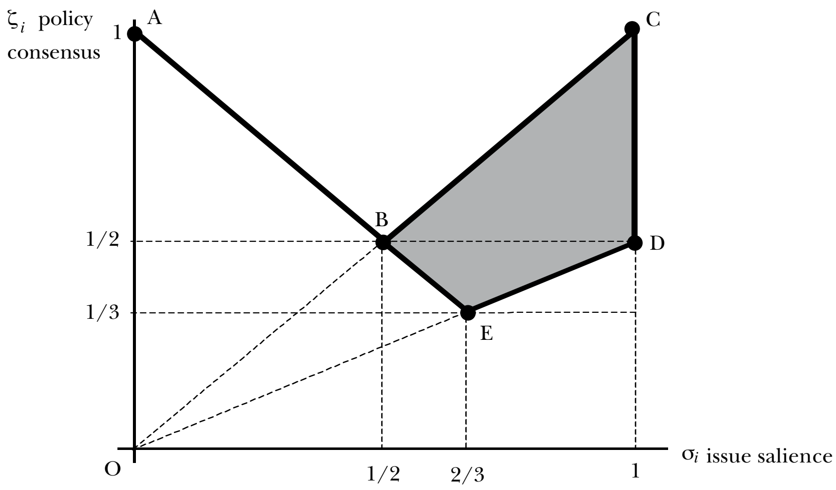
Proposition 8.1. *Let σ_i and ζ_i be the pre-campaign salience and the degree of consensus on issue i, respectively. Then,* $\max\left\{\frac{\sigma_i}{2}, 1-\sigma_i\right\} \leq \zeta_i \leq \max\{\sigma_i, 1-\sigma_i\}$.

Proof. Because $\mathbf{F}_i^x \geq \mathbf{F}_i^y \geq 0$ and $\sigma_i = \mathbf{F}_i^x + \mathbf{F}_i^y$ (Definition 8.2), $\frac{\sigma_i}{2} \leq \mathbf{F}_i^x \leq \sigma_i$. It follows then that

$$\max\left\{\frac{\sigma_i}{2}, 1-\sigma_i\right\} \leq \max\{\mathbf{F}_i^x, 1-\sigma_i\} = \zeta_i \leq \max\{\sigma_i, 1-\sigma_i\}$$

Figure 8.3 describes the set of feasible pairs of values for issue salience and policy consensus. First, the declining diagonal $\overline{AB}$ captures all those issues which take relatively low salience among voters' pre-campaign concerns ($\sigma_i < 1/2$) because there is relatively high consensus with the status-quo policies on the issues ($\zeta_i = 1 - \mathbf{F}_i^x - \mathbf{F}_i^y > 1/2$). Second, the smaller downside triangle $\overset{\Delta}{BDE}$ includes all those issues with relatively high salience ($\sigma_i > 1/2$), but relatively low policy consensus ($\zeta_i = \mathbf{F}_i^x < 1/2$). Finally, the larger rectangular triangle in the upper-right corner $\overset{\Delta}{BCD}$ encompasses all those issues which take relatively high salience ($\sigma_i > 1/2$) and on which a policy alternative to the status-quo obtains relatively high consensus ($\zeta_i = \mathbf{F}_i^x < 1/2$).

FIGURE 8.3: The set of feasible pairs of issue salience and policy consensus



8.4. Examples

We provide a few numerical examples to illustrate how parties competing in setting the electoral agenda can overlook the concerns of the electorate, as represented by issue salience, by either choosing to defend the unpopular status-quo on the issue or not talking about it at all.

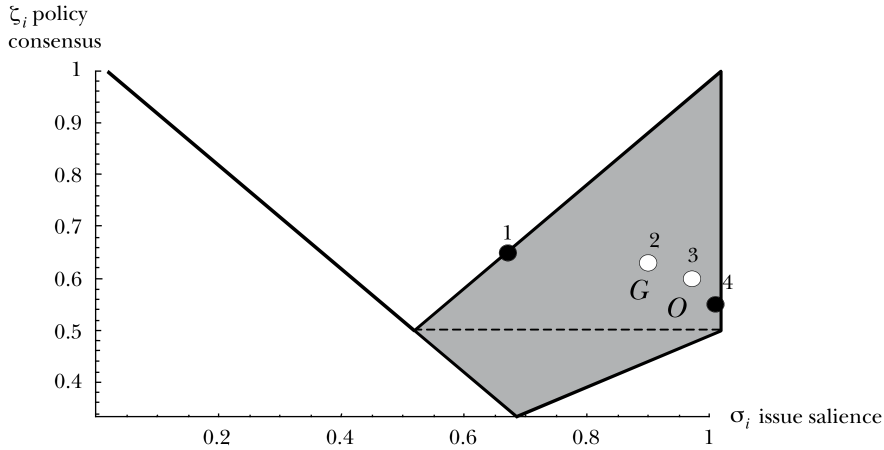
For the computation of the examples, we take a simple approach and make the probability of victory on an issue (Definition 8.3) equal the fraction of the vote obtained. Thus, $\pi_i^x = \mathbf{F}_i^x$ and $\pi_i^y = \mathbf{F}_i^y$.[4]

8.4.1. Example 1

Consider an election in which four potential issues have the following salience and consensus values:

$(\sigma_1, \zeta_1) = (0.65, 0.65)$; $(\sigma_2, \zeta_2) = (0.88, 0.63)$; $(\sigma_3, \zeta_3) = (0.95, 0.60)$; $(\sigma_3, \zeta_3) = (0.99, 0.55)$, as represented in figure 8.4.

FIGURE 8.4: Example showing that neither the most salient issue nor the issue with the highest consensus may be chosen at equilibrium



[4] The *Mathematica* program used to compute equilibria can be obtained from the authors.

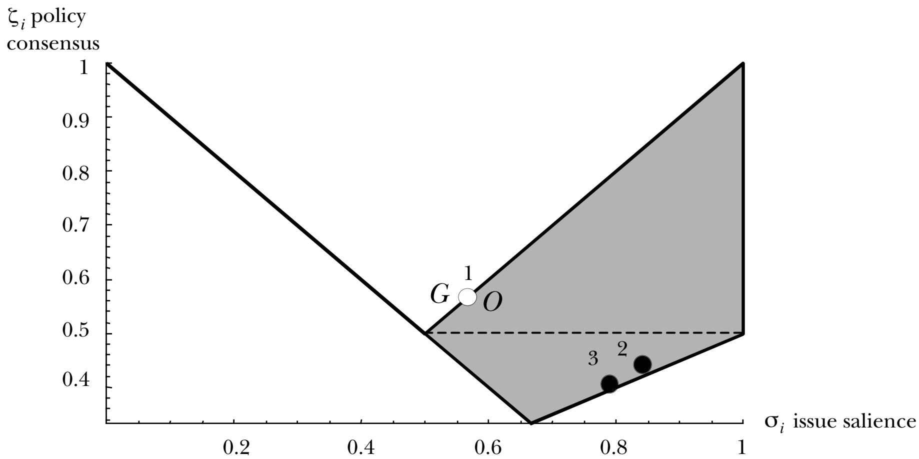
In equilibrium, government and opposition focus on different issues, 2 and 3 respectively. Specifically, the government takes the initiative and announces x_2 and the opposition responds by choosing x_3. Both parties overlook issue 4, which is the most salient issue, and issue 1, which is the one with highest consensus. However, they focus on issues with higher consensus than issue 4 and more salience than issue 1.

8.4.2. Example 2

Consider an election in which three potential issues have the following salience and issue values, as shown in figure 8.5:

$(\sigma_1, \zeta_1) = (0.56, 0.55)$, $(\sigma_2, \zeta_2) = (0.84, 0.44)$, $(\sigma_3, \zeta_3) = (0.79, 0.40)$.

FIGURE 8.5: Example showing that parties may focus on the least salient issue



The government chooses issue 1, the only one with both salience and consensus above 1/2. The opposition does not choose issues 2 or 3 because of the lack of consensus on the best policy on those issues, but it rather challenges the government on the same issue 1. The electoral campaign focuses on the least salient issue. If the issue chosen by the government, 1, benefited from significantly broader policy consensus and were, thus, a more secure issue for the government, the opposition would choose another issue (issue 2 in the example).

These are just specific examples to show possible occurrences. To approach more general results, we can note that parties will not choose the issue with both the lowest salience and the highest controversy or lowest consensus on the appropriate policy. But regarding highly salient issues, if there is not sufficiently broad consensus on a policy alternative, the opposition party may choose not to challenge a highly unsatisfactory status-quo policy and the incumbent government may survive in spite of its bad policy performance.

8.5. Conclusion

We have presented an agenda-setting model of electoral competition in which parties choose to give salience and campaign on those issues on which they expect their policy proposals will obtain voters' broad support.

Parties have to trade off the pre-campaign salience of each issue in voters' concerns and the voters' support or consensus in favor of the policy alternatives on the issue. We have found that, although parties will not compete on irrelevant issues (those with both low salience among voters and divisive policy proposals), indeed the issues which are considered the most important ones by a majority of voters may not be given salience during the electoral campaign.

This may be a surprising result, as remarked at the beginning of this paper, but it may be a reasonable one after all. Even if there is extensive public concern on some issue, if there is not a single policy proposal on the issue which can attract broad consensus, focusing on that issue might produce high division and polarization among both parties and voters. Important issues in people's concerns can, thus, be solved through electoral competition only when a policy alternative appears as clearly superior to voters' eyes. In the absence of a likely successful policy alternative, parties can choose not to give salience to the issue, thus maintaining the status-quo policy even if it is unsatisfactory for voters.

In the short term, mediocre policies broadly rejected by the electorate, as well as incumbent parties with no good performance in government, may survive for lack of a sufficiently convincing alternative. In the long term, broad policy consensus can be accumulated on an increasing number of issues, but not in the order of importance in voters' concerns.

References

BURDEN, B. C. and J. N. R. SANBERG. "Budget rhetoric in presidential campaigns from 1952 to 2000." *Political Behavior* 25 (2003): 97-118.

LANGER, G. and J. COHEN. "Votes and values in the 2004 election." *Public Opinion Quarterly* 69, no. 5 (2005): 744-759.

MCCOMBS, M. and J.-H. ZHU. "Capacity, diversity, and volatility of the public agenda: trends from 1954 to 1994." *Public Opinion Quarterly* 59 (1995): 495-525.

NATIONAL ELECTION POOL. *Exit Poll for ABC-AP-CBS-CNN-Fox News-NBC News.* Edison Media Research & Mitofsky International, provided by Voter News Service VNS (2004). Available on Internet at http://edition.cnn.com/ELECTION/2004/pages/results/states/US/P/00/epolls.0.html.

REPASS, D. E. "Issue salience and party choice." *American Political Science Review* 65 (1971): 389-400.

STROUD, N. J. and K. KENSKI. "From agenda setting to refusal setting." *Public Opinion Quarterly* 71, no. 4 (2007): 539-559.

WEISBERG, H. F. "The US presidential and congressional elections, November 2004." *Electoral Studies* 24 (2005): 741-784.

WEISBERG, H. F. and D. P. CHRISTENSON. "Changing horses in wartime? The 2004 presidential election." *Political Behavior* 29, no. 2 (2007): 279-304.

WLEZIEN, C. "On the salience of political issues: The problem with 'most important problem'." *Electoral Studies* 24 (2005): 555-579.

9

A Brief Survey on Rational Choice Models of Polling

Dan Bernhardt
Department of Economics and Department of Finance
University of Illinois

John Duggan
Department of Political Science and Department of Economics
University of Rochester

Francesco Squintani
Department of Economics
Essex University, University of Brescia
and ELSE, University College London

9.1. Introduction

The last years has seen new contributions to rational choice theory exploring the implications of polling in elections and policy choice of office holders. These papers are motivated by the observation that both electoral candidates and office holders devote substantial resources to gathering information about voters through private polling. Eisinger (2003) finds that since the Roosevelt administration, private polls have been an integral part of the White House modus operandi. Nixon had polls routinely conducted, but did not disclose results even to the Republican National Committee; and F.D. Roosevelt described private polling as his "secret weapon" (Eisinger, 2003). President Kennedy famously kept his polling numbers locked away in a safe in his brother's house rather than admit to using them. President Reagan, who is often famously viewed as making policy based mainly on his ideology, polled obsessively, taking polls "prior to his inauguration, while he was being inaugurated, and the day after he was inaugurated" (Green 2002, 4). The close relationship between President Clinton and his chief pollster, Richard Morris, is an acknowledgement of the importance of polls in determining policy outcomes in that administration. Indeed, Medvic (2001) finds that fully 46 percent of all spending on U.S. Congressional campaigns in 1990 and 1992 was devoted to the hiring of political consultants, primarily political pollsters. In addition, the major parties provide polling services to their candidates. While polls conducted by office-holders and candidates are often kept secret, there is also a vast amount of information gathered by public polls prior to elections or while an office holder is in power.

Recent rational choice models of polling largely consider three distinct lines of research: (i) the strategic behavior of polled citizens, (ii) the adverse effects of public polling, and (iii) strategic platform choice when candidates are privately informed due to their polling.

Strategic behavior of polled citizens. There is a growing literature on the strategic incentives of polled citizens in delivering their responses to pollsters. Morgan and Stocken (2007) study a simple model of information transmission via polling, based on the classic work by Crawford and Sobel (1982) on strategic information transmission. Before choosing a policy that affects all constituents, a policy maker polls a subset of her constituents to obtain information about a payoff relevant state variable. Constituents receive private signals about the state variable, but may choose to misreport their information in order to influence the policy choice. A tradeoff between truthful revelation in the polls and precision of the polls is uncovered: full revelation can be an equilibrium when relatively few constituents are polled, but as the poll size grows large, full revelation becomes impossible. The paper then investigates whether full information aggregation can arise even when truthful revelation becomes impossible. The paper considers the most informative equilibria, and finds that full information aggregation can arise in an equilibrium if constituents and the policy maker have similar ideologies. In these equilibria, constituents endogenously sort themselves: moderates answer truthfully while extremists bias their responses to the pollster. If, instead, the policy maker is ideologically isolated, full information aggregation is impossible.

In contrast to Morgan and Stocken (2007), who consider the policy choice of an office-holder, Meirowitz (2005) embeds his analysis of strategic information transmission via polling in a full-fledged model of electoral competition. In addition, he considers the possibility that pollsters report only a summary statistic from polls that ask respondents their preferences. The analysis focuses on settings with a unidimensional policy space, single peaked preferences and two office-seeking candidates. Meirowitz (2005) supposes that all the information received via polling is common knowledge among the candidates. Hence, following Calvert (1985), candidate platforms converge to the median of the distribution of the median policy in the electorate. Meirowitz (2005) confirms Morgan and Stocken's (2007) finding that truthful revelation of information via polling cannot generically occur in equilibrium. However, simple partially-revealing equilibria exist when the poll only asks respondents which party or candidate they prefer. Such equilibria evidently persist when the candidates learn the sample average, or see all the data but ignore all information received beyond the basic ranking of voters between the two parties.

Adverse effects of public polling. The literature on the adverse effects of public polling emerged in an attempt to motivate the ban on release of public polling found in many countries. Goeree and Grosser (2007) and Taylor and Yildirim (2005) study models of costly voting that predict that elections are more likely to be close and voter turnout is more likely to be high when public polling information is released to citizens prior to an election. The models suppose that there are two alternatives, one minoritarian and the other one supported by the majority. When the distribution of preferences is common knowledge among citizens, it is known from Palfrey and Rosenthal (1983, 1985) that costly voting leads to equilibria where the probability that either alternative wins the election is exactly one half. In fact, each voter chooses to participate in the election if and only if her probability of being pivotal is strictly positive. Thus, equilibrium requires that members of the minority vote with higher frequency in order to compensate exactly for their smaller group size.

Goeree and Grosser (2007) and Taylor and Yildirim (2005) consider a setting where, by contrast, voters' preferences are private information and, ex-ante, each alternative is majoritarian with *equal* probability. They identify two types of inefficiencies that may obtain. First, candidates and/or issues may win elections even though they were preferred by only a minority of the citizens. Second, aggregate voter turnout may be excessive in the sense that too many citizens expend resources in casting votes. They find that in this symmetric setting, each voter votes with the same probability in equilibrium. As a result, the majority is more likely to win the election and expected voter turnout is lower. In fact, when the population is large and voting costs are small, the majority wins with probability arbitrarily close to one in equilibrium. Welfare is, therefore, unambiguously higher when public poll results are not released prior to elections.

Private polling by parties. A third line of research explores the strategic platform location of parties that are privately informed about voters' preferences through polling. Bernhardt Duggan and Squintani (2007) formulate and analyze a general model of elections in which candidates receive private signals about voters' preferences prior to committing to political platforms. They fully characterize the unique pure-strategy equilibrium when it exists: After receiving her signal, each candidate locates at the median of the distribution of the median voter's location, conditional on the other candidate receiving the *same* signal. Sufficient conditions for the existence of pure strategy equilibrium are provided. Though the electoral game exhibits discontinuous payoffs for the candidates, Bernhardt Duggan and Squintani (2007) prove that mixed strategy equilibria exist generally, that equilibrium expected payoffs are continuous in the parameters of the model, and that mixed strategy equilibria are upper hemicontinuous. This allows them to study the robustness of the median voter theorem to private information: Pure strategy equilibria may fail to exist in models "close" to the Downsian model, but mixed strategy equilibria must, and they will be "close" to the Downsian equilibrium.

Bernhardt, Duggan and Squintani (2008a) specialize the model of Bernhardt, Duggan and Squintani (2007), to obtain explicit closed form calculations of mixed strategy equilibria, which permits comparative static and welfare analyses. In the essentially unique equilibrium, candidates who receive moderate signals adopt more extreme platforms than their information suggests, but candidates with more extreme signals may moderate their platforms. Policy convergence does not maximize voters' welfare. Although candidates' platforms diverge in equilibrium, they do not do so as much as voters would like. Bernhardt, Duggan and Squintani (2008a) find that the electorate always prefers less correlation in candidate signals, and thus private over public polling. They further find that some noise in the polling technology always raises voters' welfare, which highlights other possible adverse welfare effects of public polling, and the welfare benefits of spending limits.

9.2. Strategic information transmission via polling

9.2.1. Morgan and Stocken (2007)

This paper studies how strategic motives affect the information content of polls and, ultimately, policy outcomes. The model is a straightforward modification of the classic piece by Crawford and Sobel (1982) on strategic information transmission. A polity consists of a continuum of individuals, who differ in ideologies. It is commonly known that the policy maker has the median ideology. While the constituents are uninformed about the realized state, each constituent receives a conditionally independent private binary signal that is correlated with the state. The policy maker does not observe the state nor receive a signal about it. However, the policy maker can obtain information about the state by polling voters. Unfortunately, for the policy maker, voters internalize how their responses may influence policy and choose whether to report honestly. In particular, before choosing a policy, the policy maker conducts a poll consisting of a commonly known (finite) sample of the constituents. Each polled constituent simultaneously sends a binary message—the message is pure cheap talk. After learning the results of the poll, the policy maker selects a policy, and payoffs are realized. All agents' utilities satisfy the regularity conditions of Crawford and Sobel (1982): they are concave in the policy, and single-crossing across policy and state. Agents with different ideologies may be upward-biased or downward-biased relative to the median of the constituency, and hence relative to the policy maker.

The main results of the paper are as follows. First, Morgan and Stocken (2007) find that truthful information revelation is an equilibrium if and only if the number of constituents polled is relatively small and the ideology of citizens is relatively homogeneous. Since the size of the poll is relatively small, however, the amount of information the policy maker obtains is limited. As the size of the poll sample grows large, truthful communication ceases to be an equilibrium. Indeed, they show that there is a finite upper bound on the size of a poll for which truthful information revelation is an equilibrium. These results are intuitive: due to the concavity of preferences, a single citizen may find it optimal to truthfully reveal her signal if the information of the policy maker is imprecise, so that her report moves the policy maker's action significantly; and at the same time, prefer to pool her signals according to her ideology bias when the information of the policy maker is very precise, so that her report moves the policy maker's action only by a slight amount. In the first case, in fact, misreporting a signal may move the action beyond the bliss point of the citizen, but this cannot happen when the effect of the report on the action is sufficiently small. In sum, there is a tradeoff between polling precision and truthful revelation: precisely as the sample becomes more informative, truthful revelation ceases to be an equilibrium.

Second, Morgan and Stocken (2007) determine whether polls aggregate information under non truth-telling strategies. They find that it depends on the distribution of ideologies in the polity. When the policy maker is moderate, in the sense that there are sufficiently many upwardly-biased citizens and sufficiently many downwardly-biased citizens, they show that full information aggregation can arise in equilibrium. The citizens polled endogenously sort themselves into centrists, who answer truthfully, and extremists, who pool their answers according to their ideology. As the size of the poll grows large, the fraction of centrists among those polled becomes vanishingly small, because the ideological bounds on centrism converge to the median ideology. However, the number of centrists grows without bound, so that full information aggregation occurs in the limit.

Third, Morgan and Stocken (2007) show that ignoring strategic motives and using classical statistical inference leads to biased estimators of the state variable as well as a mischaracterization of confidence intervals for the value of the state variable. The authors propose estimators that correct for strategic effects in polls. Finally, the authors show that policy outcomes arising from a poll differ from those obtained when policies are determined by voting, as in a referendum. Suppose that the policy space is constrained to be binary so that meaningful comparisons between the two mechanisms can be undertaken. The authors show that citizens will convey some information when voting in a referendum. In contrast, when policies are determined following a poll it may be impossible for constituents to credibly convey information in any equilibrium.

9.2.2. Meirowitz (2005)

In contrast to Morgan and Stocken (2007), this paper studies polling in a complete electoral competition set-up. Candidate positions are strategic variables chosen by candidates after observing polling data. In the model, there is an initial polling stage in which a sample of voters announce their ideal points to a polling service, followed by an electoral stage in which the two candidates learn polling statistics and take policy stances, and finally by a voting stage in which the electorate chooses between the candidates. After learning the polling outcomes, each candidate takes a position on a closed interval of the real line. Voters' preferences are single-peaked and symmetric: hence they vote for the candidate whose platform is closer to their ideal point. The distribution of voter preferences is parametrized in an unknown random variable that represents the distribution's realized median. A randomly chosen finite subset of voters is simultaneously polled. This subset is gathered by an odd number of independent draws from the uniform distribution of voter preferences. Poll respondents simultaneously announce a message. The candidates observe the messages and update their beliefs about the unknown random parameter, and then choose their policy platforms.

Following Calvert (1985), in the equilibrium of the electoral game, the candidates choose the platform that corresponds to the median of the median of the distribution of voters' preferences conditional on the polling results. The key general result of this paper is that truthful communication occurs only for non-generic parametrizations of the polling game. The logic of the result is similar to that uncovered by Morgan and Stocken (2007), and relies on single-peakedness and concavity of the citizens' policy preferences. In fact, given that candidate platforms converge to the median of the median of the distribution of voters' preferences conditional on the polling results, the strategic choice in the electoral game can be subsumed into the choice of a single decision-maker (the receiver in the parlance of cheap-talk games). This result is robust to specifications where the candidates only receive a summary statistic of polling results, such as the median of the polled voters' bliss-points.

Meirowitz then proceeds to consider a setting where polls have small message spaces: specifically, poll respondents are asked only which party or candidate they prefer. Evidently, this message space is too small to support truthful equilibria, but Meirowitz finds that partially-revealing perfect Bayesian equilibria exist. One interesting implication of partially revealing equilibria is that the set of people who say they would vote for a particular candidate may not correspond to the set of people who would really vote for the candidate at the election stage. That is, some respondents may misrepresent themselves to try to influence candidate platforms: Specifically, respondents who expect to be unsatisfied with the policy outcome (say a right of center respondent) can manipulate the inferences and policy selections of candidates by lying about their preferences (e.g., claiming to be even more right of center). Hence, as in Morgan and Stocken (2007), naive interpretations of polling statistics are problematic. Finally, Meirowitz (2005) shows that equilibria in this binary message game can be reinterpreted as partially revealing equilibria of games where the polled respondents report their bliss-points and candidates observe either the whole polling data or the sample average. Because the content of information transmission in these equilibria mimics the model where the polled respondents are only asked which party or candidate they favor, these equilibria exhibit stark polarization as all respondents claim to be maximally extreme.

9.3. The perverse effects of public polling

9.3.1. Goeree and Grosser (2007) and Taylor and Yildirim (2005)

These papers independently highlight possible unpleasant implications of public polling in settings with costly voting. Specifically, they show that the release of public polls can give rise to two types of inefficiency. First, candidates and/or issues may win elections even though they were preferred by only a minority of the citizens. Second, aggregate voter turnout may be excessive in the sense that too many citizens expend resources in casting votes.

In the models by Goeree and Grosser (2007) and Taylor and Yildirim (2005), there are two fixed electoral alternatives, which could represent a referendum or a two-party election with differentiated parties. Citizens possess private valuations over electoral outcomes and voting is costly. These papers contrast two polar informational scenarios. In the first scenario, the distribution of political preferences is common knowledge. In the second scenario, a citizen only knows her own private valuation, and ex-ante each alternative is majoritarian with equal probability. These two polar cases are related to the issue of public polling release: when public polls are released to citizens, their information about each other's preferences is enhanced. In each regime the authors characterize the unique symmetric Bayesian Nash Equilibrium (BNE) in which all citizens randomize between voting for their preferred alternative and abstaining. This equilibrium can be derived also as a pure-strategy equilibrium where voters differ in their voting costs or their intensity of preferences.

If citizens are informed about each other's preferences, then following Palfrey and Rosenthal (1983, 1985), the probability that either alternative wins the election under the mixed-strategy BNE is one-half regardless of the distribution of political preferences or the cost of voting. In fact, each voter chooses to participate in the election if and only if her probability of being pivotal exceeds zero. Thus, equilibrium requires that the members of the minority vote with higher frequency in order to compensate exactly for their smaller group size. The unpleasant implication is that the minoritarian alternative can be adopted with probability one half, resulting in an aggregate utilitarian inefficiency.

If, instead voters do not know each other's preferences, then they cannot base their voting decisions on the distribution of political preferences, since they know only their own types. Because the common prior over the parameter governing the distribution of tastes is symmetric, all citizens believe their type to be in the majority, and vote with the same probability regardless of type. Because each citizen believes he/she is in the majority, expected equilibrium voter turnout is lower than when citizens know each other's preferences.[1] Furthermore, because each citizen votes with the same probability, the majority group, therefore, wins the election with probability strictly exceeds one-half.

Since many (if not most) important elections involve a large number of potential voters, it is important to understand whether the uninformed-voter

[1] This result obviously hinges on the ex-ante symmetric distribution over which outcome is likely to be preferred; it would be worthwhile to investigate the robustness of voter turnout in the generic asymmetric settings, where one outcome is ex-ante more likely to be favored by a majority of voters than another.

setting continues to yield higher welfare in the limit as the number of citizens tends to infinity. In this context, Taylor and Yildirim (2005) show that the equilibrium number of votes for each alternative correspond to independent random variables following Poisson distributions with endogenously determined means. As a result, in the limit as the number of citizens tends to infinity and the relative cost of voting approaches zero, the alternative favored by the majority wins the election with probability arbitrarily close to one when citizens are uninformed of each other's preferences, but only with probability one-half when citizens know the size of the majority and minority supports.

9.4. Strategic electoral platform choices by privately informed candidates

9.4.1. Bernhardt, Duggan and Squintani (2007)

Bernhardt, Duggan and Squintani (2007) develop a general model of elections in which candidates receive private polling signals. Each candidate receives a signal drawn from an arbitrary finite set of possible signals about the location of the median voter's ideal policy; each candidate updates about both the location of the median voter and her opponent's platform before choosing a platform from the real line; and the candidate whose platform is closest to the median voter wins. Bernhardt, Duggan and Squintani (2007) consider a very general setting, in which there can be arbitrary correlations in the polling signal structure and arbitrary (finite) numbers of signal realizations. The authors consider any family of conditional distributions of the random median policy such that the conditional distributions are continuous with connected supports. While results are given for a baseline model in which candidates have identical polling technologies, the most general results allow candidates to have different polling technologies, as might be expected when an incumbent runs against a challenger. Within this framework, the authors derive the existence and continuity properties of electoral equilibria, and determine the ways in which the classical median voter theorem is and is not robust to the introduction of small amounts of asymmetric information.

The introduction of private polling to the model generates subtle informational incentives for candidates, and logic of the median voter theorem does not extend to the general private-information environment in the expected way. In particular, a candidate does not target the median voter conditional on his own signal. In the symmetric model, there is at most one pure strategy equilibrium: After receiving a signal, a candidate updates the prior distribution of the median voter, *conditioning on both candidates receiving that same signal,* and locates at the median of *that* posterior distribution. In the probabilistic voting model, where candidates have symmetric information, conditioning on one candidate receiving a signal is the same as conditioning on both receiving it, so the traditional probabilistic version of the median voter theorem is obtained as a special case. With private information, however, this paper's result shows that strategic competition leads candidates to take positions that are more extreme than their own estimates of the median voter's ideal policy: Asymmetric information obviously leads to policy divergence, and the strategic effect magnifies the policy divergence already inherent in private information.

The paper gives sufficient conditions for existence of the pure strategy equilibrium, the key being that conditional on a candidate receiving a signal, the probability that the opponent receives a signal weakly to the "left" should exceed the probability that the opponent receives a signal strictly to the "right," and vice versa. This limits the incentive for a candidate to move away from the equilibrium platform after any signal, and together with other background conditions, it ensures the existence of the pure strategy equilibrium. This key condition is actually necessary for existence in some environments. It becomes quite restrictive, however, when the number of possible signals is large, and it is concluded that the pure strategy equilibrium typically fails to exist in elections with fine polling information. In fact, it is shown that adding arbitrarily small amounts of asymmetric information to the Downsian model can cause the pure strategy equilibrium to cease to exist, highlighting the issue of robustness of the median voter theorem with respect to even small amounts of private information.

These considerations lead one to analyze mixed strategy equilibria. Despite discontinuities inherent in candidate payoffs, it is proved that mixed strategy equilibria exist. Bernhardt, Duggan and Squintani (2007) show that the (unique) mixed strategy equilibrium payoffs vary continuously in the model parameters, and this result implies upper hemicontinuity of equilibrium mixed strategies. Imposing only minimal functional form restrictions, the paper obtains characterization results for mixed-strategy equilibria. The supports of mixed strategy equilibria lie in the interval defined by the smallest and largest conditional medians; this implies the corollary that the equilibrium of the traditional probabilistic voting model is unique within the class of all *mixed* strategy equilibria. Furthermore, it is shown that the only possible atoms of equilibrium mixed strategies are at conditional medians. As a consequence, if there is a positive probability that the candidates converge on the same policy platform in equilibrium, then that platform must belong to the finite set of conditional medians.

Finally, the paper returns to the issue of robustness of the median voter theorem. The paper's continuity results apply to the traditional probabilistic voting model and immediately yield robustness of the probabilistic version of the median voter theorem: When candidate beliefs about the median voter's location are "close" to some common distribution, mixed strategy equilibria must be "close" to the median of that distribution. Furthermore, even though the Downsian model is marked by fundamental discontinuities, the robustness result extends. Thus, in the Downsian model, the median voter theorem is fragile in terms of pure strategies, but robust in terms of mixed strategies: Mixed strategy equilibria exist and must be close to the median when small amounts of asymmetric information are added to the model. Lastly, the paper gives examples showing the robustness result for the Downsian model relies critically on complete information: It does not extend to general models with discontinuous conditional distributions.

9.4.2. Bernhardt, Duggan and Squintani (2008a)

This paper specializes the general model of elections in which candidates receive private polling information about voters' preferences developed in Bernhardt, Duggan and Squintani (2007). In Bernhardt, Duggan and Squintani (2008), the median policy is given by $\mu = \alpha + \beta$, where α is independently and uniformly distributed, and candidates receive signals about β, which is symmetrically distributed around the ex-ante median. One interpretation of this median policy decomposition is that voters are unwilling or unable to provide pollsters accurate summaries about all of their views, as is suggested by the empirical work of Gelman and King (1993). Another interpretation is that candidates learn about the position β initially preferred by the median voter, after which electoral preferences may shift by α during the electoral campaign.

The observation in Bernhardt, Duggan and Squintani (2007) that the necessary and sufficient conditions for pure-strategy equilibrium existence are implausible unless there are few possible signals, or unless signals are so precise that the probability that the opponent receives the same signal (rather than just a near-by signal) exceeds one half, lead the authors to prove that, even when a pure strategy equilibrium does not exist, there always exists a unique mixed-strategy equilibrium in which the locations of the candidates follow a strong order with respect to their signals. The authors derive the closed-form solution of this equilibrium and generate several empirical predictions. First, they show that candidates with sufficiently moderate signals adopt their pure strategy equilibrium platforms, locating more extremely than their information suggests, while candidates who receive more and more extreme signals mix over policy positions, tempering their positions by more and more toward the ex-ante median policy. This result reflects that a politician whose pollster predicts greater shifts in the median anticipates that she is more likely to compete against an opponent with a more moderate signal, who will take a more moderate platform. The result is broadly consistent with the empirical evidence that candidates' platforms significantly diverge from the median voter's preferred policy, and yet are not too extreme.

The paper then turns to the effect of the statistical properties of the polling technology on equilibrium platforms. It is shown that an increase in the precision of the candidates' signals leads candidates to locate more extremely, in the sense of first order stochastic dominance. This finding is consistent with the concurrent trends of platform polarization (see the NES data as reported in Budge et al., 2001) and technological improvement in polling. The effect of increased signal correlation across candidates (which can be induced by public polling, for example) is ambiguous for candidates with extreme signals, but it unambiguously moderates their locations following moderate signals.

The paper then provides a thorough analysis of the welfare properties of private polling and equilibrium outcomes. The analysis builds on the observation that in a model with office-motivated candidates who share symmetric information on the unknown median policy à la Wittman (1983) or Calvert (1985), candidates' platforms converge to the median of the median policy distribution and do not offer voters enough choice (see Bernhardt, Duggan and Squintani (2008b)). If one were to introduce exogenously a small amount of dispersion in candidate platforms, then each candidate's individual platform would target the median less accurately. Collectively, however, the platform closest to the realized median would generally be more accurate than the median of the median policy distribution. Because candidates care only about winning, they do not internalize this externality. As a result, candidates do not provide enough platform dispersion from the standpoint of the electorate. This paper identifies conditions under which this insight extends endogenously to the asymmetric information setting considered in this paper: Candidates' platforms diverge in equilibrium due to private polling, but not by as much as voters would like.

The welfare analysis then proceeds to show that greater signal correlation makes voters worse off: Correlation reduces both the degree by which candidates "extremize" their platforms given their signals, as well as the probability that candidates receive different signals, choose distinct platforms, and thus provide more variety to the electorate. In contrast, the effect of signal precision on welfare is non-monotonic. Increased polling accuracy raises the probability that candidates correctly identify the median voter's preferred policy, raising the welfare from any one candidate's platform. However, increased polling accuracy also raises the probability that the candidates adopt similar platforms, reducing the choice that candidates give voters. The net effect is that up to some point, raising precision raises welfare, but too much precision has the opposite effect.

These final two results have implications for public policy. First, the electorate prefers private to public polling, because sharing information raises the correlation between candidates' information and adversely reduces platform diversity. This finding provides support for public polling bans that does not rest on claims that public polling may distort elections because of bandwagon effects or effects on voter participation. Second, because greater precision eventually reduces voter welfare, campaign spending caps that limit resources devoted to polling may raise voter welfare, even when campaign advertising is truly informative and beneficial to the electorate.

9.5. Conclusion

This paper reviews recent contributions to rational choice models of polling in three areas: (i) the strategic incentives of polled citizens to report honestly, when citizens internalize how candidates will use that information to formulate policy, (ii) the possibly adverse welfare effects of public polling, when voting is costly and (iii) strategic platform location, when candidates are asymmetrically-informed about the preferences of voters due to their private polling of voters.

A common theme of this literature is that polling can interact with strategic behavior of agents to confound selection of optimal policies in equilibrium. We saw how polls that convey information to voters can lead to under- or over-participation by voters in elections, participation rates that are influenced by voter perceptions of the popularity of their positions, and hence to the "wrong" policy sometimes being adopted. We also saw how polls that convey information to candidates can be manipulated by voter responses, again sometimes leading to the wrong policy being adopted; while public polling can reduce heterogeneity in candidate information sets, which can give rise to too little variety in candidate platforms, and hence implementation of a platform far from the preferred platforms of most voters.

By omission, this literature also highlights exciting and important open issues for future researchers. Two features that these rational choice models of polling have in common are that (i) they are largely static in nature—candidates simultaneously choose policy positions, and (ii) the issue space has a single dimension. One important direction into which to take this research is to investigate how political polling affects the dynamics of political campaigns. How does political polling influence the timing of when incumbent and challenger candidates take positions? And, in both static and dynamic environments with many (perhaps binary) issues, how does polling influence *which* issues candidates choose to take positions on, and which issues candidates choose to ignore? *What* are those positions? And how do these choices depend on the information that voters start out with about the policy preferences of candidates? We hope soon to uncover the answers to some of these fundamental questions.

References

BERNHARDT, D., J. DUGGAN and F. SQUINTANI. "Electoral Competition with Privately Informed Candidates." *Games Economic Behavior* 58 (2007): 1-29.

—. "Private Polling and Voter Welfare." *Journal of Economic Theory* (2008a), forthcoming.

—. *A Vindication of Responsible Parties.* Mimeo, 2008b.

BLACK, D. *Theory of Committees and Elections.* Cambridge: Cambridge University Press, 1958.

BUDGE, I., H. D. KLINGEMANN, A. VOLKENS, J. BARA and E. TANENBAUM. *Mapping Policy Preferences; Estimates for Parties, Electors, and Governments 1945-1998.* Oxford: Oxford University Press, 2001.

CALVERT, R. "Robustness of the Multidimensional Voting Model: Candidate Motivations, Uncertainty, and Convergence." *American Journal of Political Science* 29 (1985): 69-95.

CRAWFORD, V. and J. SOBEL. "Strategic Information Transmission." *Econometrica* 50 (1982): 1431-1452.

DOWNS, A. *An Economic Theory of Democracy.* New York: Harper and Row, 1957.

EISINGER, R. M. *The Evolution of Presidential Polling.* Cambridge: Cambridge University Press, 2003.

GELMAN, A. and G. KING. "Why Are American Presidential Election Campaign Polls So Variable When Votes are So Predictable?." *British Journal of Political Science* 23 (1993): 409-451.

GOEREE, J. K. and J. GROSSER. "Welfare Reducing Polls." *Economic Theory* 31 (2007): 51-68.

GREEN, J. "The other war room." *Washington Monthly* (April, 2002).

HOTELLING, H. "Stability in Competition." *Economic Journal* 39 (1929): 41-57.

MEDVIC, S. *Political Consultants in U.S. Congressional Elections.* Columbus: Ohio State. University Press, 2001.

MERRILL, S. and B. GROFMAN. *A Unified Theory of Voting.* Cambridge: Cambridge University Press, 1999.

MEIROWITZ, A. "Polling Games and information revelation in the Downsian framework." *Games Economic Behavior* 51 (2005): 469-489.

PALFREY, T. and H. ROSENTHAL. "A Strategic Calculus of Voting." *Public Choice* 41 (1983): 7-53.

—. "Voter Participation and Strategic Uncertainty." *American Political Science Review* 79 (1983): 62-78.

TAYLOR, C. and H. YILDIRIM. *Public Information and Electoral Bias.* Mimeo Duke University, 2005.

WITTMAN, D. "Candidate Motivation: A Synthesis of Alternatives." *American Political Science Review* (1983).

10

Caucuses and Primaries under Proportional Representation

David Epstein

Massimo Morelli

Sharyn O'Halloran

Department of Political Science
Columbia University

10.1. Introduction

On March 4, 2008, the state of Texas held both a primary and a caucus to choose delegates for the Democratic National Convention, a unique arrangement nicknamed the "Texas Two-Step". Despite being held on the same day, in the same place, open to the same set of voters, and with the same two major candidates contesting the vote,[1] these two elections produced different results: Hillary Clinton won the primary with 50.9% of the vote to Barack Obama's 47.4%, while Obama won the caucuses with 56.2% of the vote to Clinton's 43.7%.[2] The turnout in these two events also differed widely: 2,868,454 voters cast ballots in the primary, over two and a half times as many as the estimated 1.1 million who participated in the caucus.[3]

Rarely do real world events present researchers with such a perfect natural experiment of competing electoral institutions, and rarer still do the contrasting institutions yield different results. Nor was this result exceptional; through May 20, Obama had won 14 out of the 16 caucus states, while Clinton had won

[1] Edwards, Richardson, Biden and Dodd were still on the ballot as well, but they received a combined 1.72% of the vote in the primary and no delegates in the caucuses.

[2] All election results in the paper are taken from the New York Times website: http://politics.nytimes.com/election-guide/2008/results/.

[3] Exact turnout results for the caucuses are not available.

20 out of 34 primaries, with both candidates receiving almost identical numbers of total votes. Obama's relative success in caucuses was explained by most commentators as due to his aggressive grass-roots organizing strategy[4], but it clearly also has much to do with his ability to mobilize a number of supporters with very intense preferences for him relative to other candidates. This intensity was of significant importance in winning caucuses, where voters must go through a process taking two, three, four or more hours, as opposed to the (relatively) quick process of casting a vote in a primary.

In this paper we explore in detail the implications of the observation that, even though each voter can only cast one vote, intensity of preferences matters when voting is costly. We use a simple voting model to derive a non-monotonic relationship between the cost of voting and electoral outcomes between two candidates, each of whom has the same total support and the same intensity of support, but one candidate has a higher variance in intensity than does the other. In particular, the high-variance candidate will do best in contests with the highest and lowest costs of participation, while his opponent will do best in the intermediate-costs contests. We also show circumstances in which the candidate with the higher variance in support has an advantage over his opponent, being able to win the nomination while spending less money, despite having the same total support as his opponent. We then test the implications of the model on the results of the 2008 democratic presidential nominating race between Clinton and Obama, and find support for the non-monotonic relationship derived from the theoretical model.

Our work fits into the broader discussion of the impact of electoral institutions on voting outcomes. This literature can be broadly divided into two categories: those studies that emphasize the impact of institutions on aggregating voters' preferences—presidential vs. parliamentary systems, proportional representation vs. plurality winners, candidate slates, and so on[5]—and those studies that examine the impact of institutions on the composition of the electorate itself, including work on extending the franchise, on raising or lowering the cost of voting, and on raising the cost of non-voting, i.e., compulsory voting systems.[6] Our work falls in the latter category, as the costs of participation are significantly higher in caucuses as opposed to primaries, and this affects the size and compo-

---

[4] See for instance http://www.boston.com/news/nation/articles/2008/05/04/small_state_plan_pays_dividends_for_obama.

[5] See for instance Shugart and Carey (1992), and Lijphart (1999).

[6] The literature on costly voting is voluminous; see Feddersen (2004) for a summary. The more specific question of the political consequences of expanding or contracting the electorate is also receiving renewed attention; see Lizzeri and Persico (2004). And a recent study of compulsory voting (Helmke and Meguid 2008) also shows consistent patterns of enfranchisement in democracies.

sition of the electorate state by state. In the face of costs to participation, we ask how differences in the intensity of preferences across voters and costs of voting across states or districts determines the winner of a multi-district simultaneous or sequential election, with the idea that these considerations may matter more than campaign strategies, fund raising, and/or election timing per se.

Our work also relates to studies of the primary system, most of which have been empirical, including classic works by Polsby (1983) and Bartels (1988). The former describes in detail the rise of the primary system as opposed to the former nominating system in which deals were worked out in "smoke-filled rooms", arguing that the switch probably hurt the Democratic party, on balance. The latter describes the role of momentum in the primary system; those candidates who do well in early primaries are apt to do well later too, and small fluctuations in the early vote can have major consequences for the entire nominating process.

Issues of momentum and information aggregation in sequential elections are now becoming quite well studied in the theoretical literature. Starting with Feddersen and Pesendorfer (1996), game theorists have made significant contributions to our understanding of how informational cascades affect vote choice. Recent work in this tradition includes Fey (2000), Callander (2007), Iaryczower (2007), and Ali and Kartik (2007). Empirical studies of sequences of costly voting include early work by Lohmann (1994) and a more recent experimental study by Battaglini, et. al. (2007). These studies have done much to illuminate the role of strategic voting in aggregating information from sequential contests. Unlike this literature, though, we focus on the impact of different voting institutions on primary outcomes, and on the ability of primaries to produce a general election winner. Sequential elections are important for us not because they make the $n+1$ st voter more confident of which is the better candidate, but because they allow a candidate with heterogeneous supporters, competing in contests with differential costs of voting, to edge nearer to the nomination. We thus add to the growing literature on primaries as unique political institutions in their own right—see for instance Gerber and Morton (1998), Meirowitz (2005), and Jackson, et al (2007)—with an emphasis on the impact of differences in intensity of preferences across supporters of the various candidates.

10.2. The model

Consider two exogenous candidates, i, j who run for election (or nomination), of value b to each candidate. The winner is the one who obtains the majority of total delegates from n states, of equal size, and the assignment of delegates to each candidate is proportional to the percentage of votes received, state by state. In other words, the total delegates supporting candidate i are a fraction $\sum_{k=1}^{n} v_k^i \ / \ n$ of the total available delegates (ignoring the integer problem), where $v_k^i \in [0,1]$ denotes the percentage of votes obtained by candidate i in State k, $k = 1,2,..., n$. Normalize the size of the electorate in each of the states to 1, so that v_k^i is both a percentage and the number of votes, so we will use these terms interchangeably.

Before the campaign, each voter ℓ is characterized by a parameter $d_i(\ell) \in R$, which denotes the expected utility difference for ℓ if i wins against j (so that if it is positive it means that ℓ prefers i to win). Since we want to highlight the role of the distribution of preference intensities, we assume that the two candidates are "equal" on all other fronts, and also in terms of "total" intensity. In other words, assume

$$\int_0^1 d_i^k(\ell)\, d\ell = 0 \ \forall k \tag{10.1}$$

and

$$\# \{\ell{:}\ d_i^k\ (\ell) > 0\} = \# \{\ell{:}\ d_i^k\ (\ell) < 0\} \ \forall k \tag{10.2}$$

We assume (10.1) and (10.2) equal total intensity of support ℓ across candidates in each State and equal number of supporters per candidate in each state respectively only because we wish to *isolate* the effect of the variance, but they are in no way essential for the qualitative results.

The positive d's thus have the same mean as the absolute value of the mean of negative d's, but the variance for positive numbers is higher. For simplicity, assume that the supporters of candidate i are divided in two groups in every State, with α of them being high intensity and $(1 - \alpha)$ low intensity, whereas the supporters of candidate j have all the same intensity in every State, such that the distribution of intensities for i is a mean preserving spread: $d_i^h >| d_j |> d_i^l$ and

$$\alpha d_i^h + (1 - \alpha)\ d_i^l = |\ d_j |$$[7]

Candidates are assumed to maximize their vote share, state by state. Each candidate has by assumption unlimited funds for the campaign[8] and here is the simplest

---

[7] For instance, in a simple spatial model with linear utilities and candidate platforms $x_i = 0$ and $x_j = 1$, the d_i^h voters could be a point mass of weight 1/3 with ideal points $y_i^h = .1$, the d_i^l voters a point mass of weight 1/6 with ideal points $y_i^l = .4$, and the d_j voters a point mass of weight 1/2 with ideal points $y_j = .8$.

[8] The results do not depend on this simplifying assumption. Intuitively, introducing exogenous budget constraints or an additional stage of fund raising may allow us to add other results on spen-

possible way to describe what the campaign does in this model: each candidate decides what voters to "target," under full information about their d type. Targeting one voter means spending one dollar talking to him or her, and what this targeted contact does is raise his or her absolute value of the initial d by a fixed amount g, so that if voter ℓ with $d_i(\ell) > 0$ is targeted by i his posterior perception of utility differential is $d_i(\ell) + g$. No more than one dollar can be spent on each supporter.

We will later extend the analysis to negative campaigns, but for now we assume that candidate i can only target effectively (that is, add the to their excitement) for the voters who are initially inclined towards him or her.

The final two assumptions concern cost of voting and voting behavior: states have voting costs C_k, where to assure the possibility of positive turnout we assume that $C_k < d_i^h + g$ for all k.

The assumption on voters' behavior that we make is similar to that of "conditionally sincere voting" or "as if pivotal" voting that can be found in Alesina and Rosenthal (1995, 1996) and Caselli and Morelli (2004): voters turn out and vote for their favorite candidate *if and only if* their after-campaign perceived utility differential from the two candidates is greater than the cost of voting in their state.[9] In other words, each agent decides to turn out (not to turn out) if the benefit from doing so is higher (lower) than the cost in case his or her vote (or non vote) is decisive for the outcome.[10]

The game, then, consists of the following stages. First, candidates simultaneously choose which potential voters to target in their campaign in every state; then any supporters for whom $|d_i(\ell)| \geq C_k$ vote for their preferred candidate. The candidate receiving more delegates across the states wins the nomination and receives the benefit b; the other receives zero.

We search for *least cost participation equilibria (LCPE)*: A strategy profile of spending decisions by the candidates and turn-out decisions by the voters is an *LCPE* of the game if and only if (1) it is a subgame perfect Nash equilibrium and (2) there is no other subgame perfect Nash equilibrium that makes the two candidates obtain the same vote shares State by State spending less money in the campaign.[11]

ding outcomes, but the role of variance in intensity of preferences, our main focus, would not be altered.

[9] Note that here the cost of voting is a common cardinal measure of for example time needed to go to vote and participate in the election, since the subjective and personal or emotional things can all be lumped in the d parameters.

[10] Another way to describe this behavioral assumption is to say that voters are "minimizing potential regret".

[11] The second condition is basically a selection criterion, due to the fact that with infinite campaign funds there would obviously be multiple equilibria with different amounts of useless spending. This

10.2.1. State-by-state equilibrium

We first analyze equilibrium spending and voting patterns state by state.

Lemma 10.1. *In equilibrium (LCPE), candidates spend resources campaigning towards all voters ℓ in all states k for whom* $C_k > | d_i(\ell) | \geq C_k - g$.

Proof. Vote shares v_i^k for each candidate i in each state k increase with the number of voters ℓ casting ballots for the candidate. Therefore in order to maximize the vote share candidates will maximize their turnout in each state. Given our equilibrium refinement, candidates will spend resources only when necessary, so they campaign those voters who would not vote without campaigning but do vote with campaigning. These are exactly the voters for whom $C_k > | d_i(\ell) | \geq C_k - g$.

This characterization result allows us to establish the following proposition:

Proposition 10.1. *There exist voting costs* $\overline{C}$ *and* $\underline{C} < \overline{C}$ *such that candidate i wins in all States for which* $C_k > \overline{C}$, *candidate j wins when* $\overline{C} > C_k > \underline{C}$, *and the election is a tie when* $C_k < \underline{C}$.

Proof. Take $\overline{C}$ to be $d_j + g$. Then for all $C_k > \overline{C}$, candidate j cannot convince her voters to vote, so turnout in her favor is zero. Candidate i, on the other hand, can convince his high-intensity supporters to turn out, possibly by campaigning, since by assumption $C_k < d_i^h + g$.

Now take $\underline{C} = d_i^l + g < \overline{C}$. For all $C_k \in (\underline{C}, \overline{C})$, candidate j can now entice her supporters to vote, possibly by campaigning, so she gets full turnout, while candidate i only has his high-intensity supporters turn out. Thus j wins all elections in this range.

Finally, for $C_k < \underline{C}$, both candidates can generate full turnout, so all voters cast ballots and the election is a tie.

The proposition establishes a basic non-monotonic relation between the cost of voting and the electoral outcome: for high-cost states, the candidate with the greater number of intense supporters will do well, as more voters will, for instance, spend three, four, or more hours at a caucus to vote. In the middle range, candidate j does better, as her voters will all pay moderate costs to cast their bal-

---

selection criterion would probably be superfluous and we could simply work with subgame perfect Nash if we introduced costly fund raising.

lots. And at the low end of cost all voters go to the polls; as we assume that the overall support for the candidates is equal, these elections end in ties.

Table 10.1 illustrates the various possible combinations of voting, expenditures, and outcomes. It is constructed by noting that if a voter with intensity d_1 will vote for her candidate with no campaign expenditures, then another voter with intensity $d_2 > d_1$ will vote as well.

TABLE 10.1: Possible combinations of voters and campaign expenditures

Case	Who votes	*i* Pays	*j* Pays	Winner
1	d_i^h	0	0	i
2	d_i^h	d_i^h	0	i
3	d_i^h, d_j	0	0	j
4	d_i^h, d_j	0	d_j	j
5	d_i^h, d_j	d_i^h	d_j	j
6	d_i^h, d_j, d_i^l	0	0	tie
7	d_i^h, d_j, d_i^l	d_i^l	0	tie
8	d_i^h, d_j, d_i^l	d_i^l	d_j	tie
9	d_i^h, d_j, d_i^l	d_i^h, d_i^l	d_j	tie

The table reveals some interesting regularities regarding spending and voting patterns. First, in equilibrium, notice that the winning candidate spends weakly more funds getting their supporters to turn out than does the losing candidate.[12] This is true even though the cost of advertising is zero, so the losing candidate is refraining from spending only because he knows that it will not change the outcome.

Also, whenever the high-intensity candidate wins an election and spends money, he spends on his highest intensity supporters. This echoes the conventional wisdom in the primaries literature (e.g., Polsby 1983) that successful candidates "mobilize a faction", rather than appeal to a broad segment of the party faithful. Furthermore, in the elections where the outcome is a tie the high-variance candidate spends on his low-intensity supporters. So we have the counter-intuitive

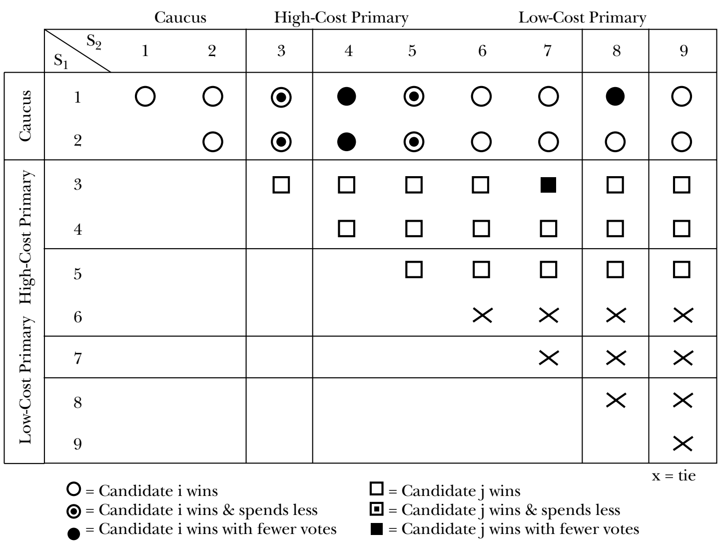
[12] That is, the winner spends more funds per state. However, as we show below, the overall winner in a multi-state election may still spend less overall than his opponent.

prediction that this type of candidate spends more funds in elections where he does less well; spending is a sign of weakness rather than strength in low cost of voting states.

10.2.2. Aggregating across states

Given these state-by-state results, how do elections aggregate voter preferences across states to produce outcomes? To investigate these issues, we consider two-state electoral contests, where each state is drawn from one of the categories defined in table 10.1. For ease of reference, cases 1 and 2 from table 10.1 are labeled "Caucus", cases 3 to 5 are labeled "high-cost primaries", and cases 6 through 9 are labeled "low-cost primaries".

FIGURE 10.1: Possible outcomes in two-state primary elections



As the table indicates, if at least one of the two states has a caucus, then candidate *i* wins the nomination. When both states hold primaries and at least one primary is of the high-cost variety, then candidate *j* wins. If both states hold low-cost primaries, the outcome is a tie.

Also indicated are whether the winning candidate spends less money than the losing candidate, and whether the winner receives fewer votes. The chart shows that there are seven cases in which the high variance candidate falls into one of these categories, as compared with only one for the low-variance candidate. We summarize these findings in the following proposition.

Proposition 10.2. *Assume that the election is contested in two states, which have costs of voting $C_1 > C_2$. There exist generic parameter values under which candidate i wins the election while (1) spending less money in the campaign; (2) losing the popular vote; and (3) focusing his campaign in the state with higher cost of voting.*

Proof. The proof is by example. Suppose

$$d_i^h + g > C_1 > d_j + g > C_2 > d_i^l + g$$

In this example candidate *j* can mobilize all her supporters in the state with the lower cost, but not in the other, while candidate *i* can effectively mobilize only the high intensity supporters but in both states. This contrast can be sufficient to generate the result: clearly *i* wins the state with high cost and candidate *j* wins the other state, but candidate *i* loses in state 2 by "less" (in percentage terms) than he wins by in state 1, hence he wins the elections. If $d_i^h > C_2 > d_j$ then he doesn't really have to spend money in state 2 while candidate *j* does, so he spends less money overall. Furthermore, when $2d_i^h < d_j$ he will win the primaries while losing the popular vote.

In the example above, the equilibrium in the campaign game is clearly unique and does not depend on the order in which the states are addressed nor on any sophisticated strategy. If the two states had equal cost of voting, namely both primaries or both caucuses, then the equilibrium would have to be symmetric, and the candidate with lower variance of intensities in her support could have an advantage if the cost of voting remains too high to mobilize the least intense supporters of the high variance candidate.

The key to candidate *i*'s success is that, although he may get the same or fewer votes than his opponent, in percentage terms he wins the caucus state by more than he loses the primary state. He therefore takes a majority of delegates due to the fact that the tail of his distribution of support is large compared to the tail of candidate *j*'s distribution.

10.2.3. Multiple states

The results above show that the high-variance candidate will have an advantage in caucus states and the low-variance candidate will win high-cost primaries, while low-cost primaries are a tie. To determine the winner of a multi-state election, let us assume that the N states are divided into N_c caucus states, N_h high-cost primaries, and $N - N_c - N_h$ low-cost primaries. Then overall outcomes are given in the following proposition:

Proposition 10.3. *The high-variance candidate wins the election (nomination) in a multi-state race iff* $N_h < N_c \frac{1+\alpha}{1-\alpha}$.

Proof. The high-variance candidate wins all N_c caucus states with 100% of the vote. The low-variance candidate wins each high-cost primary with $\frac{1}{1+\alpha}$ of the delegates, to $\frac{\alpha}{1+\alpha}$ for her opponent. So the low-variance candidate wins the overall election if:

$$\left(\frac{1}{1+\alpha} - \frac{\alpha}{1+\alpha}\right) N_h < N_c$$

$$N_h < N_c \frac{1+\alpha}{1-\alpha}$$

The high-variance candidate thus has a built-in advantage in terms of needing to win fewer states than the low-variance candidate to win the nomination. Equivalently, if the states were to have different populations then candidate i could offset a high-cost primary victory by candidate j in a state of size 1 by winning a caucus in a smaller state, one with population $\frac{1-\alpha}{1+\alpha}$.

It is also clear from these results that candidate i's utility is increasing in α, his proportion of high-intensity voters.[13] The reason for this is clear: candidate i will win the caucus states with 100% of the vote no matter what, and he will always tie the low-cost primaries. But the higher the value of α, the better he does in the high-cost primaries, and hence the more delegates he receives overall. We will return to the related incentive to endogenously create a larger set of high intensity supporters through strategic policy choices in the extensions section.

---

[13] If we want to keep condition (10.1) satisfied, this comparative statics exercise would have to be conducted by reducing accordingly d_i^h.

10.2.4. Negative campaigning

We now add the possibility that, in addition to the positive campaigning characterized above, either candidate can spend one dollar in a negative campaign, lowering the voter's intensity of preferences of one of her opponent's supporters by an amount h, which can be greater than, less than, or equal to g. For simplicity, assume $h = g$. All other elements of the game remain constant and, as before, our equilibrium selection criterion chooses the equilibrium with the lowest amount of spending.

Proposition 10.4. *In the game with positive and negative campaigns, positive campaign funds are spent on those voters for whom* $C_k \leq | d_i(\ell) | < C_k + g$. *Negative campaign funds are spent on those voters for whom* $C_k - g \leq | d_i(\ell) | \leq C_k$.

Proof. Notice first that no voter will be subject to both positive and negative campaigns in equilibrium. Suppose otherwise, so that both candidates i and j spend a dollar of campaign funds trying to sway the vote of a given voter ℓ. In equilibrium this voter will either vote for his preferred candidate or not. If he votes for his candidate, then the negative campaigning was superfluous; if he does not vote then the positive campaign was superfluous. In either case, our equilibrium selection criterion means that the candidate spending the superfluous dollar would stop doing so.

Next, consider a voter for whom $C_k + g > | d_i(\ell) | \geq C_k$, and assume that $d_i(\ell) > 0$, so that the voter supports candidate i. Here if i doesn't spend, j has a best response to spend in negative campaign. Hence i makes positive reinforcement campaign to avoid that, and no other subgame perfect Nash equilibrium exists where there is less spending and equal vote shares. If $C_k - g < d_i(\ell) \leq C_k$, then if j spends in negative campaign i cannot compensate, while if j doesn't spend on such a negative campaign i would target them effectively, hence the unique LCPE profile has negative campaign spending by j in this range. Finally, it should be clear that neither for intensities above $C_k + g$ nor for intensities below $C_k - g$ there can be any spending in equilibrium.

It is interesting to compare spending patterns in this equilibrium with the patterns in the game without negative campaigns. With only positive campaigns, candidates target their spending towards those voters with intensities $d_i(\ell)$ between $C_k - g$ and C_k in absolute value; that is, voters who support them but would not vote for them absent the campaign spending.[14] Voters with intensities above

[14] This equilibrium is similar to lobbyists' targeting those legislators who mildly oppose their proposals, as in Snyder (1989).

this range would vote in any case, and below this range they cannot be induced to go to the polls, even with campaigning.

On the other hand, with negative campaigns this pattern is reversed. Voters with intensities between $C_k - g$ and C_k are the subject of *negative* campaigns by the opposing candidate, while those with intensities between C_k and $C_k + g$ are the subject of positive campaigns; in the former case to insulate non-voters against possible positive campaigns, and in the latter to insulate voters against possible negative campaigns. Thus in equilibrium each candidate targets their message toward voters who are already inclined to take the action that the campaigning then reinforces. This would be important, for instance, in empirical studies of where campaign dollars are targeted; it would be tempting to conclude *ex ante* that campaign funds that do not change a voter's intended action are being misused.

10.3. Evidence from the 2008 Democratic primaries

The theoretical model above predicts that, under certain circumstances, primary returns for the candidate with a higher variance in the intensity of his support will show a non-monotonic relation relative to the costs of voting. In particular, the high-variance candidate should do best when the cost of voting is highest or lowest, and his opponent should do best in states with intermediate costs of voting.

We now examine whether these patterns are apparent in the 2008 Democratic primary election contest between Barack Obama and Hillary Clinton. To a first approximation, the requirements of the model are met: both have nearly equal levels of support in the public at large; as of this draft, Clinton led 50.1% to 49.9% in the popular vote. And Obama's followers for much of the primary season included both a set of ardent supporters, giving rise to the "Obama-mania" phenomenon, and independents, with perhaps a lower level of attachment to any Democratic candidate.

Some limitations are also apparent. First, we have no way to directly measure the intensity of support of Obama followers as opposed to Clinton's, and the latter certainly has her share of voters highly dedicated to her cause. Second, many primaries were contested with other candidates in the field, particularly John Edwards, who gathered a significant number of votes. How the Edwards voters would have cast their ballots were he not in the race is difficult to determine. And third, data availability limits us to testing only the predictions of relative voter support across states, rather than the predictions made on spending patterns.

The data used for this section comprise, first, the rules by which each state held its primary and/or caucus for the Democratic presidential nomination in 2008.[15] The contests differed by their method (primary or caucus), their eligibility requirements, and the minimum number of days prior to the contest that voters could register to participate. Both the method and registration rules are straightforward; the definitions of the eligibility categories are as follows:

— Open: An open contest is one in which any registered voter may participate.
— Closed: A closed contest is one in which a voter may only vote if he is registered with that political party (you must be a registered Democrat to vote in the Democratic contest and a registered Republican to vote in the Republican contest.)
— Modified: A modified contest is one in which voters may participate in if they are either registered with that party, or are a registered voter with no party affiliation (e.g. you may vote in your party's contest, or either party's contest if you are independent.)
— Affiliation Change: In a modified election, if you are a registered voter but not registered with a party, your party affiliation must be changed to the party of the primary you vote in.

From these categories, we create a variable "costrank" indicating in an ordinal manner how costly each contest is to participate in, from voters' point of view. We used the following rules:

1. All caucuses were deemed to be more costly than all primaries.
2. Within each category, closed contests were deemed to be the most costly, as they allow only party members to vote.
3. Within each category, modified contests with affiliation change came next, followed by modified contests without affiliation change, under the view that some psychic costs are involved for independents to declare themselves members of one party or the other.
4. Finally, within each category, came open contests.

The resulting set of costrank levels is given in table 10.2. As defined, of course, the distance between adjacent categories is equal, which is certainly a simplifica-

---

[15] These rules can be found on the web and are summarized in the Appendix.

tion of reality. In particular, the potential gulf between caucus and primary states is not captured directly in our costrank variable.

TABLE 10.2: Definition of "costrank" categories

Costrank	Method	Eligibility	Δ Aff?
1	Primary	Open	
2	Primary	Modified	N
3	Primary	Modified	Y
4	Primary	Closed	
5	Caucus	Open	
6	Caucus	Modified	N
7	Caucus	Modified	Y
8	Caucus	Closed	

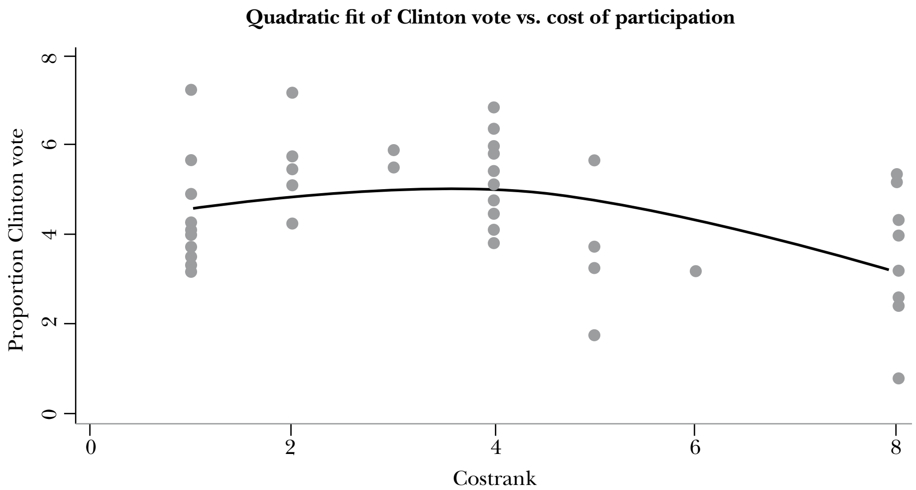
For each contest we also calculated the percent of the two-candidate Clinton vote as the number of votes for Clinton divided by the sum of the Clinton and Obama votes. Summary statistics for all variables are given in table 10.3, where the eligibility variable is coded as 1 for open, 2 for modified and 3 for closed.

TABLE 10.3: Summary statistics

Variable	Mean	Std. Dev.	Min.	Max.	N
Clinton vote	0.45	0.14	0.08	0.73	48
Caucus state	0.27	0.45	0	1	48
Eligibility	2.1	0.88	1	3	48
Days prior	22.33	10.3	0	38	46
Costrank	4.04	2.57	1	8	48

Our model predicts that the Clinton vote will be non-monotonic in the cost of voting in various primary and caucus contests. As a first look at the data, graph 10.1 shows a scatterplot and quadratic fit of Clinton's results. As predicted, Clinton does better in states with intermediate costs and worse at either end, although especially so in closed caucuses.

GRAPH 10.1: Relation between cost of participation and Clinton vote



We now subject the data to a simple regression analysis. Were one to naively approach the data as a set of independent variables, one might run a regression similar to that in Model 1 of table 10.4, treating each aspect of the caucus or primary contest separately. As shown, only the caucus vs. primary variable is significant here; the more specific type of contest (open, modified or closed) has no extra impact.

TABLE 10.4: Regression results

	(1)	(2)	(3)	(4)	(1)
Caucus state	–0.186 (0.041)***		–0.131 (0.053)**	–0.123 (0.06)**	
Eligibility	0.03 (0.019)			0.009 (0.029)	
Days prior	–0.003 (0.002)				0.014 (0.008)*
Days prior^2					–0.0004 (0.0002)*

TABLE 10.4 *(cont.)*: **Regression results**

	(1)	(2)	(3)	(4)	(1)
Costrank		0.053	0.061	0.054	
		(0.031)*	(0.03)**	(0.037)	
Costrank^2		–0.008	–0.007	–0.007	
		(0.003)**	(0.003)**	(0.003)*	
Obs.	46	48	48	48	35
R^2	0.338	0.249	0.339	0.341	0.086

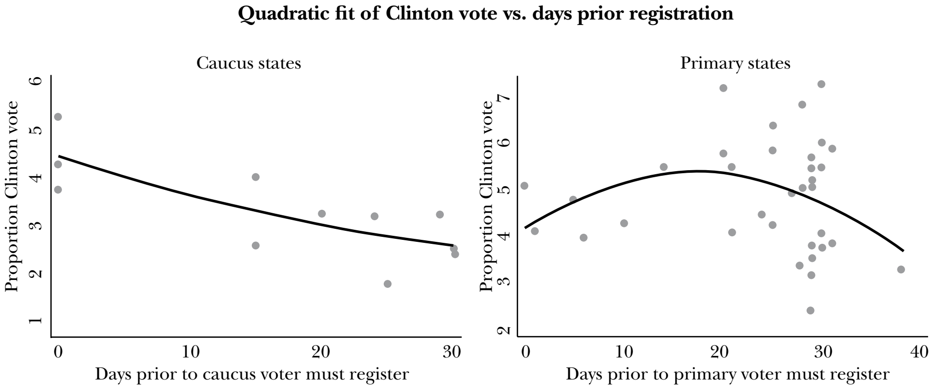
But our model cautions us to look for nonlinear impacts of these variables. For instance, for a caucus state, making the primary open rather than closed lowers the cost of participation and should thus favor Clinton. But in a primary state, lowering the cost of participation should help Obama. These considerations are wrapped up in our costrank variable, and Model 2 in the table looks for direct and quadratic effects of costrank alone.

As predicted, the quadratic term is negative and significant, while the linear term is insignificant. When one adds back in the caucus variable in Model 3, it is still significant and negative, indicating that, as hypothesized above, the linear formulation of costrank downplays the significant difference between primary and caucus states. And Model 4 shows that adding the eligibility requirements back in yields no additional explanatory power.

Various specification tests were performed to examine these results. No one case was classified as an influential outlier, as would be indicated by a Cook's distance greater than 1. Neither did any state have particularly high leverage, and collinearity concerns entered only in the expected high correlation between the costrank variable and its square.

We have not yet incorporated the days prior registration information to our analysis, as it is not clear how it relates to the elements of costrank. However, graph 10.2 shows a scatterplot of days prior and the Clinton vote for caucus and non-caucus states separately, with Clinton doing better in the low-cost caucus states and a curvilinear relation between cost and results in the primary states. Model 5 in the regression table, run for primary states only, confirms that this latter curvilinear relation is statistically significant, though not highly so.

GRAPH 10.2: Clinton vote vs. registration requirements



10.4. Extensions

If candidates benefit from the presence of high-intensity supporters, it stands to reason that they would invest resources to create them. One type of resource, campaign spending, is already present in the model. An extension of the model is to allow candidates to use policy positions to create intense supporters, even at the cost of a candidate's overall number of supporters. We thus examine a model of strategic candidate locations, where one candidate strategically separates himself from his opponent in order to create a cadre of high-intensity supporters.

Take a simple spatial model where voter ideal points are uniformly distributed in the [0,1] interval, voters have linear utilities, and the policy platform x_j of candidate j is fixed at $x_j > 1/2$.[16] Again assume a two-state competition with voting costs $C_1 > C_2$, ignore for the moment the possibility of advertising, and allow candidate i to pick any policy platform within the space.

Then traditional (costless) voting theory would predict candidate i would locate at x_j, or $x_j - \varepsilon$ for some arbitrarily small ε, thereby winning the election. In the presence of costly voting, though, such a strategy would leave the utility difference between the candidates at zero (or ε), and hence turnout would be zero as well.

[16] Candidate j may, for instance, have a longer track record of votes which defines her policy positions to a greater extent than her opponents'. We use linear preferences here to make the calculations simple, but our general point holds even more strongly when voters have strictly concave utility functions.

As long as $C_1 < 2x_j - 1$, candidate i can win the election by locating at $x_i = x_j - C_1$. Then voters with ideal points between 0 and x_i have utility difference $d_i = C_1$, as do voters between x_j and 1, so they all vote in state 1's caucus. But by construction $x_i > 1 - x_j$, so candidate i wins the caucus state, and he will win the primary state by the same margin as well. So candidates will strategically position themselves, separating from their opponents just to the point that their most fervent supporters will turn out in the states with the highest cost of voting.

This leads to another reason why real-world caucuses would favor an extremist candidate, given the actual distribution of voters within a party. Consider two candidates, like Obama and Clinton, symmetric about the median of their party, but one of whom (Obama) is more extremist in the policy space.[17] If the entire distribution of voters is single-peaked, or at least has tails on either end, then those voters within the Democratic party favoring Obama include the tail of the distribution, while those favoring Clinton consist of the voters between the median Democrat and the overall median voter in the country. This means that Obama will have more voters whose ideal points are distant from either candidate, and if preferences are strictly concave then these voters will have a higher intensity of preference in favor of Obama than even the most fervent Clinton supporters. If the cost of voting in a caucus is high enough, then, the extremist candidate has a built-in advantage.

10.5. Conclusion

This paper presented a model of primaries and caucuses with heterogeneous voters and different costs of voting across states. We derived a nonlinear relationship between costs of voting and election outcomes in an environment where two candidates have equal overall support, but one candidate has a higher variance of support than the other; more zealots, but more apathetic supporters as well. We showed that in a certain set of parameter values the high variance candidate will have an advantage over his rival, winning high-cost caucuses by more than he loses low-cost primaries. We analyzed patterns of electoral competition from the Clinton-Obama primary race in 2008 and found them to be consistent with the predictions of our model.

[17] Extremist in this context means relative to one's own party, so it denotes a relatively liberal Democrat or a relatively conservative Republican.

We found as well that candidates may strategically locate themselves in a policy space, differentiating themselves from their opponent, in order to galvanize their most intense supporters. The next question to ask is whether these candidates may have more difficulty appealing to the median national voter in what is, after all, a low-cost general election. This raises the questions regarding optimal electoral institutional design, and if differing the costs of participation at the electoral and primary stage makes sense. These issues are becoming increasingly relevant to real-world events, as the primary system is in a state of flux and may undergo significant revisions before the next presidential election cycle, and we leave these important questions to future work.

Appendix

TABLE 10.5: Primary contest rules

State	Date	Eligibility	Δ Aff?	Method	Registration	Days Prior
AL	5-Feb	Open		Primary	26-Jan	10
AK	5-Feb	Closed		Caucus	6-Jan	30
AZ	5-Feb	Closed		Primary	7-Jan	29
AR	5-Feb	Open		Primary	6-Jan	30
CA	5-Feb	Modified	N	Primary	22-Jan	14
CO	5-Feb	Closed		Caucus	7-Jan	29
CT	5-Feb	Closed		Primary	31-Jan	5
DE	5-Feb	Closed		Primary	12-Jan	24
DC	12-Feb	Closed		Primary	14-Jan	29
FL	29-Jan	Closed		Primary	31-Dec	30
GA	5-Feb	Open		Primary	7-Jan	29
HI	19-Feb	Closed		Caucus	5-Jan	30
ID	5-Feb	Open		Caucus	02-may	25
IL	5-Feb	Open		Primary	8-Jan	28
IN	6-May	Modified	N	Primary	7-Apr	29
IA	3-Jan	Closed		Caucus	3-Jan	0
KS	5-Feb	Closed		Caucus	21-Jan	15
KY	20-May	Closed		Primary	22-Apr	28
LA	9-Feb	Closed		Primary	9-Jan	31

TABLE 10.5 *(cont.)*: **Primary contest rules**

State	Date	Eligibility	Δ Aff?	Method	Registration	Days Prior
ME	10-Feb	Closed		Caucus	26-Jan	15
MD	12-Feb	Closed		Primary	14-Jan	29
MA	5-Feb	Modified	N	Primary	16-Jan	20
MI	15-Jan	Open		Primary	17-Dec	30
MN	5-Feb	Open		Caucus	16-Jan	20
MS	11-Mar	Open		Primary	10-feb	30
MO	5-Feb	Open		Primary	9-Jan	27
MT	3-Jun	Open		Primary	05-may	29
NE	19-May	Modified	N	Caucus	25-Apr	24
NV	19-Jan	Modified	Y	Caucus	19-Jan	0
NH	8-Jan	Modified	N	Primary	8-Jan	0
NJ	5-Feb	Modified		Primary	15-Jan	21
NM	5-Feb	Closed		Primary	8-Jan	28
NY	5-Feb	Closed		Primary	11-Jan	25
NC	6-May	Modified	N	Primary	11-Apr	25
ND	5-Feb	Open		Caucus	5-Feb	0
OH	4-Mar	Modified	Y	Primary	3-Feb	30
OK	5-Feb	Closed		Primary	11-Jan	25
OR	20-May	Closed		Primary	29-Apr	21
PA	22-Apr	Closed		Primary	24-Mar	29
RI	4-Mar	Modified	Y	Primary	2-Feb	31
SC	26-Jan	Open		Primary	20-Dec	38
SD	3-Jun	Closed		Primary	19-May	15
TN	5-Feb	Open		Primary	7-Jan	29
TX	4-Mar	Open		Mixed	4-Feb	29
UT	5-Feb	Modified		Primary	6-Jan	30
VT	4-Mar	Open		Primary	27-Feb	6
VA	12-Feb	Open		Mixed	12-Jan	29
WA	9-Feb	Modified	N	Caucus	4-Feb	5
WV	13-May	Modified	N	Primary	22-Apr	20
WI	19-Feb	Open		Primary	18-Feb	1
WY	8-Mar	Closed		Caucus	22-Feb	15

References

ALESINA A. and H. ROSENTHAL. *Partisan Politics, Divided Government and the Economy.* Cambridge University Press, 1995.

ALESINA A. and H. ROSENTHAL. "A Theory of Divided Government." *Econometrica* 64 (1996): 1311-1342.

ALI S. N. and N. KARTIK. *Social Learning in Elections.* Manuscript, University of California, San Diego, 2007.

BARTELS, L. M. *Presidential Primaries and the Dynamics of Public Choice.* Princeton, NJ: Princeton University Press, 1988.

BATTAGLINI, M., R. MORTON, and T. PALFREY. "Efficiency, Equity, and Timing in Voting Mechanisms." *American Political Science Review* 101 (2007): 409-424.

BORGERS, T. "Costly Voting." *American Economic Review* 94, no. 1 (2004): 57-66.

CALLANDER, S. "Bandwagons and Momentum in Sequential Voting." *Review of Economic Studies* 74 (2007): 653-684.

CASELLI, F. and M. MORELLI. "Bad Politicians." *Journal of Public Economics* (2004).

FEDDERSEN, T. and W. PESENDORFER. "The Swing Voter's Curse." *The American Economic Review* 86 (1996): 408-424.

FEDDERSEN, T. "Rational choice theory and the paradox of not voting." *Journal of Economic Perspectives* 18, no. 1 (2004): 99-112(14).

FEY, M. *Informational Cascades and Sequential Voting.* Working Paper, University of Rochester, 2000.

GERBER, E. and R. MORTON. "Primary election systems and representation." *Journal of Law, Economics, and Organization* 14 (1998): 304-324.

HELMKE, G. and B.M. MEGUID. *Endogenous Institutions: The Origins of Compulsory Voting Laws.* Manuscript, University of Rochester, 2008.

IARYCZOWER, M. *Strategic Voting in Sequential Committees.* Working paper, Caltech, 2007.

JACKSON, M., L. MATHEVET, and K. MATTES. "Nomination Processes and Policy Outcomes." *Quarterly Journal of Political Science* 2 (2007): 67–92.

LIJPHART, A. *Patterns of Democracy: Government Forms and Performance in Thirty-Six Countries.* Yale University Press, 1999.

LIZZERI, A. and N. PERSICO. "Why Did the Elites Extend the Suffrage? Democracy and the Scope of Government, With an Application to Britain's 'Age of Reform." *The Quarterly Journal of Economics* 119, no. 2 (2004): 707-765.

LOHMANN, S. "The Dynamics of Informational Cascades: The Monday Demonstrations in Leipzig, East Germany." *World Politics* 47, no. 1 (1994): 42-101.

MEIROWITZ, A. "Informational Party Primaries and Strategic Ambiguity." *Journal of Theoretical Politics* 17, no. 1 (2005): 107-136.

PALFREY, T. and D. LEVINE. "The Paradox of Voter Participation? A Laboratory Study." *American Political Science Review* (2006), forthcoming.

PALFREY, T. and H. ROSENTHAL. "Voter Participation and Strategic Uncertainty." *American Political Science Review* 79, no. 1 (1985): 62-78.

POLSBY, N. W. *Consequences of Party Reform.* New York: Oxford University Press, 1983.

SHUGART, M. and J. CAREY. *Presidents and Assemblies: Constitutional Design and Electoral Dynamics.* Cambridge University Press, 1992.

11

Electoral Incentives, Political Risk-Taking and Policy Reform

Alessandro Lizzeri
Department of Economics
New York University

Nicola Persico
Department of Economics
New York University

So unpredictable are the reactions of the people to governmental action that politicians prefer to be sure that if they act they will not find themselves far out on a limb."
V.O. KEY, *Public Opinion and American Democracy,* p. 424

It's hard to get people under normal circumstances to take risk, electoral risk. I think you have to take electoral risk to take on Medicare or Social Security to do it right.
U.S. SEN. BOB KERREY, (D.) Nebraska.[1]

11.1. Introduction

This paper provides a model of risk-taking by candidates in the choice of electoral platforms. We compare electoral systems with respect to the incentives they give candidates to take electoral risks, and provide a welfare analysis of the effects of risk-taking.[2]

The premise of this paper is that policy platforms are risky at the time when they are chosen, and so politicians often face a trade-off: whether to opt for bold policies, which carry greater electoral risk but offer potentially large rewards, or instead to choose safer policies. A memorable instance of a risky platform adoption in U.S. politics is the so-called "Contract With America," which went on to

[1] Cited from "The lesson: Anything's possible, even likely" by David Dahl. *The St. Petersburg Times,* February 13, 1999.

[2] In this paper, as in much of the literature on electoral competition, we use the term "party" and "candidate" interchangeably.

deliver a remarkable electoral upset in the 1994 congressional elections. At the time of adoption, the electoral appeal of this platform was viewed with extreme skepticism by many.[3] The trade-off between electoral risk and return has largely been overlooked in the formal political-economy literature.[4] This trade-off, and the way it is resolved across different political systems, is the focus of this paper.

Suppose we proxy electoral risk-taking by the variability of a party's vote share around its long-term trend. Then a look at cross-country data reveals a systematic relationship between electoral risk-taking and the "degree of proportionality" of the electoral system. The disproportionality index (Taagepera and Shugart 1989), measures the degree to which a party's vote share is reflected in the proportion of seats in the assembly.[5]

Figure 11.1 plots electoral systems according to their index of disproportionality (vertical axis) and the variability of the largest party's vote share over time (horizontal axis).[6] The figure shows that more proportional systems exhibit less variability in vote shares. Conversely, in systems that are closer to winner-take-all (plurality voting), we observe greater variability in vote shares.[7]

In this paper we analyze how the risk-return trade-off is resolved in electoral competition, and obtain comparative statics consistent with the relationship uncovered in figure 11.1. In the model, candidates choose the degree to which their platform embodies a "reformist" agenda, and they are uncertain about how voters perceive reformist platforms. More reformist platforms are more likely *on average* to be well received by voters—but they also carry a greater risk of being rejected by the electorate.

---

[3] "Some Democrats had greeted the Contract With America with enthusiasm, seeing it as a return to Ronald Reagan's policies and seeing that as a massive political blunder. [...] Many reporters and political commentators dismissed the Contract With America as a political promise, worth nothing on the day after Election Day." Thomas Donlan "Contract With America." *Barrons.* 74(46): 70. 1994 Nov 14.

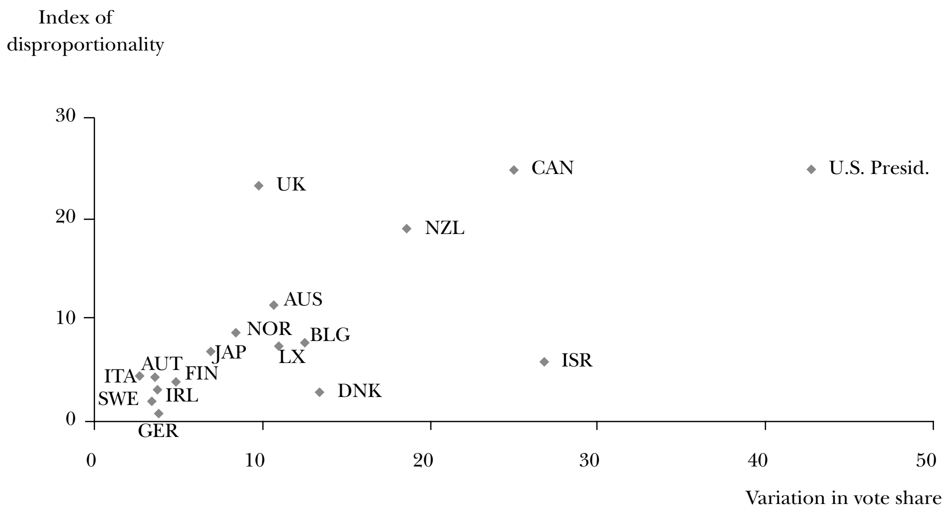
[4] The literature on incumbency advantage discusses the decision to run for office at the risk of not being re-elected (see Rohde (1979) for a discussion of the risk-taking attitudes of candidates). That literature takes electoral risk as given, and furthermore is not concerned with electoral institutions.

[5] Cox (1997) and Lijphart (1999) also discuss measures of proportionality of electoral systems. See Milesi-Ferretti, Perotti, and Rostagno (2000) for a discussion of the merits of different proportionality measures and an alternative way to measure proportionality.

[6] Note that the relation is between the degree of proportionality and the vote share of the largest party, *not* the share of seats captured by this party. The former is an endogenous consequence of voter's behavior and risk-taking by candidates, the latter is a mechanical consequence of the electoral system.

[7] Appendix B describes the index of disproportionality and the way in which we compute the variability of vote shares.

GRAPH 11.1: Electoral variability



We contrast the candidates' willingness to select risky platforms under alternative electoral incentives. We consider two extreme objectives for candidates: they either care only about winning or they want to maximize their vote share. We have previously argued (Lizzeri and Persico 2001) that these alternative objectives can capture some elements of the degree of proportionality of the electoral system. A very disproportional system leads to winner-take-all incentives where candidates only care about obtaining more than 50% of the votes. In an electoral system that is highly proportional, increases in vote shares translate into proportional increases in rewards to parties, such as seats in a legislature. We show that a winner-take-all system induces candidates to take on more electoral risk relative to a proportional system. This difference in risk-taking behavior stems from the different ways in which the variability in vote shares is evaluated in the two systems. In the proportional system, candidates maximize expected vote shares, which in our model translates into effectively maximizing the expectation of a concave function which is an increasing transformation voters' utility functions. Thus, in this system candidates endogenously inherit voters' risk attitudes; when voters are risk averse, policies that entail risk are discounted by the candidates. In contrast, in the winner-take-all system a candidate cares only about whether his vote share exceeds 50 percent. Filtering the vote share through this highly nonlinear (and discontinuous) function works to eliminate the effect of voters' risk aversion on candidates' payoffs. In particular, in the winner-take-all system a candidate is not concerned about the dispersion of vote share around the 50 percent mark, therefore the riskiness of a platform is not taken into account. This explains why riskier platforms will prevail under a winner-take-all system.

To highlight the effects of electoral incentives on candidates risk-taking behavior, we purposely assume that candidates are risk neutral with respect to the outcome of the election. This guarantees that, whatever attitudes the candidates have towards risky policies, they are not *assumed* by building them into the candidates' utility functions: rather, they are *derived* as the equilibrium outcome of the game with two different electoral systems. The assumption of risk-neutrality serves to highlight the fact that electoral systems have effects on risk-taking independent of any risk aversion on the part of candidates. Our results are robust to introducing risk aversion on the part of candidates (see Section 11.5.3). It is important to recognize the difference between the force we highlight and a logic based on a "loser-risk-all" attitude whereby a winner-take-all system encourages highly disadvantaged candidates to generate variability because, absent any variation in vote shares, they would lose with certainty. This loser-risk-all explanation for electoral variability is not necessarily compelling, because it ignores the behavior of the advantaged candidate. The advantaged candidate is harmed by variability and therefore will tend to choose platforms that minimize electoral risk. Once the behavior of the advantaged candidate is taken into account, it is not clear that a winner-take-all system will produce greater electoral variability. Our symmetric model, in contrast, does not rely on the presence of a disadvantaged candidate, and in equilibrium the incentive to take risk is higher in the winner-take-all system for *both* candidates.

An empirical implication of this model is that there is greater variability in the vote shares of parties competing under electoral systems that are highly disproportional. This implication is consistent with graph 11.1.

The fact that different electoral systems induce differences in the candidates' attitudes towards risky policies has implications for the welfare of voters. Two effects must be distinguished. First, the proportional system leads to platforms that reflect voters' risk aversion, whereas in the winner-take-all system this risk aversion is neutralized by the incentive structure for candidates and risk plays no role. Thus, if voters are very risk-averse, this effect will result in overly risky platforms in winner-take-all elections.

The second welfare effect relates not to risk aversion, but to the optimal variety of electoral platforms. In our model, risk-taking on the part of candidates results in variation among the realized platforms. Voters will choose between policies with widely disparate appeal. If we ignore risk aversion, voters benefit from the variety of offered platforms. This is because they get to vote for the best platform, the one with the most favorable realization. The worst policies do not get implemented because candidates supporting them do not get elected. Thus, voters do not bear the losses from the worst possible realiztions of a policy. However, when choosing the amount of risk in his platform, a candidate does take into account the negative consequences of having a very bad realization because these lead to very low vote shares. Thus, a wedge is driven between the politician's incentives to take electoral risk and the social value of risky platforms. If voters are not very risk averse, this wedge leads politicians to choose too little risk relative to the social optimum.

Following Hotelling (1929), we call this effect "(excessive) sameness." In the spatial model, the fact that both parties locate at the same (median) point in the policy space has been interpreted as a failure to offer voters any real choice among policies. However, this reasoning is not well grounded in a welfare criterion since it overlooks the fact that in elections—unlike in markets—the choice of each agent has an externality on all other agents since all voters "consume" the same policy. As a result, the equilibrium where both candidates position themselves at the median is typically ex-ante Pareto-superior to a situation where two candidates choose positions symmetrically around the median. Thus, contrary to Hotelling's intuition, sameness is not necessarily excessive in the spatial model.[8] In contrast, in our model, voters' preferences over policies are roughly aligned. Therefore, in our model "excessive sameness" has a meaningful welfare interpretation: voters are better off for being offered variety in their choice set. Thus, our model affords a formalization of the idea of "excessive sameness."[9]

In our analysis, the question of whether "sameness" is "excessive" depends on the electoral system as well as on the risk attitudes of voters. When voters are not very risk averse, both the winner-take-all and the proportional systems lead to insufficient differentiation of electoral platforms, and hence to excessive sameness. In this case, the winner-take-all system is preferable since it leads to riskier platforms, and so suffers less from excessive sameness.[10]

---

[8] See Myerson (1997) for this point.

[9] See Chan (2000), who independently proposes a different model that leads to the possible emergence of excessive sameness. Also, see Carillo and Mariotti (2001), which we discuss in the related literature.

[10] This property of our model contrasts with what happens in the spatial model when candidates have policy preferences and there is uncertainty about the ideal point of the median voter. In that model, the high-power incentives of the winner-take-all system generate more pronounced convergence to the median policy, and hence more sameness than a proportional system.

We view these findings as relevant for the issue of policy reform. As the quotations in the epigraph suggest, we frequently observe that major policy reform entails substantial electoral risk.[11] To the extent that major policy reforms are associated with a high degree of electoral risk, our findings can be interpreted as laying some initial groundwork for a comparative politics of reform. Although empirically difficult to assess, it is apparent that countries exhibit large differences in the speed with which reforms, and new policies, are introduced. This suggests that some political systems are more conducive than others to policy reforms. We propose that these differences may be due, at least in part, to the incentives provided by different electoral systems.

11.2. The model

Agents and endowments. There are two parties (or candidates), L and R. There is a continuum of voters of measure one.[12] Each voter is endowed with units of money.

Platforms: the "electoral portfolio". Candidates can choose to run on platforms that embody different degrees of risk. The least risky platform simply leaves voters with their endowment. Risky (reformist) platforms take the form of taxing voters some amount t in order to invest in a public project. The public project is produced using the technology $g(t)$ which is increasing, strictly concave, and differentiable. We assume $g'(0) > 1$, and $g'(\omega) < 1$.

To introduce electoral risk, we assume that the public project is perceived by the electorate as the realization of a random variable. If candidate L chooses a tax of t_L and promises to invest the proceeds in the public project, candidate L's **electoral portfolio** is perceived by voters to be

$$\omega - t_L + g(t_L) + t_L Z_L$$

where Z_L is a zero-mean, symmetrically distributed random variable. Candidate R's portfolio is similarly perceived. At the time of the election, voters observe the realization of Z_L and Z_R. However, when candidates commit to platforms, they regard the Z_i's as random variables. The random variable Z_i captures the candi-

[11] See e.g. Alesina and Drazen (1991); Fernández and Rodrik (1991); and Coate and Morris (1999). Roland (2000) surveys this literature.

[12] We build on the model studied by Lindbeck and Weibull (1987) and Dixit and Londregan (1996).

dates' uncertainty about how voters will perceive the public project offered by candidate *i*.

The tax t captures the intensity of reform. Pursuing the financial analogy, the term $g(t) - t$ can be interpreted as the expected return of a candidate's electoral portfolio, while the term tZ_i represents the risk for candidate i of choosing a portfolio with expected return $g(t) - t$. Observe that the electoral risk associated with reform increases with the intensity of reform t.

Let t^{RN} be such that $g'(t^{RN}) = 1$. We refer to this as the *risk-neutral* level of reform, because it is the level of reform that would be implemented by a (risk-neutral) decision-maker who maximizes the expected value of the electoral portfolio.

Voters' behavior. We assume that voters are heterogeneous, and we follow Lindbeck and Weibull (1987) in positing that the source of heterogeneity is an ideological bent: some voters ex-ante favor the ideological position represented by the L candidate, some favor the position represented by the R candidate. This could be due to some difference between the candidates along some other policy dimension, unrelated to the reform issue. One example could be the issue of abortion, with one candidate being pro-choice and the other pro-life. We implicitly assume that on this dimension, candidates' positions are exogenous.[13]

Following Lindbeck and Weibull we assume that voters' ideological preferences enter linearly in their utility function: Voter *i*'s utility if candidate j is elected is given by:

$$U\,[\omega - t_j \left(1 - Z_j\right) + g(t_j)] + x_j^i$$

where U is strictly increasing, concave, and differentiable. Thus, the voter's utility is composed of two elements: one is the enjoyment of the policy platform proposed by candidate j, and the other is the ideological utility of the voter if the position of candidate j is implemented. Denote

$$x^i \stackrel{\text{def}}{=} x_R^i - x_L^i$$

The number x^i captures the ideological slant of voter i. Voter i votes for candidate L if and only if

[13] We could allow for candidates to offer platforms along the ideological dimension as well. All that would be needed is that candidates are not strictly office seeking but care about the ideological dimension as well. As in Calvert (1983), we would obtain some divergence along the ideological dimension which is all that is needed for our results.

$$U[\omega - t_L(1 - Z_L) + g(t_L)] - U[\omega - t_R(1 - Z_R) + g(t_R)] > x_i \tag{11.1}$$

We assume that, for each voter i, the variable x^i is realized from a cumulative distribution function F with density f symmetric around zero. This implies that the distribution of ideology in the population as a whole favors neither candidate.

Given a pair of policy platforms t_L and t_R, the vote share of candidate L is given by

$$S_L(t_L, t_R) = F(U[\omega - t_L(1 - Z_L) + g(t_L) - U[\omega - t_R(1 - Z_R) + g(t_R)])$$

Observe that, from the candidates' perspective, this vote share is a random variable because Z_L and Z_R are random. As in Lindbeck and Weibull (1987), in order to guarantee existence of a pure strategy equilibrium, we assume that the vote share of each candidate is strictly concave in his action. A sufficient condition is that the function $F(U(p)) - U(q))$ be strictly concave in p, which we assume hereafter (see Lindbeck and Weibull (1987) for details).

Electoral Systems. We focus on an important way in which political systems differ. At one extreme are countries where policy decisions are made through a bargaining process in an assembly. In these more proportional systems, vote shares translate into considerable influence even if they fall short of plurality. Countries with more proportional systems include Italy and Belgium. At the opposite extreme, majoritarian systems are thought to favor the candidate with the highest share of the vote, in the sense that more power of policy setting is conferred upon that candidate; this seems to be the case in Britain, for example. In majoritarian systems, small vote shares do not translate into much influence. For the purpose of studying the electoral incentives of politicians, we project this institutional complexity onto one dimension, namely the rewards that accrue to vote shares. In the proportional system, vote shares are valued according to their size: politicians maximize the expected plurality. In contrast, in the winner-take-all system all the power goes to the candidate with the highest vote share, and candidates maximize the probability of winning.

Proportional System: candidates maximize their vote share.

Winner-Take-All System: candidates maximize the probability that their vote share exceeds 50 percent.

Of course, this characterization of electoral systems is a highly specific one and does not capture several additional distinguishing features of proportional systems such as coalition governments. See the conclusion for some discussion of this point.

This description does not specify how policy is implemented in each system. For the moment we assume that in both systems the policy of the candidate with larger number of votes is implemented. This assumption has the virtue of limiting the difference between the two systems to the incentives to politicians, thus highlighting the focus of our analysis. While this assumption is uncontroversial in the winner-take-all system, in the proportional system one may think of alternative rules for policy implementation, perhaps reflecting policy compromise. This issue is discussed further in Section 11.5.4.

11.3. The two-policies example

To generate intuition about the basic trade-off between electoral risk and return, in this section we discuss a special case where candidates can choose between only two policies, the status quo ($t = 0$) and reform ($t = 1$).[14] This is the simplest model in which this trade-off can be discussed. In the next section we analyze the general model in which candidates can choose the degree to which their platforms implement reform.

We want to show that reform is implemented more frequently (as a function of $g(1)$) in the winner-take-all system. In that system, if both parties choose $t = 1$ their expected payoff is 1/2. If candidate L deviates to offering $t = 0$ his expected payoff is

$$\Pr\left(S_L > \frac{1}{2}\right) = \Pr\left(F\left(U(\omega) - U(\omega - 1 + g(1) + Z_R)\right) > \frac{1}{2}\right)$$

$$= \Pr\left(U(\omega) - U(\omega - 1 + g(1) + Z_R) > 0\right)$$

$$= \Pr\left(g(1) - 1 + Z_R > 0\right)$$

Whenever $g(1) > 1$ this probability is less than 1/2, and therefore the unique equilibrium is that both candidates promise reform.[15]

Consider now the proportional system. We want to show that the condition $g(1) > 1$ does not guarantee that both candidates promise reform. In fact, we show that unless $g(1) > K > 1$, there exists an equilibrium where both candidates

---

[14] That one of the policies is $t = 1$ is simply a normalization, what matters is the value of $g(1)$.

[15] Analogously, when $g(1) < 1$ the unique equilibrium has both candidates promising the status quo.

promise (0, 0). To verify that this is the case, start from a (0, 0) strategy combination and suppose candidate L deviates and offers $t = 1$. Candidate L's payoff at this strategy combination is

$$E(S_L) = E\left(F(U(\omega - 1 + g(1) + Z_L) - U(\omega))\right)$$

The function $F(U(p)) - U(q))$ is concave in p by assumption, and so by Jensen's inequality we have

$$E\left(F(U(\omega - 1 + g(1) + Z_L) - U(\omega))\right) < F\left(U(\omega - 1 + g(1) + E(Z_L)) - U(\omega)\right)$$
$$= F\left(U(\omega - 1 + g(1)) - U(\omega)\right)$$

When $g(1) \leq 1$ the last expression is less than 1/2, showing that the deviation makes candidate L strictly worse off. In order to make a deviation profitable, $g(1)$ has to be larger than some $K > 1$. Whenever $1 < g(1) < K$, it is an equilibrium for both candidates to promise $t = 0$ in the proportional system but not in the winner-take all system.

This argument shows that in the proportional system candidates need a higher "expected rate of return" $g(1)$ in order to promise the risky policy. In the winner-take-all system, candidates are more likely to adopt risky policies because they only care about the average value of the reform policy relative to the status quo. In the proportional system, however, candidates also care about the dispersion of the distribution because, in order to maximize vote shares, candidates incorporate into their objective function the concavity of voters' utility functions.

11.4. The extent of reform

In order to study the question of the extent of reform, we now turn to the general case where candidates choose the degree to which their platforms incorporate reforms, i.e., they can choose any $t \geq 0$.

We now impose more structure on the Z_i's. We assume that $Z_i = Z \cdot X_i$ where Z, X_L, and X_R are independent random variables with zero mean and symmetric distribution. In addition, we assume X_L and X_R are identically distributed. Thus Z_L and Z_R are two identically distributed (but not necessarily independent) symmetric random variables. Note that each candidate's uncertainty about voters' preferences is the product of a component that is common to both candidates

(Z) and an idiosyncratic component (X_i). This formulation allows for the possibility that, even if candidates propose the same level of reform, voters' perceptions of the policy may depend on the identity of the candidate. This dependence on the candidate's identity could be due to residual differences between the proposed policies reflecting ideological components or some valence dimension.

Theorem 11.1. *Let $t \in [0, \infty]$. In the winner-take-all system, in the unique pure-strategy equilibrium, both candidates choose the platform $t^{WTA} = t^{RN}$ that maximizes the expected value of reform. In the proportional system, a symmetric equilibrium exists and in the symmetric equilibrium each candidate chooses platform $t^{PR} < t^{WTA}$. Thus, there is less reform in the proportional system.*

Proof. *Winner-take-all.* In the winner-take-all system, candidate L cares about the probability that his vote share exceeds 1/2. Thus, he solves

$$\max_{t_L} \Pr\left(F\left(U[\omega - t_L(1 - Z_L) + g(t_L)] - U[\omega - t_R(1 - Z_R) + g(t_R)]\right) > 1/2\right)$$

$$= \max_{t_L} \Pr\left(\omega - t_L(1 - Z_L) + g(t_L)] > \omega - t_R(1 - Z_R) + g(t_R)\right)$$

$$= \max_{t_L} \Pr\left(g(t_L) - t_L - (g(t_R) - t_R) > t_R Z_R - t_L Z_L\right)$$

First, observe that for any t_L and t_R the random variable $t_R Z_R - t_L Z_L = Z(t_R X_R - t_L X_L)$ has zero mean and is symmetric in view of Lemma 8 of the Appendix, because Z and $t_R X_R - t_L X_L$ are independent symmetric random variables with zero mean. We begin by showing that, in equilibrium it must be $g(t_L^*) - t_L^* = g(t_R^*) - t_R^*$. Suppose not. Then the equilibrium probability of winning for the candidates is not 1/2 (remember, for any t_R, t_L we know that $t_R Z_R - t_L Z_L$ has median zero). So, one candidate must win with probability smaller than 1/2, which is impossible in equilibrium.

Now consider a strategy combination t_L^*, t_R^* such that $g(t_L^*) - t_L^* = g(t_R^*) - t_R^*$. For any $\tilde{t} \geq 0$, $t_R^* Z_R - \tilde{t} Z_L$ is symmetric and has mean zero (hence zero median), so

$$\Pr\left(g(t_L^*) - t_L^* - (g(t_R^*) - t_R^*) > t_R^* Z_R - \tilde{t} Z_L\right) = \frac{1}{2} \tag{11.2}$$

We want to show that if t_L^*, t_R^* constitute an equilibrium then it must be $g'(t_i^*) = 1$. Suppose not, and to fix ideas, suppose $g'(t_L^*) > 1$. Denote $\tilde{t}_L = t_L^* + \Delta$ for some small Δ, so that $g(\tilde{t}_L) - \tilde{t}_L > g(t_L^*) - t_L^*$. Using equation (11.2) we have

$$= \Pr\left(g(t_L^*) - t_L^* - (g(t_R^*) - t_R^*) > t_R^* Z_R - t_L^* Z_L\right)$$

$$= \Pr\left(g(t_L^*) - t_L^* - (g(t_R^*) - t_R^*) > t_R^* Z_R - \tilde{t}_L Z_L\right)$$

$$< \Pr\left(g(\tilde{t}_L) - \tilde{t}_L - (g(t_R^*) - t_R^*) > t_R^* Z_R - \tilde{t}_L Z_L\right)$$

which contradicts the optimality of t_L^*. A symmetric argument holds in the case where $g'(t_L^*) < 1$. Thus, in equilibrium it must be $g'(t_L^*) = 1$ and, similarly, $g'(t_R^*) = 1$. The same argument shows that a strategy combination such that $g'(t_L^*) = g'(t_R^*) = 1$ is an equilibrium.

Proportional. In the proportional system, candidate L maximizes.

$$E\left(F\left(U[\omega - t_L(1 - Z_L) + g(t_L)] - U[\omega - t_R(1 - Z_R) + g(t_R)]\right)\right)$$

Denote $Y(t_L, Z_L) \overset{\text{def}}{=} \omega - t_L(1 - Z_L) + g(t_L)$. Notice that $Y(t_L, Z_L)$ is a concave function of t_L for all realizations of Z_L. Since $F(U(p) - U(q))$ is concave in p by assumption, it follows that $F(U(Y(t_L, Z_L)) - U(Y(t_R, Z_R))$ is concave in t_L. Thus, candidate L's objective function is a concave transformation of t_L, and so the first-order conditions identify the solution to the agent's problem. Any pair t_L^*, t_R^* that solves the first-order conditions constitutes a Nash equilibrium. The first-order conditions for an interior equilibrium are

$$E\Big[-(1 - Z_L) \cdot f(U(Y(t_L^*, Z_L)) - U(Y(t_R^*, Z_R))) \cdot U'(Y(t_L^*, Z_L))$$
$$+ g'(t_L^*) \cdot f(U(Y(t_L^*, Z_L)) - U(Y(t_R^*, Z_R))) \cdot U'(Y(t_L^*, Z_L))\Big] = 0 \quad (11.3)$$

Rewriting,

$$g(t_L^*) = 1 - \frac{E\left[Z_L \cdot f(U(Y(t_L^*, Z_L)) - U(Y(t_R^*, Z_R))) \cdot U'(Y(t_L^*, Z_L))\right]}{E\left[f(U(Y(t_L^*, Z_L)) - U(Y(t_R^*, Z_R))) \cdot U'(Y(t_L^*, Z_L))\right]}$$

The denominator in the above fraction is positive because $F(U(p) - U(q))$ is increasing in p. Thus, the sign of the fraction is determined by its numerator. We have

$$E\left[Z_L \cdot f(U(Y(t_L^*, Z_L)) - U(Y(t_R^*, Z_R))) \cdot U'(Y(t_L^*, Z_L))\right]$$

$$= E_Z\Big\{E_{X_R}\Big\{E_{X_L}\Big[Z_L \cdot f(U(Y(t_L^*, Z_L)) - U(Y(t_R^*, Z_R))) \cdot U'(Y(t_L^*, Z_L)) \mid X_R, Z\Big] \mid Z\Big\}\Big\}$$

$$= E_Z\Big\{ZE_{X_R}\Big\{E_{X_L}\Big[X_L \cdot f(U(Y(t_L^*, Z_L)) - U(Y(t_R^*, Z_R))) \cdot U'(Y(t_L^*, Z_L)) \mid X_R, Z\Big] \mid Z\Big\}\Big\}$$

$$= E_Z\Big\{ZE_{X_R}\Big\{Cov_{X_L}\Big[X_L \cdot f(U(Y(t_L^*, Z_L)) - U(Y(t_R^*, Z_R))) \cdot U'(Y(t_L^*, Z_L)) \mid X_R, Z\Big] \mid Z\Big\}\Big\}$$

Now, decompose the expectation with respect to Z into the sum of two parts, the part where $Z > 0$ and the part where $Z < 0$. When $Z > 0$ then $f(U(Y(t_L^*, Z_L)) - U(Y(t_R^*, Z_R)))\ U'(Y(t_L^*, Z_L)))$ is a decreasing function of X_L (recall that $F(U(p) - U(q))$ is concave in p), and so

$$Cov_X\Big[X_L \cdot f(U(Y(t_L^*, Z_L)) - U(Y(t_R^*, Z_R))) \cdot U'(Y(t_L^*, Z_L)) \mid X_R, Z\Big] < 0$$

Then, the expectation with respect to X_R is negative, and multiplication by a positive Z preserves the sign. Taking expectation over the values $Z > 0$ shows that the part of the expectation where $Z > 0$ is negative. When $Z < 0$ then $f(U(Y(t_L^*, Z_L)) - U(Y(t_R^*, Z_R))) \cdot U'(Y(t_L^*, Z_L)))$ is an increasing function of X_L, and the same reasoning shows that the part of the expectation where $Z < 0$ is also negative. In sum, the expectation with respect to Z has a negative sign. This shows that if t_L^*, t_R^* constitute a symmetric equilibrium, then $g'(t_L^*) > 1$ (and $g'(t_R^*) > 1$). Since $g'(t^{WTA}) = 1$, we have proved that if a symmetric interior equilibrium t^{PR} exists, it is smaller than t^{WTA}.

If a symmetric equilibrium is not interior then $t^{PR} = 0 < t^{WTA}$. Thus, it remains to show that a symmetric equilibrium exists. Set $t_L^* = t_R^* = t^*$ and suppose there is a $t^* > 0$ that solves equation (11.3). Then we are finished. Suppose there is no such t^*. Since the right-hand side of equation (11.3) is continuous in t^*, it must be that the right-hand side is either always positive or always negative for all $t^* > 0$. However we know that at $t^* = t^{WTA}$ the right-hand side is negative. Thus, it can only be that for all $t^* > 0$ the right-hand side is negative. But then continuity guarantees that at $t^* = 0$ the right-hand side is nonpositive, hence $t^{PR} = 0$ is an equilibrium.

The result that $t^{WTA} > t^{PR}$ can translate into testable implications on variability for the vote share, of the kind discussed in the introduction. To see why, observe that at a symmetric equilibrium t^*, candidate L's vote share equals

$$F(U[\omega - t^*(1 - Z_L) + g(t^*)] - U[\omega - t^*(1 - Z_R) + g(t^*)])$$

The variability of this expression is due solely to the fact that $t^* Z_L \neq t^* Z_R$. This effect is strongest when t^* is large. In particular, when t^* approaches zero this random variable converges in probability to its mean 1/2: in the limit there is no uncertainty and the vote shares equal 1/2. Therefore, when the value of t^{PR} is close to zero relative to the value $t^{WTA} = t^{RN}$ we are guaranteed that the variability of vote shares is greatest in the winner-take-all system.[16] This result is consistent with the stylized implication about the variability of vote shares that was presented in the introduction.

11.5. Welfare implications

Our welfare measure is the expected utility of voters, evaluated prior to the resolution of the uncertainty. Thus, we take an *ex-ante*, behind the veil of ignorance perspective. We argue that this is the appropriate perspective for the purpose of evaluating the performance of political systems: the electoral system is presumably put in place to last for several elections, and at the time of its selection, voters have imperfect information about their future preferences for policy. Observe also that the ideological motives of voters, which in principle should affect the computation of welfare, can (in equilibrium) be ignored. This is because the distribution F of ideology in the population is symmetric. So, while each voter's ideological bias may lead him to regret or rejoice the election of a particular candidate, in the aggregate the contributions of the ideological component cancel out. Consequently, in this setup the voters' ideological motives do not influence welfare comparisons between policies.

In order to analyze welfare, we need to take a stand on policy implementation. Our default model is that the implemented platform is the one of the winning candidate. This assumption puts both systems, the proportional and the winner-take-all, on equal footing in terms of policy implementation, so that any differences between the two systems are due solely to the different incentives to propose risky policies in the two systems. In Section 5.4 we extend the analysis to discuss the case in which the policies implemented in the proportional system represent some compromise between the winner's and the loser's platforms.

At an equilibrium in which candidates choose $t_1 = t_2 = t^*$, under our assumption the implemented policy is the same in either electoral system,

[16] Equilibria in which t^{PR} is close to zero relative to t^{WTA} arise when the function g is such that g' is close to 1 on $[0, \omega]$ (see equation 11.3)). In the extreme, when $g(t) \equiv t$ and so $g'(t) \equiv 1$, the proof of Theorem 11.1 yields that $t^{PR} = 0$ and t^{WTA} can be any value in $[0, \omega]$, so taking small perturbations $\tilde{g}$ of the g function such that $\tilde{g}' > 1$ on $(0, \omega)$ can ensure that $t^{WTA} = \omega$ while t^{PR} remains close to zero.

$$\omega - t^* + g(t^*) + t^* \max \{Z_L, Z_R\} \tag{11.4}$$

and the ex ante welfare is the utility associated with this portfolio. The value of t that maximizes the ex ante welfare depends on the statistical relationship between Z_1 and Z_2. We study the two welfare effects associated with an increase in the amount of reform. The first effect reflects the value of variety associated with reform. The second reflects the risk that reform entails. The first effect dominates if risk aversion is small, in which case both systems produce too little reform and the winner-take-all system is preferable to the proportional system. If risk aversion is large, the second effect is the more important and the winner-take-all system leads to excessively risky platforms.

11.5.1. Excessive sameness in both electoral systems

A key point about the implemented policy described by expression (11.4) is that, even though $E(Z_i) = 0$, $E \max \{Z_1, Z_2\} > 0$. This reflects the fact that voters vote on the *realization* of platforms and select the platform with the better realization. In this case, the value t that maximizes the expected value of (11.4) solves $g'(t) + E \max \{Z_1, Z_2\} = 1$ (take expected value of (11.4) and differentiate). This level is higher than $t^{WTA} = t^{RN}$, the reform level implemented in the winner-take-all system. Thus, from the point of view of risk neutral or mildly risk averse voters, the equilibrium level of reform is too low under both electoral systems. As explained in the introduction, this inefficiency results from the fact that individual candidates do not internalize the positive effect of offering variety to voters. We call this effect *excessive sameness* following Hotelling (1929).

11.5.2. Excessive risk-taking in the winner-take-all system

When voters are sufficiently risk averse, the level of reform t^{RN} implemented in the winner-take-all system entails excessive risk. To show this, we refer to expression (11.4) and show that if t^* were equal to t^{RN}, then slightly lowering t^* would be beneficial. A slight decrease in t^* below t^{RN} has two competing effects. The first is negative, due to the fact that the term $g(t^*) - t^*$ goes down–but this is a second-order effect because, by definition, we have $g'(t^{RN}) = 1$. The second effect is that reducing t^* reduces the effect of the variance of the random variable $\max \{Z_L, Z_R\}$. If voters are sufficiently risk averse this positive effect dominates. The dominance of this second effect shows that voters who are very risk averse find the level of reform excessive: in this case the winner-take-all system unambiguosly leads to excessive risk taking.

11.5.3. Candidates' risk aversion

One may wonder whether the conclusion of excessive risk-taking in the winner-take-all system reflects our assumption that, while voters are risk averse, candidates are risk neutral. This is not so: even risk averse candidates would choose $t^{WTA} = t^{RN}$. Indeed, the curvature of the candidates' utility function does not affect the equilibrium under the winner-take-all system because in that system there are only two outcomes that matter to candidates: victory and defeat.

In the proportional system, although the precise value of t^{PR} would depend on the shape of the candidates' utility function if candidates were risk averse, it would remain true that $t^{PR} < t^{WTA}$. Indeed, suppose candidates each had a risk-averse utility function $u(S)$ defined over their vote share. In this case their objective function would be

$$E\left[u(F(U[\omega - t_L\,(1 - Z_L) + g(t_L)] - U[\omega - t_R\,(1 - Z_R) + g(t_R)]))\right]$$

Denoting $\tilde{F}\,(x) = u\,(F(x))$ we can rewrite this as

$$E\tilde{F}\,(U[\omega - t_L\,(1 - Z_L) + g(t_L)] - U[\omega - t_R\,(1 - Z_R) + g(t_R)])$$

Now observe that the only property of F that was used in the proof of Theorem 11.1 is that $F(U\,(p) - U(q))$ is concave in p. If F has this property then *a fortiori* $\tilde{F}\,(\cdot) = u\,(F(\cdot))$ has the property, because u is concave. This shows that the proof of Theorem 11.1 goes through unchanged when candidates are risk averse over vote shares.

11.5.4. Policy implementation and compromise in the proportional system

So far, we have discussed the contrast between the winner-take-all and proportional systems only in terms of the different objectives that these systems induce in office-motivated candidates. In particular, we have assumed that in each system the implemented policy is that of the winning candidate. However, in reality, and especially in a proportional system, minority parties may have an influence on policy, and the final outcome may depend in a complicated way on the outcome of some post-election bargaining game. We now discuss two alternative models of post-election policy compromise.

The first version model is one of probabilistic compromise.[17] Consider a function $\pi(s) \rightarrow [0, 1]$. This function represents the probability that the policy proposed by candidate L is the implemented policy, as a function of candidate L's share of the vote s. We assume that the same weight π is assigned to the ideological component of the candidate's position.

We also assume that π is symmetric, differentiable, non decreasing in s, and that $\pi(1/2) = 1/2$. Assume further that this function is not constant everywhere. The function π may be thought of as a reduced form of a bargaining game between the two candidates.

We now show that given any such π, optimal behavior for voters is unchanged relative to our previous analysis. In order to see this, assume that candidate j proposes position t_j. Recall that voter i's utility if the candidate's entire platform is implemented is given by $U[\omega - t_j(1 - Z_j) + g(t_j)] + x_j^i$. With probabilistic compromise, the expected utility of a voter is:

$$\pi(s)\ (U[\omega - t_L(1 - Z_L) + g(t_L)] + x_L^i) + (1 - \pi(s))\ (U[\omega - t_R(1 - Z_R) + g(t_R)] + x_R^i)$$

Differentiating this expression with respect to s, we obtain

$$\pi'(s)\ ((U[\omega - t_L(1 - Z_L) + g(t_L)] + x_L^i) - (U[\omega - t_R(1 - Z_R) + g(t_R)] + x_R^i))$$

This expression represents the marginal effect of a vote by voter i on his expected utility in the election. Hence, voter i will vote for candidate L if this marginal effect is positive. Since $\pi'(s) > 0$, voter i will vote for candidate L if and only if

$$U[\omega - t_L(1 - Z_L) + g(t_L)] - (U[\omega - t_R(1 - Z_R) + g(t_R)] > x_R^i - x_L^i$$

This expression is the same as the one in equation (11.1) that describes voting behavior in the case in which the policy of the candidate with more than 50 percent of the votes is implemented with probability one.

Since voters' behavior is unchanged, we may conclude that the candidates' equilibrium strategies are unchanged relative to our previous analysis (in this Downsian model candidates only care about shares of the vote, and not about

[17] Fishburn and Gehrlein (1977) study the properties of a variety of notions of probabilistic compromise for a large class of voting games.

policy). Thus, under the proportional system, candidates will still choose policies that are less risky than under the winner-take-all system.

There is, however, a negative welfare consequence of this compromise. The compromise limits the ability of voters to choose the best policy on offer given that they observe the realization of the uncertainty concerning the value of policies. This is similar to the excessive sameness logic described above. Thus, the possibility of compromise can worsen the effectiveness of the proportional system relative to the winner-take-all system.

The second model of compromise we discuss involves a physical blending between the policies proposed by the two candidates. As before, consider a function $\phi(s) \rightarrow [0, 1]$. This function now represents the weight of the policy proposed by candidate L in the implemented policy, as a function of candidate L's vote share s. Thus, if candidate L chooses t_L and candidate R chooses t_R, and these platforms result in a vote share of s for candidate L, then the implemented policy is a compromise between the policies proposed by the two candidates represented by $t^{comp} = \phi(s)\ t_L + (1 - \phi(s))\, t_R$. In addition, the compromise is also implemented for the ideological component of policy (with the same weight).[18]

The same properties we assumed for the π function are now assumed for the ϕ function. The notion of physical compromise is different from probabilistic compromise in two ways. First, the welfare consequences are different. Second, voting behavior becomes much more complex. We will ignore the second aspect here to focus on the first. Thus, we assume that voting behavior is still described by equation (11.1). In the present context, this is equivalent to assuming sincere voting. Again, this assumption implies that the candidates' equilibrium strategies are unchanged relative to our previous analysis.

However, the implemented policy will now reflect this compromise. The compromise will have the following contrasting effects on welfare. On the one hand, as in the case of probabilistic compromise, there is a reduction in the ability of voters to choose the best policy on offer upon the realization of the values of the different policies. The new effect is that from an ex-ante perspective this policy compromise will provide some insurance against risk, insurance that is valuable to risk averse voters. The overall effect is clear if in the absence of compromise the proportional system leads politicians to be too "cautious" in their choice of a risky platform. In this case this bias is exacerbated by the presence of compromise. More generally however, whether the overall effect of compromise on wel-

---

[18] Grossman and Helpman (1996) proposed a similar notion of policy compromise to study a different issue.

fare is positive or negative depends on voters' preferences and on the specific form of the compromise function.

Clearly, these two ways of thinking about policy compromise are only examples of the many possible ways to think about post-electoral bargaining. In our context an interesting possibility is that such bargaining might lead to efficient policy choice. This efficient choice may result because the election outcome might resolve some of the uncertainty that politicians have about voters' preferences. However, such analysis would be quite complex.

11.6. Related literature

The differences between majoritarian (winner-take-all) and proportional systems have been addressed by a number of authors. Aranson et al. (1974) argue that in a symmetric environment these two systems lead to the same equilibrium outcomes. Lindbeck and Weibull (1987) reach similar conclusions in a different (but symmetric) model of electoral competition. In the absence of aggregate uncertainty, if a deviation does not pay in the proportional system, then it does not increase the vote share; hence, this deviation cannot lead to an increased probability of winning, and thus cannot be beneficial in the winner-take-all system (see Aranson et al.). Thus, absent uncertainty (a key ingredient in our model), the two systems yield the same equilibrium outcomes. Snyder (1989) is concerned with the effects of electoral incentives on redistribution across districts. Departing from the assumption of symmetry, Snyder shows that, while in a proportional system swing districts receive more resources, this feature need not be true in a winner-take-all system. Taken together, these papers show that systematic differences between the two systems are not easy to find. In our paper, electoral uncertainty implies a difference between the proportional and winner-take-all electoral systems even in a completely symmetric world, where voters are ex ante identical and neither candidate has an electoral advantage.

Lizzeri and Persico (1999) compare proportional and winner-take-all systems in terms of public project provision. In that paper, what generates the difference between the two systems is the lack of targetability of the public project, relative to transfers. In the present paper all policies are equally non targetable. Thus, in contrast with our previous paper, the relative targetability of policies plays no role. Additional work on the comparison between majoritarian and proportional systems has been done by Persson and Tabellini (1999). They construct a model of redistributive politics *à la* Lindbeck and Weibull in which a majoritarian system generates less public project provision than a proportional system. Persson and Tabellini also examine cross-country data from around 1990 and find weak support for the prediction that majoritarian elections are associated with less public projects.

Milesi-Ferretti, Perotti, and Rostagno (2000) consider an alternative model and empirically examine the relationship between the degree of proportionality of electoral systems and the composition of public spending. They find that the share of transfers in GDP is positively related to the degree of proportionality and, in contrast with Persson and Tabellini, find a negative relation between the degree of proportionality and spending on public goods.[19] Case (2001) empirically contrasts winner-take-all and proportional systems of incentives in their ability to explain the pattern of electoral redistribution in Albania.

Coate and Morris (1995) consider the effect of asymmetric information on the choice of platforms. In their model, it is citizens who are less informed than politicians. This leads politicians to prefer means of redistribution that, although inefficient, are not clearly perceived by voters to be inefficient. In our model it is politicians who, at the time they commit to their platform, are uncertain about how voters will perceive the platform.

Austen-Smith and Banks (1988) compare a three-candidate proportional system with a two-candidates winner-take-all system, in the context of a model of spatial competition. They find that, under plurality rule, both candidates adopt the policy preferred by the median voter, while under proportional representation the equilibrium electoral platforms are symmetrically distributed around the median. The government is formed between the candidate that adopts the median position (which receives the fewest votes) and one of the other parties. The policy outcome that emerges from the legislative process is a compromise between the platforms of these two parties and is different from the median voter's preferred policy.

Carillo and Mariotti (2001) present a model where parties select candidates to run in an election. Candidate quality is uncertain; however, electoral campaigns serve as signals of this quality. Therefore, at election time, voters have better information about candidate quality than parties do when they select the candidates. They discuss the potential asymmetries that result if incumbents are better known than challengers. They only consider a winner-take-all system and furthermore, in their setup a proportional system would lead to the same outcomes, so our basic question could not be addressed in their model. However, a point of similarity between the two models is that in their context excessive conservatism arises for reasons similar to our excessive sameness result.

---

[19] The two papers look at different data and have different definitions of public goods.

Finally, our model is distinct from models of electoral ambiguity (see Shepsle (1972) and, more recently, Aragones and Postlewaite (1999)). In those models, candidates purposely leave voters in a state of uncertainty about the policy that the candidate will implement once in office. In our model, at election time voters are not uncertain about the value of the policy proposed by a given candidate.

11.7. Conclusion

In this paper we have analyzed an important aspect of political campaigning, namely, the risk-taking behavior of political candidates. We have suggested that electoral systems differ in terms of variability of the vote share that parties receive in elections, and we have ascribed this difference to the candidates' risk-taking behavior. We have shown that two different electoral systems, the proportional and the winner-take-all, give candidates different incentives for electoral risk-taking. The proportional system penalizes policies of uncertain popular appeal. In contrast, risky policies are not penalized in the winner-take-all system. This difference in incentives has interesting welfare consequences, which we have discussed.

We view our findings as indicating that different electoral systems offer different incentives for the adoption of risky reform projects. In this paper, we have pointed out one dimension of the political environment which can determine the ease of introducing reforms. Thus, one interpretation of our analysis is that it offers a basis for a comparative political analysis of reform with a focus on the specific rules of the electoral contest. In recognizing the importance of uncertainty and in emphasizing the role of political constraints in the implementation of reform, we follow the existing literature. In this literature the uncertainty is equally perceived by candidates and voters and it involves the ultimate effects of reform. While we take a somewhat different perspective, focusing on the uncertainty that comes from the imperfect information that candidates have about the electorate, our conclusions do not rest on the simplifying assumption that voters know their value for each policy at the time of voting. Indeed, our model could be reinterpreted to allow voters to be uncertain about the future value of policy platforms, in which case voters would choose based on their expectations. This is to say that the forces on which we focus should survive changes in the fine details of the information structure, so long as the key assumption is preserved that candidates, at the time that they choose their platforms, know less about the voters' preferences than the voters themselves will know at election time.

One special feature of our model is that it assumes the ability to commit to policies, and ignores any ideological consideration in the politicians' choice of risky platforms. We believe that this stylized representation captures some important aspects of electoral competition. Other authors have discussed the efficiency of democracies when politicians are motivated by ideological considerations (see Besley and Coate (1998), and Dixit and Londregan (1998)).

An important limitation of our analysis is that we restrict attention to elections with two candidates. This has some appeal because it shows that some interesting differences across electoral systems may arise even in the case of two candidates. Our analysis shows that proportional and winner-take-all systems can generate different outcomes just from the different electoral incentives that are generated for candidates. In Section 5.4 we showed that some differences in post-election policy implementation across the two systems can be incorporated in our model. However, we do not capture the richness of coalitional politics that takes place in real electoral systems.

An interesting aspect of elections with more than two candidates can be discussed in the context of our welfare analysis. In our model, a beneficial effect of having many (N) candidates is the expansion in the choice set available to voters. Now we would have an analogue of Equation (11.4) that involves a maximum among N random variables instead of just two, thereby increasing the value of the realized policy. Thus, a first order effect of increasing the number of candidates is beneficial. Of course, the equilibrium choices of candidates will be affected by the number of competitors and by the details of the electoral system.

Appendix

Technical Lemma

The proof of Theorem 11.1 uses the following property of symmetric zero-mean random variables.

Let X_1 and X_2 be two independent symmetric random variables with zero mean. Then

1. their weighted sum $t_1 X_1 + t_2 X_2$ is distributed symmetrically around zero for all values of t_1; as a consequence, $\Pr(X_1 < X_2) = \Pr(X_1 - X_2 < 0) = 1/2$.
2. their product $X_1 \cdot X_2$ is distributed symmetrically around zero.

Proof: (1) The result is immediate if we show that $X_1 + X_2$ is distributed symmetrically. To this end, denote by f_i the density of X_i; because X_i is symmetric around zero, $f_i(x) = f_i(-x)$. By the convolution formula, the density of $X_1 + X_2$ is $f_{1+2}(x) = \int f_1(x-z) f_2(z)\, dz = \int f_2(x-z) f_1(z)\, dz$. Then we have

$$\begin{aligned} f_{1+2}(-x) &= \int f_1(-x-z) f_2(z)\, dz \\ &= \int f_1(x+z) f_2(z)\, dz \\ &= \int f_1(y) f_2(y-x)\, dy \\ &= \int f_1(y) f_2(x-y)\, dy \\ &= f_{1+2}(x) \end{aligned}$$

(2) We have

$$F_{1\cdot 2}(x) = \Pr(X_1 \cdot X_2 \leq x) = E_{X2}\left(F_1\left(\frac{x}{X_2}\right)\right) = \int F_1\left(\frac{x}{y}\right) dF_2(y)$$

so

$$f_{1\cdot 2}(x) = \int \frac{1}{y} f_1\left(\frac{x}{y}\right) dF_2(y) = \int \frac{1}{y} f_1\left(-\frac{x}{y}\right) dF_2(y) = f_{1-2}(-x)$$

The index of disproportionality for each electoral system (the vertical axis in graph 11.1) is the index D from Taagepera and Shugart (1989), p. 106, 107. This index measures the deviation from a perfectly proportional system in which the proportion of seats in the assembly exactly equals each party's vote share. Algebraically, we have $D = (1/2) \sum |s_i - v_i|$, where s_i and v_i denote the percentage of seats in the assembly and votes, respectively, for party *i*, and the summation ranges over all parties. See Taagepera and Shugart (1989) for a discussion of this index.

The vote shares are taken from Mackie and Rose (1991), and refer to the period from 1950 (inclusive) to 1985. Extending the analysis to later elections would be desirable, but would necessitate altering the index of proportionality for those countries (such as Italy) which have had changes in the electoral system after the publication of Taagepera and Shugart (1989).

The variability in vote share is computed by taking, for each country, the largest party and computing the variability of its vote share over time relative to a linear trend. Almost always there is a party which has the largest vote share consistently throughout the period. The parties for each electoral system are:

Country	**Party**
Australia	Labor
Austria	Socialist
Belgium	Christian Socialist[20]
Canada	Liberal
Denmark	Social Democratic
Finland	Social Democratic
Germany	CDU/CSU
Ireland	Fianna Fail
Israel	Labour
Italy	Christian Democratic
Japan	Liberal Democratic
Luxenbourg	Christian Social
New Zealand	National
Norway	Labor
Sweden	Social Democratic
UK	Conservative

For example, for Italy we have the following pattern of vote shares over time.

Election year	**Vote share DC**
1953	40.1
1958	42.4
1963	38.2
1968	39
1972	38.7
1976	38.7
1979	38.3
1983	32.9

To compute the variability of the vote share, we regress the vote share on the election year, and take the sum of squared residuals of the regression. This is a measure of the variability around a trend. Dividing by the number of elections

[20] Includes Christian People's Party from 1968.

provides the appropriate normalization. This is the number that is reported on the horizontal axis of graph 11.1.

The R^2 value of the regression of the variability in vote shares on D is 0.31.

One may be concerned that the variability in vote share appears to be larger in winner-take-all systems simply because these systems have larger parties. We performed the same exercise by computing a "normalized" index of variability by dividing the previously obtained variability index by the average vote share for each party. The results are somewhat weaker in this case but the qualitative features are preserved.

References

ALESINA, A. and A. DRAZEN,. "Why Are Stabilizations Delayed?." *American Economic Review 81*, no. 5 (1991): 1170-1188.

ARAGONES, E. and A. OSTLEWAITE. *Ambiguity in Election Games*. Mimeo, University of Pennsylvania, 1999.

ARANSON, P. H., M. J. HINICH, P. C. ORDESHOOK. "Election Goals and Strategies: Equivalent and Non-equivalent Candidate Objectives." *American Political Science Review 68*, no. 1 (1974): 135-152.

AUSTEN-SMITH, D. and J. BANKS. "Elections, Coalitions, and Legislative Outcomes." *American Political Science Review 82*, no. 2 (1988): 405-422.

BESLEY, T. and S. COATE. "An Economic Model of Representative Democracy." *Quarterly Journal of Economics 112*, no. 1 (1997): 85-114.

—. "Sources of Inefficiency in a Representative Democracy: a Dynamic Analysis." *American Economic Review 88*, no. 1 (1988): 139-156.

CARILLO, J. and T. MARIOTTI. "Electoral Competition and Politician Turnover." *European Economic Review* 45 (2001): 1-25.

CASE, A. "Election goals and income redistribution: Recent evidence from Albania." *European Economic Review* 45, no. 3 (2001): 405-423.

CHAN, J. "The Under- and Over-Reaction to Private Information in Political Competition" manuscript, Johns Hopkins University (2000).

COATE, S. and S. MORRIS. "On the Form of Transfers to Special Interests." *Journal of Political Economy 103*, no. 6 (1995): 1210-1235.

—. "Policy Persistence." *American Economic Review 89*, no. 5 (1995): 1327-1338.

COX, G. W. *Making Votes Count: Strategic Coordination in the World's Electoral Systems*. Cambridge: Cambridge University Press, 1997.

DIXIT, A. and J. LONDREGAN. "The Determinants of Success of Special Interests in Redistributive Politics." *The Journal of Politics 58*, no. 4 (1996): 1132-1155.

—. "Ideology, Tactics, and Efficiency in Redistributive Politics." The *Quarterly Journal of Economics 113*, no. 2 (1998): 497-529.

FERNÁNDEZ, R. and D. RODRIK. "Resistance To Reform: Status Quo Bias in the Presence of Individual-Specific Uncertainty." *The American Economic Review 81*, no. 5 (1991): 1146-1155.

FISHBURN, P. C. and W. V. GEHRLEIN. "Towards a Theory of Elections with Probabilistic Preferences." *Econometrica 45* (November 1977): 1907-1923.

GROSSMAN, G. and E. HELPMAN. "Electoral Competition and Special Interest Politics," *Review of Economics Studies* (April 1996).

HOTELLING, H. "Stability in Competition." *Economic Journal 39* (1929): 41-57.

HUANG, C. and R. H. LITZENGERGER. *Foundations of Financial Economics.* New York: North Holland, 1988.

KEY, V. O. *Public Opinion and American Democracy.* New York: Alfred Knopf Inc., 1961.

LINDBECK, A. and J. WEIBULL. "Balanced-Budget Redistribution as the Outcome of Political Competition." *Public Choice 52*(1987): 273-297.

LIJPHART, A. *Patterns of Democracy: Government Forms and Performance in Thirty-Six Countries;* New Haven: Yale University Press, 1999.

LIZZERI, A. and N. PERSICO. "The Provision of public projects under Alternative Electoral Incentives." *American Economic Review* (March 2001).

MACKIE, T. T. and R. ROSE. *The International Almanac of Electoral History,* third edition. Washington, DC: Congressional Quarterly Press, 1991.

MILESI-FERRETTI, G-M., R. PEROTTI, and M. ROSTAGNO. *Electoral Systems and the Composition of Public Spending.* Mimeo Columbia University, 2000.

MYERSON, R. "Analysis of Democratic Institutions: Structure, Conduct, Performance." *Journal of Economic Perspectives* 9, no. 1 (1995): 77-89.

OSBORNE, M. and A. SLIVINSKI. "A Model of Political Competition with Citizen-Candidates." *Quarterly Journal of Economics 111,* no. 1 (1996): 65-96.

PANTZALIS, C., D. A. STRANGELAND and H. J. TURTLE. "Political Elections and the Resolution of Uncertainty: The International Evidence." *Journal of Banking and Finance* 24 (2000): 1575-1604.

PERSSON, T. and G. TABELLINI. *Political Economics and Public Finance.* Mimeo, forthcoming in the *Handbook of Public Economics,* 1998.

—. Alfred Marshall Lecture "The Size and Scope of Government: Comparative Politics with Rational Politicians." *European Economic Review 43, no.* 4-6 (1999): 699-735.

PERSSON, T., G. ROLAND and G. TABELLINI. "Comparative Politics and Public Finance." Mimeo (1997), Institute for International Economic Studies, Stockholm.

RODRIK, D. "The Positive Economics of Policy Reform." *The American Economic Review 83,* no. 2 (1993): 356-361.

ROHDE, D. "Risk-Bearing and Progressive Ambition: The Case of Members of the US House of Representatives." *American Journal of Political Science 23,* no. 1 (1979): 1-26.

ROLAND, G. *Transition and Economics: Politics, Markets, and Firms.* MIT Press, Cambridge, MA, 2000.

SHEPSLE K. A. "The Strategy of Ambiguity: Uncertainty and Electoral Competition." *American Political Science Review 66* (1972): 555-568.

SNYDER, J. "Election Goals and the Allocation of Campaign Resources." *Econometrica 57,* no. 3(1989): 637-660.

TAAGEPERA, R., and M.S. SHUGART. *Seats and Votes: The Effects and Determinants of Electoral Systems.* New Haven: Yale University Press, 1989.

12

The Choice of Political Institutions

Josep M. Colomer
Spanish National Research Council (CSIC)

12.1. Introduction

Different political institutions can be judged as being better or worse for the degree of political satisfaction or utility they produce to the citizens. Political utility can be estimated for the inclusiveness of citizens in the participation processes and the fit between policy-making decisions and citizens' preferences. From this perspective, institutions favoring the diffusion of power, such as universal suffrage, multiple governments and institutions dealing with different issues, and multiple political parties with opportunities to access or share power, can be considered relatively good to the extent that they create wider opportunities for people's participation and influence in decision-making than those favoring the concentration of power into a single government, group or political party.

At the same time, the assumption that people seek their own interest not only when making private or public policy decisions, but also when choosing the institutional rules for making those decisions, is broadly shared and analytically fruitful. Institutional choosers often aim at putting levers of rule at their easy disposal in order to concentrate, rather than check power. The crucial point is, however, that a socially efficient institutional design can result from circumstances in which no actor has sufficient influence to impose its own project and diverse ambitions counterweight each other. Not surprisingly, this is a relatively frequent situation in a complex world, which may explain why major institutional choices are increasingly made in favor of formulas able to produce the diffusion of power and to satisfy broad groups of people.

The diffusion of power can thus be both a criterion for good governance and a prudent choice by power-seeking actors. In the current world, as we will see in this paper, the number of small, relatively homogeneous communities increases; the number of democracies also increases; institutional choices tend to favor the division of powers rather than concentration into a single body or party; and electoral rules are increasingly chosen to permit multiple parties to participate in and share government. As actors' self-interested behavior leads to broadly efficient and satisfactory institutional choices, it seems that a kind of "invisible hand" in the field can be identified.

A basic model of institutional choice based on people's self-interested motives can be valid both for the transition between non-democratic and democratic regimes and for institutional change and reform within democratic regimes, as is illustrated in this paper. The model basically assumes that political actors, whether they have strong policy or ideology motivations or not, are interested in power. Then, in situations of uncertainty regarding the future, political actors tend to prefer institutions promoting power-sharing or likely alternation in power rather than risky formulas creating permanent absolute winners and absolute losers.

More specifically, a situation of uncertainty appears when the incumbent rulers are challenged by demands launched by new groups. If the existing institutional formulas permit only the absolute victory of one actor at the expense of all the others, whether the winner is defined as a social class, an ethnic group, or a political party, the incumbent rulers risk becoming absolute losers. The emerging challengers may feed expectations of becoming new absolute winners and of replacing the incumbent rulers under the existing institutions. Yet if some degree of uncertainty regarding future outcomes is shared by the challengers, they may also prefer institutions favoring the diffusion of power. Changes in favor of broader diffusion of power include the broadening of voting rights, the creation of several polities from a previous single government, the division of powers between central and territorial governments, the introduction of separate elections for the presidency and the assembly able to produce multiple winners, and the adoption of proportional representation inducing power sharing by multiple parties in parliaments and coalition governments.

The subsequent institutional formulas can reinforce themselves. The very key actors whose existence is viable under the existing institutional framework tend to support those institutions and resist the introduction of adverse changes. If the institutional formulas favor or permit the emergence and survival of multiple actors accessing power positions, they can obtain wide, endogenous support, and generate resistance toward changes favoring the concentration of power.

In the rest of the paper I survey worldwide, long-term tendencies in major institutional choice and design which can be interpreted according to this view. Specifically, in successive sections I review the number and size of the countries in the world, the number of democracies, the political regime formulas chosen for democratic experiences, and the choice of electoral rules, as well as some connections between choices in the different fields.

12.2. The number of countries increases

In recent worldwide developments, the classical building of large sovereign states has mostly been replaced by a proliferation of small countries. Specifically, while there were only 50 independent countries in the world in 1870 and about the same number in 1900, there are 192 members of the United Nations in 2008.

The creation of small political communities can be evaluated for their benefits and costs. Regarding the benefits, it is old knowledge that small communities can be more appropriate than large, populous territories for soft, democratic forms of self-government. In large states, local majorities can become state-wide minorities and see their preferences rejected from binding collective decisions. In contrast, the proliferation of small communities increases the number of people whose preferences become collective decisions.

Regarding the costs for small countries, they have decreased dramatically during the last few decades. Specifically, small countries can be economically viable in an international context of free trade. Indeed increasing trade has developed since the mid-twentieth century in large areas of the world. Reductions in the costs of transport, especially by air, and of communications, especially by telephone and the internet, have greatly favored this development. The new institutional setting also favors stability, including the International Monetary Fund and the World Trade Organization, as well as numerous regional transnational agreements and a few common currencies.

All this makes the traditional protection of markets by relatively large states less necessary and even insufficient. Indeed small countries tend to develop more foreign trade relative to their domestic product than large units. The political implication is that a small territorial unit seceding from a large empire or state can be economically viable if, once separated, its individuals and companies can maintain the same amount of external trade, including with traders in its former regional counterparts (Colomer 2007).

12.3. The number of democracies increases

Democratic regimes have been widely diffused across the world and at an increasing pace during the last few decades. In the late nineteenth century, competitive elections to legislative assemblies were regularly held in only nine of about 50 empires and states existing at the time, an area inhabited by less than ten percent of the total population. In contrast, by the early twenty-first century, democracy characterized by high levels of civil liberties and competitive elections in which both men and women vote exists in 90 countries. This is the highest number ever and represents almost half of the 192 currently recognized countries, inhabited by almost half (48 percent) of the world's population. (Data revised and updated from Colomer 2003, partly based on Polity IV and Freedom House series).

There is a positive correlation between the spread of democracy and the increase in the number of independent countries, which implies a decrease in their size, as well as with the concurrent decentralization of large states and empires. At the beginning of the twenty-first century, there is democracy in almost all recognized micro-countries with less than 300,000 inhabitants, in more than two thirds of those with less than one million inhabitants (including the former group), and in more than one half of all small countries with less than 10 million inhabitants (including the two former groups). In contrast, only one third of large countries with more than 10 million inhabitants enjoys democratic regimes. As a result, the number of small democracies is twice the number of large democracies (specifically there is democracy in 63 of 111 countries with less than 10 million inhabitants and in 27 of the 81 countries with more than that population).

12.4. Divided and multiparty governments proliferate

The spread of democracy is linked not only to changes in country size, but also to choices of democratic institutional formulas. Specifically, democratic regimes tend to endure when they adopt institutional formulas favoring divided and multiparty governments rather than a concentration of power in a single political party.

Updated calculations show that of all attempts to establish a democratic regime (including elections by adult male suffrage) in countries with more than one million inhabitants since the nineteenth century, those having initially adopted the British model of parliamentarism with majority electoral rule have survived in only 37 percent of cases, while the rate of success for parliamentarism with proportional representation is 72 percent, and for presidential and semi-presidential regimes 54 percent (with high variance in duration). In a long-term perspective, while the number of democratic regimes with parliamentary-majority rules has stalled and their proportion has declined, those with either parliamentary-proportional formulas or division of powers have surged (updated from Colomer 2003).

By the early twenty-first century, out of 64 democratic regimes in countries with more than one million inhabitants, only one sixth are parliamentary regimes with majority electoral rules, while one third are parliamentary regimes with proportional representation, and one half are presidential or semi-presidential regimes.

12.5. Proportional representation expands

More specific institutional choices involve the rules for assembly elections. In a global perspective they have evolved from indirect elections to direct elections by majority rule and from these to mixed systems and proportional representation rules, thus steadily enlarging the potential basis for participating and power-sharing groups.

Since the nineteenth century, we have counted 82 major changes of assembly electoral system in 41 countries with more than one million inhabitants. In consistency with the discussion above, we observe that more than 80 percent of these changes have been in the direction of more inclusive formulas. Specifically, indirect assembly elections decreased and virtually disappeared in the early twentieth century. The appeal of majority rule, which was the basic formula in the few democratic countries existing in the late nineteenth and early twentieth centuries, was replaced by proportional representation, especially after the First World War. This trend has intensified in recent democratization processes. Mixed systems have also spread in the more recent period, mostly as a result of changes from non-democratic regimes or majority rule. Nowadays, most democratic countries with more than one million inhabitants use electoral systems with proportional representation rules (Colomer 2004).

12.6. Conclusion

It is not unfounded to assume that the choice of political institutions is usually driven by politicians' and would-be rulers' ambition, the search for power, and calculations, estimates or expectations about the likely consequences of different institutional formulas to favor choosers' self-interest. However, as we have seen in the previous pages, the outcomes of such endeavors tend to be relatively favorable to formulas restricting the opportunities for a high concentration of power and permitting the broad satisfaction of people's preferences and demands. Specifically, the institutional evolution during the last decades have moved in favor of producing small countries, more democracies, division of powers, and electoral rules favoring multiparty representation. In spite of, or precisely through actors' self-interested behavior, institutional choices seem to be guided by an 'invisible hand' favoring relatively acceptable solutions.

Of course, all of this is based on long-term tendencies and is positively tested with only average values for large numbers of cases. For single-case analyses, we should take into account that many specific decisions and reforms are embedded in larger sets of institutional choices, thus entailing some trade-offs between different levels and sets of rules. For instance, federalism or territorial representation in very large countries with diverse population may work as a substitute for proportional representation. By giving different homogeneous, territorially-based groups opportunities to enter institutions, a major electoral reform can be prevented. As another example, the introduction of direct presidential elections may open a new opportunity for electoral contest. But it may also constrain the degree of multipartism in the assembly because presidential elections are always submitted to some majority rule and thus foster polarization. Specific processes may not be, therefore, as linear as the big numbers presented in this paper may suggest. All in all, however, it seems that actors' self-interested behavior tends to lead to some positive evolution of institutions.

References

COLOMER, J. M. *Political Institutions: Social choice and democracy.* New York and Oxford: Oxford University Press, 2003.

—. *Handbook of Electoral System Choice.* London: Palgrave-Macmillan, 2004.

—. *Great Empires, Small Nations. The uncertain future of the sovereign state.* London: Routledge, 2007.

GASTIL R.D. and A. KARATNUCKY. *Freedom House. Freedom in the World: The annual survey of political rights and civil liberties.* New Brunswick: Transaction, 1972-2008. Available at www.freedomhouse.org.

MARSHALL M. G., K. JAGGERS and T. R. GURR. *Polity IV: Political Regime Characteristics and Transitions, 1800-2003.* Available at www.systemicpeace.org/polity/polity4.htm

13

The Returns to U.S. Congressional Seats in the mid-19th Century

Pablo Querubin
Department of Economics
Massachusetts Institute of Technology

James M. Snyder, Jr.
Departments of Political Science and Economics
Massachusetts Institute of Technology

13.1. Introduction

An extensive literature in political economy stresses the importance of conflicts of interest between elected representatives and their constituencies. The main concern is that elected representatives, once in office, may use their political power to redistribute resources on rents to themselves or to favor certain interest groups in return for bribes or campaign contributions. These models tend to predict inefficient and/or distorted policies. Such rents may also be inconsistent with the protection of property rights and a level playing field that provide correct incentives for innovation and investment (arguments at the heart of institutional theories of comparative development).[1]

In these models incumbent politicians typically capture some of the rents in equilibrium. The rents might be small if the political environment is highly competitive and politicians do not have any special information, but otherwise they should be substantial. Thus, one way to assess the magnitude of political rents is to track the wealth of politicians. To the degree that rents are large, we should

[1] The literature includes Barro (1973); Ferejohn (1986); Banks and Sundaram (1993, 1998); Harrington (1993); Persson, Rolland and Tabellini (1997, 2000); Fearon (1999); Barganza (2000); Hindriks and Belleflamme (2005); Le Borgne and Lockwood (2001, 2006); Smart and Sturm (2003, 2004); Besley (2006); and Padro i Miquel (2007); as well as Stigler (1971); Peltzman (1976); Denzau and Munger (1986); Austen-Smith (1987); Baron (1994); Grossman and Helpman (1994, 1996, 2001); and Persson and Tabellini (2000).

observe politicians accumulating substantially more wealth while in office than they would have otherwise.

Even if the returns accruing to politicians do not imply any specific inefficiency or distortion, estimates of these returns may help assess arguments about the "quality" of politicians and the effects of quality on policy, as in Caselli and Morelli (2004); Messner and Polborn (2004); and Mattozzi and Merlo (2006, 2007, 2008). Finding that those with political power tend to accumulate wealth more than others would also help us understand the persistence of elites and the reproduction of political and economic power (e.g., Dal Bo; Dal Bo and Snyder, 2009).

In this paper we use historical census data from the U.S. to estimate the returns to holding a seat in the U.S. House of Representatives during the 1850's and 1860's. We focus on the northern states, and on representatives who served during the period 1845 to 1875. The U.S. census recorded wealth in 1850, 1860, and 1870, and we have found the individual census records of a large sample of representatives. We then employ a simple "before-and-after" design. For example, we compare the accumulation of wealth between 1860 and 1870 for representatives first elected during the five years just before 1870 with those first elected during the five years just after 1870. The first group had access to congressional rents that would appear in their 1870 wealth, while the second group did not. We describe this in more detail below, and also discuss possible weaknesses in the approach.

We find no evidence of large returns to congressional seats for the 1850's or late 1860's. We do find evidence of significant returns for the early 1860's. We are tempted to speculate that the returns to a seat in the House were low during "normal" times in the mid-19th century, but increased when federal government spending expanded sharply during the Civil War years. At a minimum, the *absence* of any evidence of large returns during the 1850's and late 1860's calls into question the frequent claims by politicians, journalists, and reformers at the time, as well as some later historians, that this was an extraordinarily corrupt era in U.S. politics.

We are in the process of extending this work to the south, and to other political offices—U.S. Senators, governors, and mayors, as well as top bureaucratic posts. We are also collecting data on individuals who run for office but *lose*. These provide an excellent control group, especially those who lose by a small margin. In future work we will employ a regression-discontinuity design that compares those who narrowly won office to those who narrowly lost. This will yield estimates that are arguably subject to less bias than those reported here. This comes at a cost of course—we must collect much more data. Moreover, finding records of census wealth for losers is more difficult than for winners, because there is less biographical information to help in matching.

Our paper contributes to a small but recently growing literature on estimating the value of public office. In another historical paper, Acemoglu et al. (2008) find that in the Colombian state of Cundinamarca, between 1879 and 1890 an additional year in power was associated with an additional 50 percent increase in the value of land. However, given that politicians may differ from non-politicians in many other respects, a naive comparison of politicians and non-politicians may confound the causal effect of politics with the effects of unobserved characteristics. Eggers and Hainmueller (2008) collect probate records of candidates to the British Parliament, and use a regression-discontinuity design to estimate the effect of holding a seat in parliament on wealth at death. They find significant positive effects for Conservative MPs but not for Labour MPs. Three papers study congress in the current era. Lenz and Lim (2009) use reported assets of U.S. members of Congress, matched with a sample from the Panel Study of Income Dynamics, and finds that members of Congress do not have higher asset returns than their matched counterparts. Using different methodologies, Groseclose and Milyo (1999) and Diermeier, Keane and Merlo (2005) estimate the returns to a career in the U.S. Congress. These papers cannot distinguish between the monetary returns to office and other sources of value, such as "ego rents." Also, they can only estimate the returns of a seat in Congress at the intensive margin, because they have no data on those who run and lose. Finally, in a study of the Ukraine, Gorodnichenko and Peter (2007) examine the difference between consumption expenditures and income for public sector employees relative to the difference for similar private sector employees, and estimate that public officials receive bribes of at least 1 percent of GDP. Our paper improves on these in some respects, but, of course, it has other limitations.

13.2. A corrupt Era?

In the second half of the 19th century, the United States was a "developing" nation, or at least an industrializing one. And by most accounts, U.S. politics at the time was highly corrupt. Railroads paid bribes for massive land grants and loans, steamship companies paid for lucrative mail routes, construction companies paid for canal contracts, and manufacturers and public utilities of all sorts paid for high tariffs and monopoly privileges. Politicians helped war profiteers sell shoddy goods to the government at inflated prices during the Civil War. Gross conflicts of interest involving officeholders were common and unpunished. Public officials sold a wide variety of services, including aid in obtaining appointments to military academies, assistance in lobbying for war claims and Indian claims, and tips about when the government was planning to sell gold. The spoils system dictated the distribution of government jobs. Electoral fraud was widespread. The press was partisan or bought off or both. Bosses increasingly dominated politics in major cities and some states. Simon Cameron summed up the political ethics of the era nicely with his famous line: "An honest politician is one who, when he is bought, will stay bought."

Reformers at the time identified two key problems: (1) politicians were no longer drawn from the pool of "the best men," and (2) as a result they treated politics simply as a way to make money for themselves and their friends. For example, *Harper's Weekly* lamented that "men of property and intelligence" had surrendered power "to men inferior in every proper recommendation... who follow politics just as any other money-making business." The magazine went on to criticize "the pecuniary corruption omnipresent in our Legislative Halls, which controls land grants and steamer contracts, and is incarnated in that gigantic corruption-fund, the public printing." The *Cincinnati Enquirer* described politicians as a "class of inferior men who have come out of public stations far richer than they went into them." Even Ralph Waldo Emerson railed against the "class of privileged thieves who infest our politics... those well dressed well-bred fellows... who get into government and rob without stint and without disgrace."[2]

Many later scholars agree with these claims. Summers (1987) writes, "In every way the decade before the Civil War was corrupt. The 1850's were as depraved as any other age, and, at least from the evidence available to historians, far more debauched than the 1840s" (page 14).[3] Writing about the events of 1857, Stampp (1990) notes, "Corruption was not a new phenomenon in American politics... but corruption had become distressingly common in this period of accelerating

---

[2] James Bryce's description in *The American Commonwealth* is even more colorful: "A statesman of this type [ward politician] usually begins as a saloon or barkeeper, an occupation which enables him to form a large circle of acquaintances, especially among the 'loafer' class who have votes but no reason for using them one way more than another... But he may have started as a lawyer of the lowest kind, or lodging-house keeper, or have taken to politics after failure at store-keeping... They are usually vulgar, sometimes brutal, not so often criminal... Above them stand... the party managers, including the members of Congress and chief men in the State legislatures, and the editors of influential newspapers... What characterizes them as compared with the corresponding class in Europe is that their whole time is more frequently given to political work, that most of them draw an income from politics and the rest hope to do so, and that they come more largely from the poorer and less cultivated than from the higher ranks of society" (page 64-66).

[3] Summers goes on to argue that corruption was a factor leading to secession. In particular, it helped bolster the arguments of both abolitionists and Southern Rights men. The former argued that corruption enabled the "Slave Power" to dominate the national government. It achieved its goals, especially the extension of slavery into the territories, by bribing weak and venal northerner politicians. The latter argued that "only disunion could keep the South from being infected with Northern corruption, just as revolution had freed the colonists from the contagion of British practice in 1776" (page 290). Greenberg (1985) makes similar arguments.

commercialization and industrial growth" (page 30). He explains the growth as follows: "Most of the financial corruption resulted from the temptations dangled before politicians by land speculators, railroad promoters, government contractors, and seekers after bank charters or street railway franchises. Often the politicians were themselves investors in western lands, town properties, railroad projects, or banking enterprises, and the distinction between the public good and private interests could easily become blurred in their minds" (page 28). The administration of Ulysses S. Grant is considered by many historians to be the most corrupt in U.S. history, and the post-Civil War period has been dubbed "the era of good stealings." In his discussion of the scandals of the Grant administration, Josephson (1938) argues, "It is high time that we cease to think of the spoilations of the General Grant Era as 'accidental' phenomena, as regrettable lapses into moral frailty... We must turn rather to examine the systematic, rational, organized nature of the plundering which was carried on at the time" (page 127).[4] Sproat (1968) argues that most liberal reformers in the late 1860's longed for a bygone era when politicians were statesman and gentlemen—"men of unbending integrity, 'sturdy independence,' and unimpeachable honesty" (page 50). They viewed the typical politician of the post-Civil War era as "a slave to organizational tyranny and a pawn of special interest" (page 51).

In spite of these widespread claims about political corruption, there is little systematic evidence that politicians in this period did indeed abuse political power for their own economic benefit. We will now provide some.

13.3. Data and estimation strategy

13.3.1. Theoretical and methodological issues

The main problem underlying the estimation of the returns of a seat in Congress is self-selection into politics. The decision to become a politician is influenced by a series of personal characteristics like talent or ability that are plausibly correlated with other personal outcomes such as economic success. Hence, a naive comparison of wealth accumulation by politicians and non-politicians will confound the causal effect of politics and the effect of other personal characteristics we may be unable to measure or observe[5].

---

[4] For a revisionist view, see Summers (1993).

[5] On the one hand, highly talented individuals may find holding office especially costly since they must sacrifice high returns in the private sector. If so, then a simple comparison of wealth accumulated

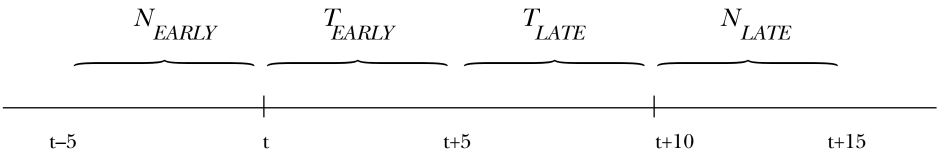
To estimate the causal effect of political office-holding on wealth accumulation one could use a regression discontinuity design based on close elections. The identifying assumption is that the outcome of very close elections is random and hence we can assume that any differences in wealth accumulation between close winners and close losers can be attributed to politics. This approach however, requires detailed information on both the winners *and* losers of congressional races. We are currently collecting data on all candidates to the U.S. Congress between 1840 and 1875 and will report the results of the regression discontinuity approach in future work. In this paper however, we report the results of a simple "before-and-after" design that relies solely on data for individuals who actually won and served.

Figure 13.1 below illustrates our approach. Suppose we can observe the wealth of members of Congress at two different years t and $t+10$. We can then create indicator functions to classify all members of congress who served in the years around this period. Let N_{EARLY} be an indicator function that takes a value of 1 for all members of Congress that served *only* during the 5 years preceding t and zero otherwise. Similarly, T_{EARLY} takes a value of 1 for members of Congress that served only during the 5 years following t and zero otherwise. We can also define similar indicator functions for congressmen who served around $t+10$. That is, T_{LATE} takes a value of 1 for all those who served only in the 5 years preceding $t+10$ and zero otherwise while N_{LATE} takes a value of 1 for congressmen who served only during the 5 years after $t+10$ and zero otherwise. We can use these indicator functions to get a rough estimate of the returns to serving in Congress in the early and late part of the decade under consideration. For example, to get an estimate of the returns to Congress in the late 1860's we can compare the accumulation of wealth between 1860 and 1870 for representatives that only served during the five years just before 1870 (i.e. all congressmen for which $T_{LATE}=1$) with those that only served during the five years just after 1870 (i.e. all congressmen for which $N_{LATE}=1$). The first group was "treated" by politics—had access to congressional rents that would appear in their 1870 wealth—while the latter group was not. Similarly, we can get an estimate of the returns from Congress during the early 1860's by comparing the accumulation of wealth between 1860 and 1870 for those individuals that only served during the five years just after 1860 (i.e. those for which $T_{EARLY}=1$) with those that only served during the five years just before 1860 (those for which

by politicians and non-politicians would tend to *underestimate* the rents from politics. On the other hand, if only the most talented individuals, who would have been very successful in the private sector anyway, manage to win elections and become politicians, then a naive comparison of politicians and non-politicians will tend to *overestimate* the rents from holding office.

$N_{EARLY} = 1$). In this case however, while only the latter group was treated by politics between 1860 and 1870, we need to consider the possibility that politicians obtain returns from Congress not only while in office but also after they have served.

FIGURE 13.1: Before and after timing



To motivate our regression framework and have a better understanding of the magnitudes we are estimating consider the following process for the accumulation of wealth of a given individual i at time t:

$$\frac{dW}{dt} = [r + r_d\, d(t) + r_a\, a(t) + X'\beta]\; W(t) + [R_d\, d(t) + R_a\, a(t)] \qquad (13.1)$$

Equation (13.1) distinguishes between two types of returns to serving in Congress—those that increase the returns on existing initial wealth, and more direct payoffs in monetary units that are independent of the politician's initial wealth. The first term shows the different factors that may affect the returns on *initial* wealth. The r corresponds to the market rate of return to which all individuals have access. The r_d corresponds to the additional return that politicians get during the time in which they are holding office. An individual only enjoys these additional returns when $d(t) = 1$, where $d(t)$ is an indicator function for whether the individual is holding office at time t. The r_d may be related to the better investment opportunities to which congressmen may have access while in office due to privileged information on the financial or real estate markets. The r_a corresponds to the additional return that politicians may enjoy *after* leaving office. This may reflect networks or connections that politicians are able to enjoy once they leave office. A congressman only enjoys this additional return when $a(t) = 1$, where $a(t)$ is an indicator function for whether the individual is out of office at time t (after having served). Finally, $X'\beta$ captures all other individual characteristics that can affect the returns an individual gets on his stock of wealth, such as occupation, age, and initial wealth (if we believe there is mean reversion). The second term in (13.1) captures direct payoffs that congressmen may get—such as direct bribes or side-payments—that increase their wealth directly and are independent of their initial wealth. The R_d corresponds to the direct payoffs a politician receives at time t while in office, while R_a correspond to the direct payoffs politicians may enjoy after leaving office.

Dividing both sides of equation (13.1) by $W(t)$ yields:

$$\frac{dW}{dt} \;/\; W(t) = \frac{d\ln(W(t))}{dt} = [r + r_d\; d(t) + r_a\; a(t) + X'\;\beta] + [R_d\, d(t) + R_a\, a(t)]/W(t) \qquad (13.2)$$

In the above analysis we modeled the evolution of wealth in continuous time for illustration purposes. However, we can do the "before-and-after" analysis described above by estimating a discrete time version of equation (13.2) above. More concretely, we can estimate:

$$\log\,(W_{i,\,t+10}) - \log\,(W_{i,\,t}) = \alpha + \gamma T_i + \phi\,\frac{T_i}{W_{i,\,t}} + X'_i\,\beta + \varepsilon_i \qquad (13.3)$$

where $W_{i,\,t}$ ($W_{i,\,t+10}$) is the wealth of congressman i in year t ($t + 10$), T_i corresponds to one of the "treatment" indicator functions defined above, and X_i corresponds to a series of individual characteristics that may influence wealth accumulation between the two years.

The specific sample on which the above regression should be estimated as well as the interpretation of the coefficients depends on whether we are estimating the returns to a seat in Congress in the early or late half of the decade under consideration.

In order to estimate the returns for the late part of the decade, we should estimate the regression on the sample of individuals that served only in the five years preceding or following year $t + 10$ (i.e. those for which either T_{LATE} or N_{LATE} equals 1). In this case, T_i will just correspond to the indicator function T_{LATE}. In terms of interpreting the coefficients, in this case γ corresponds to the estimate of r_d, ϕ corresponds to an estimate of R_d while the constant α captures the market rate of return, r. In this case, we do not have to worry about returns to Congress *after* serving since our control group (those who served in the five years after $t + 10$) did not serve between t and $t + 10$.

If we want to estimate the returns in the early half of the decade, the estimation sample should consist of all those who only served in the five years preceding or following year t (i.e. all those for which either T_{EARLY} or N_{EARLY} equals 1). In this case, T_i will correspond to T_{EARLY} and again, γ will provide an estimate of r_d and ϕ

an estimate of R_d. However, in this case the estimate of α will now confound both the market return r as well as the returns after serving in Congress r_a and R_a / W_t.

However, it is important to mention some potential drawbacks of the above framework. First, notice that in our definition of T_i we ignore the fact that some congressmen start serving at different times and serve for a different number of years within the five-year period (that is, some "treated" congressmen may have been "treated" for more years than others). Most importantly however, our estimates of γ and ϕ (i.e. of the returns and payoffs from office while serving) can be biased if congressional winners in the five years just before or just after a given year are different with respect to various characteristics that we are unable to control for and are correlated with wealth accumulation between t and $t + 10$. Selection into politics is unlikely to be a concern here since our sample consists of members of Congress (individuals that *ran* and *won* elections). Moreover, we are comparing winners in a relatively small time window across a given year which gives us further confidence that they are similar. However, there can still be some underlying differences we cannot observe, and hence, we cannot be certain that our estimates are unbiased.

In the analysis that follows we estimate equation (3) above for $t = 1850$ and $t = 1860$. This will allow us to get an estimate of the returns to serving in Congress in the early and late part of the 1850's and 1860's.[6]

13.3.2. Data

We obtained the names of all members of Congress serving between 1840 and 1875 from ICPSR and McKibbin (1997) and the *Biographical Directory of the United States Congress*. These sources also provided additional information on congressmen, including the year and place of birth, profession and career, and place of residence at different points in time. We also used Martis (1982) to match congressional districts to counties and cities. This information was useful for matching each representative to his census records.

The wealth data are from the Federal U.S. censuses of 1850, 1860 and 1870. These are the only years in which the census collected information on people's wealth. The census recorded real estate wealth in 1850, 1860 and 1870, and personal wealth in 1860 and 1870. In addition, the census recorded information on year and place of birth, county of residence and occupation. The census records

---

6 In each of the before-and-after analyses we drop congressmen who also served in the U.S. Senate during the relevant period.

in these years are available in Ancestry.com, a genealogical website that provides images of the original census records as well as a search engine that helps locate every single individual recorded in these censuses by first, middle and last name as well as year and place of birth and place of residence.

We proceeded to find the census record in each census year of every member of the House of Representatives during our period. We initially used PERL scripts to automatically locate the census records of as many congressmen as possible, using the first and last name, year of birth and county of residence. Despite the automated matching done by the scripts, the data collection process was still very labor intensive since wealth figures and occupation had to be typed manually. In addition, many records must be found by careful manual searching, because names and birth years are sometimes miss-recorded in the Ancestry.com search engine, many census records include only first and middle initials rather than full first names, some birth years are incorrect in the census, and some congressmen move to different counties or states. In some cases, we were unable to match congressmen with very common names, or unable to find them for unknown reasons.[7]

There were a total of 1,968 different non-southern congressmen in this period. So far, we have managed to find two or more census records for 1,431 of them.

The wealth data provided in census records was self-reported by the respondents, and was not checked for accuracy in other ways by government officials. Given this, there could be concerns associated with the reliability of these data. There are, however, several reasons to believe that these data are useful for our purposes. Most importantly, the information collected by census officials was, as a matter of policy, strictly confidential.[8]

Moreover, several previous studies have assessed the reliability of the census data in different ways. Soltow (1975) found that "wealth averages for the samples in the years 1850-1870 are generally in line with estimates made by various authorities on wealth distribution. Growth rates are similar to those found for GNP per worker by Kusnetz and commodity output per worker by Gallman" (page 6). Another

---

[7] There could be concerns on whether dropping congressmen with common names will introduce any bias in the analysis. Steckel (1988) and Ferrie (1996) ran, for their 1850 and 1860 samples, logit regressions of a "common name" dummy against characteristics such as location of residence (region and city size) and other personal characteristics such as real and personal wealth, ethnicity, illiteracy and occupation. Their results show that while common names occur less often in southern states and in cities with less than 75,000 inhabitants having a common name is not correlated with real or personal wealth.

[8] Even if some respondents were worried that the information provided would not in fact be kept confidential, there was no clear incentive for under-reporting or over-reporting wealth. There was no federal tax on wealth at the time, and no estate tax. Personal vanity, however, might have lead to some over-reporting.

group of studies compared wealth reported in the census sheets with taxable wealth. Particularly relevant for our purposes is Steckel (1994), who matched 20,000 households from the federal census of Massachusetts and Ohio with real and personal property tax records from 1820 to 1910. While the data from Ohio suggests that census wealth tends to exceed taxable wealth, his analysis suggests no systematic associations between the discrepancies and any individual characteristics.

Finally, even more important for our purposes, is whether politicians are more likely to misreport the true value of their wealth. In order to explore this issue, we found the 1850 and 1860 census records for all of the individuals in *The Rich Men of Massachusetts*, a book that purports to give the wealth of the richest 1,500 men in Massachusetts as of about 1851.[9] Our analysis (not reported) indicates that the correlation between wealth reported in this book and the wealth recorded in the censuses of 1850 and 1860 is relatively high. More importantly, there is no evidence of significant under-reporting or over-reporting of politicians compared to non-politicians. This provides further confidence in the reliability of the census data for our purposes.

A final measurement issue concerns the fact that it is sometimes difficult to distinguish between respondents with zero wealth and respondents who refused to provide any information to the census marshall (or instances where the marshall did not request the information).[10] In both situations census marshals left the census record fields blank, which makes it hard to distinguish "zero" wealth from "wealth figure not available." It is clear that in most cases an empty wealth field corresponds to zero or very low wealth, since they are in the census records of very young individuals, and individuals with low-paying occupations such as laborers and domestic servants.

13.4. Results

To assess the validity of our approach, in table 13.1 we test for pre-existing differences in congressmen who served before and after the different census years.

[9] The book provides information on total wealth, while the 1850 census, as note above, reported only real estate wealth. Thus, we matched individuals in the book with the 1860 census as well as the 1850 census, in order to have a measure of total wealth despite the fact that 1860 census measure is nine years later.

[10] Steckel (1994) notes that the incidence of "zero" wealth responses suggests that "some census enumerators failed to acquire accurate information on the value of wealth holdings through lack of diligence, non-compliance of the household, or ignorance of the respondent" (page 80).

Not surprisingly, congressmen who serve prior to a given census year are, on average, older than those who serve after the census year. To control for this difference, in our regressions we will always include the age and squared age of the congressman to capture the (possibly non-linear) effect that age may have on wealth accumulation. Most importantly, the table shows that treated congressmen do not differ by their initial wealth, a variable that plausibly captures other relevant characteristics such as ability, education, or occupation. In addition—just as one example—the table shows that treated congressmen are no more or less likely to be lawyers.[11] These similarities give us some confidence that the main difference between politicians at either side of the census year is their exposure to politics.

TABLE 13.1: Differences in means for observables

	Early treatment		Late treatment	
Variable	**Untreated**	**Treated**	**Untreated**	**Treated**
Initial real wealth, 1850	8.96	8.96	8.64	8.56
Age, 1850	47.99	42.88*	36.77	37.35
Lawyer Dummy, 1850	0.53	0.6	0.6	0.64
Initial total wealth, 1860	10.03	10.04	9.23	9.52
Age, 1860	47.4	43.36*	37.25	39.39*
Lawyer Dummy, 1860	0.64	0.64	0.59	0.64

Entries are cell means.
** = difference between treated and untreated is significant at the .05 level.*
Wealth is in natural logs.

Table 13.2 presents the estimates of the main coefficients of interest, i.e., γ in equation (13.3), the coefficient on T_i. In these specifications we omit the variable $T_i / W_{i,t'}$ and set $\phi = 0$. We estimated models that included the variable $T_i / W_{i,t'}$, and also experimented with other specifications that allowed the effect of treatment in Congress to vary according to initial wealth, but these interaction terms were never statistically significant. So we focus on the simpler specification here.

[11] This is true for the other major occupation groups as well.

TABLE 13.2: Before and after estimates of the returns to a seat in Congress

	Early treatment		Late treatment	
Dependent variable	**Basic controls**	**All controls**	**Basic controls**	**All controls**
Δ Real wealth	–0.03	0.05	0.09	0.11
1850-1860	(0.14)	(0.14)	(0.14)	(0.15)
Observations	193	193	257	257
Ending total	–0.08	–0.06	–0.08	–0.05
Wealth, 1860	(0.13)	(0.14)	(0.13)	(0.14)
Observations	198	198	262	262
Δ Total wealth	0.36*	0.36*	–0.07	–0.04
1860-1870	(0.16)	(0.17)	(0.11)	(0.12)
Observations	236	236	308	308

Entries give estimated coefficient on variables as described in text. Robust standard errors in parenthesis.
Basic controls = Initial Wealth, (Initial Wealth)^2, (Initial Wealth)^3, Age and Age^2.
All controls = Basic controls plus Occupation Dummies and State Fixed Effects.
** = coefficient is significant at the .05 level.*

The results are straightforward. First, we find no evidence of a large positive return to serving in Congress during the 1850's. Second, the same is true for the second half of the 1860's. Third, we do find evidence of a relatively large return to serving in Congress during the first half of the 1860's. Moreover, notice that wealth accumulation between 1850 and 1860 was similar for those congressmen who served in the late 1850's and in the early 1860's. This suggests that the additional returns we find for the latter group do not correspond to pre-treatment differences.[12]

For the first half of the 1860's, the point estimate is 0.36, which implies that serving in congress during this period yielded an additional 36 percent in total wealth accumulation between 1860 and 1870. The average growth in wealth between 1860 and 1870 of the control group—those who served in the second half of the 1850's but not the first half of the 1860's—was only 39 percent. So, the returns to a seat in congress during the period were quite large in relative terms.

[12] There was little inflation between 1850 and 1860, but prices were about 40 percent higher in 1870 than in 1860.

14.5. Conclusion

How do we reconcile our findings with the claims of widespread corruption that were so common during this period? Perhaps the claims, at least for the 1850s and late 1860s, were exaggerated or mainly political rhetoric. Another possibility is that the action was elsewhere, in state and local governments. After all, throughout the 19th century (except during the Civil War) combined state and local spending exceeded federal spending. The patterns we identify suggest that this is worth further study.

Another possible lesson is that politicians can sometimes exploit extraordinary circumstances. We find evidence that congressmen used their offices for personal gain during the early 1860s. This coincides with the Civil War, a period of extraordinarily large federal government spending. In the 1861 fiscal year (July 1860-June 1861), just before the Civil War, the federal government spent only about $67 million, about $2 per capita. Expenditures exploded during the war, to $475 million in 1862, $715 million in 1863, $865 million in 1864, and $1,298 million in 1865.[13] Spending shrank sharply after the war, though not to its pre-war levels even in real terms—the average was $292 million over the period 1867-1871. Moreover, much of the spending at the beginning of the war was done frantically under an emergency situation, with relatively little oversight and considerable chaos. There were many opportunities to make money, and politicians were well placed to take advantage of them. Perhaps they did.

References

Acemoglu, D., M. A. Bautista, P. Querubin and J. A. Robinson. "Economic and Political Inequality in Development: the case of Cundinamarca, Colombia" in Elhanan Helpman (Ed), *Institutions and Economic Performance*, Harvard University Press, Cambridge, MA, 2008.

Austen-Smith, D. "Interest Groups, Campaign Contributions, and Probabilistic Voting." *Public Choice* 54 (1987): 123-139.

Austen-Smith, D. and J. S. Banks. "Electoral Accountability and Incumbency." In Peter Ordeshook (ed.), *Models of Strategic Choice in Politics*. Ann Arbor: MI, University of Michigan Press, 1989.

Banks, J. S. and R. Sundaram. "Adverse Selection and Moral Hazard in a Repeated Elections Model." In W. Barnett et al. (eds.), *Political Economy: Institutions, Information, Competition and Representation*. New York: Cambridge University Press, 1993.

---

[13] All spending figures are from the *Statistical Abstract of the U.S., 1878,* Table 1. This excludes debt payments.

—. "Optimal Retention in Agency Problems." *Journal of Economic Theory* 82 (1998): 293-323.

BARGANZA, J. C. "Two Roles for Elections: Disciplining the Incumbent and Selecting a Competent Candidate." *Public Choice* 105 (2000): 165-193.

BARON, D. P. "Electoral Competition with Informed and Uninformed Voters." *American Political Science Review* 88 (1994): 33-47.

BARRO, R. "The Control of Politicians: An Economic Model." *Public Choice* 14 (1973): 19-42.

BESLEY, T. *Principled Agents? The Political Economy of Good Government.* Oxford: Oxford University Press, 2006.

BRYCE, J. *The American Commonwealth.* New York: The Macmillan Company, 1912.

CASELLI, F. and M. MORELLI. "Bad Politicians." *Journal of Public Economics* 88 (2004): 759-782.

DAL BO, E., P. DAL BO and J. SNYDER. "Political Dynasties," *Review of Economic Studies* 76 (1) January 2009.

DIERMEIER, D., M. KEANE and A. MERLO. "A Political Economy Model of Congressional Careers." *American Economic Review* 95 (2005): 347-373.

DENZAU, A. T. and M. C. MUNGER. "Legislators and Interest Groups: How Unorganized Groups Get Represented." *American Political Science Review* 80 (1986): 89-106.

EGGERS, A. and J. HAINMUELLER. *The Value of Political Power: Estimating Returns to Office in Post-War British Politics.* Unpublished Manuscript, Harvard University, 2008.

FEARON, J. D. "Electoral Accountability and the Control of Politicians: Selecting Good Types Versus Sanctioning Poor Performance." In Bernard Manin, Adam Przeworski, Susan Stokes, (eds.), *Democracy, Accountability and Representation.* Cambridge: Cambridge University Press, 1999.

FEREJOHN, J. "Incumbent Performance and Electoral Control." *Public Choice* 50 (1986): 5-25.

FERRIE, J. "A New Sample of Males Linked from the Public Use Micro Sample of the 1850 U.S. Federal Census of Population to the 1860 U.S. Federal Census Manuscript Schedules", *Historical Methods,* 29, 1996: 141-156.

FORBES A. and J. W. GREENE. *The Rich Men of Massachusetts.* Boston: W. V. Spencer, 1851.

GORODNICHENKO, Y. and K. SABIRIANOVA PETER. "Public sector pay and corruption: Measuring bribery from micro data." *Journal of Public Economics* 91 (2007): 963-991.

GREENBERG, K. *Masters and Statesmen: The Political Culture of American Slavery.* Baltimore: Johns Hopkins University Press, 1985.

GROSECLOSE, T. and J. MILYO. *Buying the Bums Out: What's the Dollar Value of a Seat in Congress?.* Discussion Papers Series, Department of Economics, Tufts University 9923, Department of Economics, Tufts University, 1999.

GROSSMAN, G. and E. HELPMAN. "Protection for Sale." *American Economic Review* 84 (1994): 833-850.

—. "Electoral Competition and Special Interest Politics." *Review of Economic Studies* 63 (1996): 265-286.

—. *Special Interest Politics.* Cambridge, MA: MIT Press, 2001.

INTER-UNIVERSITY CONSORTIUM FOR POLITICAL AND SOCIAL RESEARCH and C. MCKIBBIN. *Roster of United States Congressional Officeholders and Biographical Characteristics of Members of the United States Congress, 1789-1996: Merged Data (Computer file).* 10th ICPSR ed. Ann Arbor, MI: Inter-university Consortium for Political and Social Research (producer and distributor), 1997.

HARRINGTON, J. "Economic Policy, Economic Performance, and Elections." *American Economic Review* 83 (1993): 27-42.

HINDRICKS J. and P. BELLEFLAMME. "Yardstick Competition and Political Agency Problems". *Social Choice and Welfare* 24 (2005): 155-169

JOSEPHSON, M. *The Politicos.* New York: Harcourt, Brace, 1938.

LE BORGNE, E. and B. LOCKWOOD. *Candidate Entry, Screening, and the Political Budget Cycle.* Unpublished Manuscript, 2001.

—. "Do Elections Always Motivate Incumbents? Learning vs. Re-Election Concerns". *Public Choice* 129 (2006), 41-60.

Lenz, G. and K. Lim. *Getting Richer in Office? Corruption and Wealth Accumulation in Congress.* Unpublished Manuscript, Massachusetts Institute of Technology, 2009.

Martis, K. C. *The Historical Atlas of United States Congressional Districts: 1789-1983.* New York: The Free Press, 1982.

Mattozzi, A. and A. Merlo. *Mediocracy.* Unpublished manuscript, 2006.

—. "The Transparency of Politics and the Quality of Politicians." *American Economic Review, Papers and Proceedings* 97 (2007): 311-315.

—. "Political Careers or Career Politicians?" *Journal of Public Economics*, 92, 2008: 597-608.

Messner, M. and M. Polborn. "Paying Politicians." *Journal of Public Economics* 88 (2004): 2423-2445.

Mushkat, J. *Fernando Wood: A Political Biography.* Kent, OH: Kent State University Press, 1990.

Padro I Miquel, G. "The Control of Politicians in Divided Societies: The Politics of Fear." *Review of Economic Studies* 74, no. 4 (2007): 1259-1274.

Peltzman, S. "Toward a More General Theory of Economic Regulation." *Journal of Law and Economics* 19 (1976): 211-240.

Persson, T., G. Roland and G. Tabellini "Separation of Powers and Political Accountability" *Quarterly Journal of Economics*, 112 (1997), 1163-1202.

—. "Comparative Politics and Public Finance," *Journal of Political Economy*, 108 (2000), 1121-1161.

—. *Political Economics: Explaining Economic Policy.* Cambridge, MA: MIT Press, 2000.

Salant, D. J. "Behind the Revolving Door: A New View of Public Utility Regulation." *RAND Journal of Economics* 26, no. 3 (1995): 362-377.

Schlozman, K. L. and J. T. Tierney. *Organized Interests and American Democracy.* New York: Harper and Row, 1986.

Smart, M. and D. Sturm. *Does Democracy Work? Estimating Incentive and Selection Effects of U.S. Gubernatorial Elections, 1950-2000.* Unpublished manuscript, 2003.

—. *Term Limits and Electoral Accountability.* Unpublished manuscript, 2004.

Soltow, L. *Men and Wealth in the United States, 1850-1870.* New Haven: Yale University Press, 1975.

Sproat, J. G. *Liberal Reformers in the Gilded Era.* New York: Oxford University Press, 1968.

Stampp, K. M. *America in 1857: A Nation on the Brink.* New York: Oxford University Press, 1990.

Steckel, R. "Census Manuscript Schedules Matched with Property Tax Lists: A Source of Information on Long-Term Trends in Wealth Inequality", *Historical Methods*, 27 (2), 1994: 71-85.

—. "Census Matching and Migration: A Research Strategy", *Historical Methods*, 21 (2), 1988: 52-60.

Stigler, G. "The Theory of Economic Regulation." *Bell Journal of Economics* 2 (1971): 3-21.

Summers, M. W. *The Plundering Generation: Corruption and the Crisis of the Union, 1849-1861.* New York: Oxford University Press, 1987.

—. *The Era of Good Stealings.* New York: Oxford University Press, 1993.

United States, Bureau of the Census. *Statistical Abstract of the U.S., 1878.* Washington D.C.: U.S. Government Printing Office, 1879.

Van Deusen, G. *Thurlow Weed: Wizard of the Lobby.* Boston: Little, Brown and Company, 1947.

14

Non-Ignorable Abstentions in Mexico's Instituto Federal Electoral

Guillermo Rosas
Department of Political Science
Washington University in St. Louis

Yael Shomer
Department of Political Science
Washington University in St. Louis

14.1. Introduction

The purpose of this paper is to explore empirically some of the effects of assuming different abstention-generating mechanisms on the estimation of ideal points. For this purpose, we inspect a small committee, the Council-General of the Mexican *Federal Electoral Institute* (IFE, by its Spanish acronym). Though we start from common wisdom propositions about the putative ideological profiles of IFE Councilors, we do not purport to provide a theoretically-nuanced approximation to the ideological organization of this Council. Instead, we use the Council's roll-call record to gauge how inferences about ideology would be affected by different assumptions about Councilors' motivations to abstain. In this regard, our approach to the politics of IFE is unabashedly empirical, and indeed the choice of this committee as an object of study is driven by various characteristics that we deem desirable in this kind of exploratory analysis: IFE's Council-General is a committee made up of a handful of individuals known political sponsors, it decides on extremely important electoral matters, and it produces a relatively high incidence of abstentions.

In the past few decades, political methodologists have developed various techniques to circumvent what Londregan (2000) calls "the micro-committee problem". In essence, this problem arises from the relative paucity of divided votes available in collective decision-making bodies with a relatively high member-to-bill ratio, which makes it difficult to infer the ideological positions of committee members. Among the new techniques, Bayesian estimation methods have recently challenged the dominance of more traditional tools of ideal point estimation such as NOMINATE scores (Poole and Rosenthal 2001, 1997) as the most appropriate ways to study the voting behavior of individuals in small committees (Clinton, Jackman and Rivers 2004; Jackman 2001; Martin and Quinn 2002). In particular, Clinton, Jackman and Rivers (2004) have shown how ideological positions of legislators can be inferred from their responses to roll-call votes using an item-response theory (IRT) model. As in all statistical methods, however, inferences based on the IRT model depend on several assumptions with varying levels of verisimilitude. One assumption that may not always be approximated by this method is the assumption of ignorable missing values.

The paper is divided in four sections. Section 14.2 briefly comments on the notion of "ignorable missingness" and its relation to the standard Bayesian MCMC IRT model. Section 14.3 describes the inner workings of IFE's Council-General as it existed before electoral reforms in 1996. Finally, Sections 14.4 and 14.5 depict changes in the inferred distribution of ideal points of Council-General members that follow from alternative assumptions about the process that generates abstentions. Since these assumptions are not readily verifiable, analysts must acknowledge uncertainty in estimated ideological positions that follows from lack of knowledge about the process that generates abstentions.

14.2. The Clinton-Jackman-Rivers IRT model

We focus our analysis of abstentions in roll-call data on Bayesian inference about legislators' ideal-points, though some of our conclusions on ignorability of abstentions also hold for likelihood-based inference. An extremely useful framework to arrive at Bayesian inferences about ideal points is the probit analog of the Rasch two-parameter logit model commonly employed in item-response theory (IRT). Clinton, Jackman and Rivers (2004) show how this statistical model follows naturally from microfoundations about the behavior of legislators that confront an up-or-down vote on an item that might change the statu quo in policy space, and provide several examples of the flexibility and amenability of this model to accommodate extensions. Following Clinton, Jackman and Rivers (2004), we consider x_i, ζ_j, Ψ_j, $\in \mathrm{R}^1$ as the ideological position of legislator $i \in \{1... n\}$ and the YEA and NAY locations, respectively, of bill $j \in \{1... m\}$ in a one-dimensional ideological space. Legislators derive utility from the spatial distance between their own ideal point x_i and the locations ζ_j and Ψ_j of bills. In a one-dimensional setting, it is common to represent the utilities of voting YEA and NAY as:

$$U_i(\zeta_j) = -(x_i - \zeta_j)^2 + \eta_{ij}$$
$$U_i(\Psi_j) = -(x_i - \Psi_j)^2 + \nu_{ij} \quad (14.1)$$

where η_{ij} and ν_{ij} are spherical disturbances. The utility differential $y^*_{ij} = U_i(\zeta_j) - U_i(\Psi_j)$ determines whether a legislator will vote for or against proposal j:

$$y_{ij} = \begin{cases} 1 & \text{if } y^*_{ij} \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (14.2)$$

For the purpose of statistical estimation, one can model the probability that legislator i will vote in favor of proposal j as in Equation (14.3):

$$P(y_{ij} = 1) = P(y^*_{ij} \geq 0) = \Phi(\beta_j x_i - \alpha_j) \quad (14.3)$$

In Equation (14.3), $\beta_j = 2(\zeta_j - \Psi_j) / \sigma_j$, $\alpha_j = (\Psi_j^2 + \zeta_j^2) / \sigma_j$, α_j, is the standard deviation of the difference of the disturbance terms in Equation (14.1) (i.e., $\sigma_j^2 = \text{var}(\eta_{ij} - \nu_{ij})$), and Φ is the CDF of the standard normal distribution (a probit link).

In this setup, we observe a matrix **Y** of 1s and 0s corresponding to YEAs and NAYs, along with abstentions—i.e., missing values—on some cells of **Y**. Bayesian analysis proceeds by stipulating priors on parameters **a**, **b**, and **x**, sampling from the posterior distribution of these parameters and the missing data, and presenting summaries of these samples. In a nutshell, the iterative MCMC algorithm samples missing y_{ij} conditional on estimated parameters **a**, **b**, and **x**, and then samples **a**, **b**, and **x** based on observed votes and previously imputed values of missing y_{ij}.[1] This procedure guarantees reasonable estimates of **x** under the assumption that the abstention-generating mechanism is ignorable. But if this assumption does not hold, Bayesian (and likelihood-based) inference methods are in fact ignoring information that could well be used to estimate ideology parameters with greater precision.[2]

---

[1] This is a loose description, as the Gibbs sampler draws a continuous latent variable y^* (data augmentation), rather than a dichotomous y, subject to constraints implied by observed votes: $y^* > 0$ if $y = 1$ and $y^* < 0$ if $y = 0$. If y is missing, then y^* is drawn from an unconstrained normal distribution. Moreover, **a** and **b**, on the one hand, and **x**, on the other, are sampled in successive steps, not simultaneously. See Albert and Chib (1993); Clinton, Jackman and Rivers (2004).

[2] Aside from the assumption of ignorability, the CJR IRT model assumes local independence and strict monotonicity. Indeed, the one-dimensional specialization of the CJR model is part of a family of models characterized by the acronym SMURFLI(2), since they are based on the following assumptions: **s**trict **m**onotonicity, **u**nidimensional **r**esponse **f**unction, **l**ocal **i**ndependence, and **two** possible responses (YEA or NAY in our case) (Mislevy and Wu 1996).

In some legislatures and committees, it is not reasonable to ignore the process that generates abstentions. Rubin (1976) provides conditions under which we can safely ignore the process that generates missing values. We focus here on Rubin's first set of conditions. According to Rubin (1976) abstentions are ignorable if (a) the missing data are missing at random (MAR) and if (b) the item and subject parameters that drive the linear predictor of AYE/NAY votes and the parameters that drive the linear predictor of the decision to abstain are distinct (D). An intuitive way to approach the MAR condition is the following: Imagine that the analyst knew how a legislator would vote in a sequence $\mathbf{Y}$ of bills, and then compiled the actual sequence $\mathbf{Y}^{obs}$ of observed votes, where some elements of $\mathbf{Y}^{obs}$ may be empty. If the corresponding elements of $\mathbf{Y}$ are not useful in predicting the unobserved entries of $\mathbf{Y}^{obs}$, then missing values are missing at random. The MAR assumption also underlies likelihood-based inferences about ideal points.

In many settings, assuming MAR is not problematic. For example, analysts may believe that legislators that fail to register an AYE or NAY vote simply happen to be away from the floor at the time of voting and that their absences are in no way connected to the political process. Alternatively, analysts may believe that legislators abstain for idiosyncratic reasons that can be safely ignored. The MAR assumption then provides a very reasonable approximation to the process that generates missing votes. However, there are other circumstances in which such an assumption would not resonate well with the analyst's knowledge about the process that generates unregistered votes. For example, members of the Israeli Knesset are known to leave the floor in order to avoid registering a vote *against* the coalition to which their party belongs, while at the same time denying a vote to the opposition. In Argentina, Jones and Hwang (2005) suggest that deputies may choose to abstain rather than support the position of the national leadership of their party if this position is at loggerheads with that of the party leader in their province of origin. In these and similar circumstances, unregistered votes—be they abstentions or physical absences from the floor—cannot be assumed to be missing at random.

In legislatures or small committees, non-random missing values may obtain under a variety of logics. For example, members with preferences on the far right or left of an ideological continuum may abstain because they may feel *alienated* from a political process that yields centrist (from their point of view) alternatives. Conversely, these individuals may be *indifferent* between the AYE and NAY positions of a particular bill. More interestingly, legislators may feel the pull from constituencies (say, the party whip and the median voter in their home district) with different policy preferences, and choose to abstain rather than vote against the wishes of one of these. When the MAR assumption does not hold, the process that generates missing votes is no longer ignorable, and missing values must be modeled.

The second assumption is not required in likelihood-based inference to make abstentions ignorable. In Bayesian inference, however, assumptions about the prior distribution of parameters are an integral part of estimation, and in consequence ignorability of the missing process depends on the character of these assumptions. Parameters are distinct "if there are no *a priori* ties, via parameter space restrictions or prior distributions" between them.[3] In applications with large amounts of information from available data, assumptions about the prior distribution of parameters will have little impact on estimates about the posterior distribution. In small committees, however, prior distributions are likely to weight heavily on inferences about parameters, and one should therefore be explicit about all *a priori* information contained in the model.

14.3. Mexico's Instituto Federal Electoral

To explore how assumptions about the abstention-generating process affect inferences about ideal points in a voting assembly, we have chosen a small committee with a complete voting record and with non-trivial degrees of abstention among its members. Mexico's *Instituto Federal Electoral* fits this description perfectly. Before the extremely contested presidential elections of 2006, Mexico's electoral management body was widely credited for managing this country's transition to democracy in a peaceful and ordered manner. Indeed, before this watershed election IFE was trumpeted as an autonomous agency that placed major decision-making powers in the hands of non-partisan members of the Council General (Woldenberg 1995). The long tenure of its members and the stability of its operational budget were seen as at least mild guarantees of independence.

In fact, the *Instituto Federal Electoral* (IFE) that emerged from reforms in 1994 was only a semi-autonomous agency in charge of overseeing Mexico's federal elections. However, though IFE's charter originally called for a preponderant

---

[3] Technically, from Rubin's Definition 3, "the parameter ϕ is distinct from θ if their joint parameter space factorizes into a ϕ -space and a θ -space, and when prior distributions are specified for θ and ϕ, these are independent" (Rubin 1976, 585).

presence of the executive power in its board, successive reforms have led to the creation of a vigorous agency independent from Mexico's once omnipotent Presidents. Concurrent with its increasing autonomy, IFE took over the years an ever more important role in organizing all electoral aspects of Mexico's protracted transition to democracy. IFE's Council General decided on all organizational matters, including the creation and upkeeping of electoral lists, installation of electoral booths, vote counting, monitoring of campaign spending by parties, and overall regulation of political campaigns.

We inspect the voting record of IFE's Council-General from 1994 to 1996, the so-called Carpizo Council. During these years, the Council-General included four Legislative Councilors—two Senators and two Deputies, representing the two largest parties in each chamber. In effect, these four seats granted the larger political parties (PRI, PAN, and PRD) direct representation in the IFE's main executive body, fostering a situation presumably conducive to checks and balances. Mitigating the clearly partisan nature of the council, six "Citizen Councilors" nominated and ratified by two-thirds of the lower chamber of Congress completed the membership of the council. This was the first time that a non-partisan group of experts was introduced in IFE as a counterweight to political parties. Citizen Councilors could in principle reduce political bickering, grant voice and representation to the electorate, and bring into IFE extensive legal and technical know-how. The six Citizen Councilors voted in all deliberations and were able to introduce new agenda items as long as these were recognized by the Council's Chairman. The national executive's *ex oficio* representative at IFE, the Minister of the Interior, acted as chair of the Council General and concentrated agenda-setting power in this body; furthermore, the Council chair enjoyed exclusive power to nominate administrative officers for IFE's bureaucracy and could cast a tie-breaking vote.

TABLE 14.1: Descriptive statistics, IFE Council-General

Member	ABS	Y	N	Member	ABS	Y	N
Chair	49	5	0	Granados (PRD)	5	37	12
Senator PRD	28	21	5	Zertuche (PRD)	3	42	9
Senator PRI	9	32	13	Ortiz (PRD)	3	41	10
Deputy PRI	8	31	15	Pozas (PRI)	4	39	11
Deputy PAN	12	30	12	Woldenberg (PAN)	2	41	11
				Creel (PAN)	2	42	10

Because the Carpizo Council was still presided by the Interior Ministry, and because large parties were granted votes in all decisions through their Senate and congressional representatives, questions arised early on concerning the "autonomous" capacity of the first set of Citizen Councilors. The common wisdom about this period was that Citizen Councilors voted as a block *against* Legislative Councilors, thus evincing a divide between experts and politicians in the Council-General that superseded a left–right divide (PRD–PRI–PAN) or even a pro-government–opposition divide that pitted PAN and PRD vs. PRI; both of these were supposed to cleave the ideological space in the Mexican Congress at the time. A quick glance at the voting record of the Carpizo Council confirms that there are no votes on which a coalition of Citizen Councilors imposed their will on Legislative Councilors, though there are a handful of votes in which abstainers come mostly from either the Citizen or the Legislative blocs. If we interpret abstentions as "votes against the majority", then this handful of votes are consistent with the common wisdom. But are we sure that this interpretation is correct? In fact, Estévez, Magar and Rosas (2008) show evidence that in later Councils (1996–2005) Citizen Councilors behaved as 'party watchdogs' that would not act against the interests of the political parties that sponsored them to IFE's Council.[4] Partisan conflict, rather than confrontation between Citizen vs. Legislative Councilors, may also characterize the Carpizo Council. To what extent are these two alternative interpretations driven by assumptions about abstentions?

We analyze the 1994–1996 Council-General to gauge how these interpretations fare under different assumptions about the mechanism that generates abstentions. In all cases, we assume that a single ideological dimension suffices to capture relevant sources of disagreement in the Council. Because of the large degree of consensus in this body, the analysis is based on a set of fifty-four usable votes (Malo and Pastor 1996).[5] Voting behavior is collected from minutes of twelve sessions of the Council General between June 1994 and November 1996. As a first glance into the prevalence and potential interpretation of abstentions in the context of the Carpizo Council consider data in table 14.1, which breaks down the voting behavior of Councilors based on whether they voted to move a proposal, voted against it, or abstained. The Council's Chair abstained in a large majority of these votes, and none of his five votes were tie-breakers.[6] It is also

[4] The party sponsors of the six Citizen Councilors were PRD (Granados, Ortiz, and Zertuche), PAN (Creel and Woldenberg), and PRI (Pozas).

[5] Usable votes are about 35% of the total. They exclude all votes passed by a universal coalition of ten supporters. Aside from these, we excluded four votes to approve minutes from previous sessions.

[6] In fact, these five votes were among the large set of consensual votes, as the President joined majorities of seven to nine members in all of these cases.

clear that the abstention rates of party representatives are higher than those of Citizen Councilors. In particular, the Senate representative from the PRD abstained a little over half the time.[7] Excluding the Council Chair, the overall abstention rate for the Carpizo Council is approximately 14%.

An alternative look into the problem of abstentions in the Carpizo Council appears in table 14.2. This table displays fifty-four items broken down by the number of affirmative votes they received, along with the number of abstainers. For example, there are 27 items that were voted almost consensually by nine Councilors. In 24 of these, a single Councilor held out against the majority by abstaining. It is reasonable to believe that this individual would have voted *against* the item, but may have succumbed to pressures for consensual behavior built into IFE's institutional setup (see Estévez, Magar and Rosas 2008). By far, the more interesting item in this series corresponds to the one item that received the support of one Councilor and was rejected with the negative vote of three Councilors, with six abstainers. These six abstainers were the Citizen Councilors.[8] A second noteworthy item received two votes in favor, four against, and four abstentions. The abstainers this time were the Legislative Councilors.[9]

TABLE 14.2: Breakdown of votes by size of enacting coalition, President excluded

		Members abstaining						
		0	**1**	**2**	**3**	**4**	**5**	**6**
Members voting YEA	0	0	1	0	1	1	0	0
	1	3	1	2	1	0	0	1
	2	0	0	0	0	1	0	0
	3	1	0	0	0	0	0	0
	4	0	0	0	0	0	0	0
	5	0	0	0	0	0	0	—
	6	0	2	2	0	0	—	—
	7	0	1	0	3	—	—	—
	8	1	0	5	—	—	—	—
	9	3	24	—	—	—	—	—

[7] Because the dataset does not distinguish between abstentions and absences, I have checked that missing values do not correspond to entire sessions; this is not the case.

[8] This bill was introduced by the PRD representation (June 8, 1994) and aimed to provide political parties the right to obtain the original documentation used to build the voter registry.

[9] This bill was introduced by the PFCRN representation (this party was eventually subsumed within the PRD) and sought to force IFE to publish electoral rapid counts as soon as they were received by IFE.

All Council-General members are required to vote on all matters and all items are decided by simple plurality. These instances of "bloc non-voting" in which all Citizen Councilors or all Legislative Councilors abstain are worthy of attention because the abstaining bloc could have in both cases swayed the result either way, therefore leaving us to wonder about the motives behind this behavior.[10] It cannot be the case that non-voters were *against* the motion, because they could have easily formed a counter majority. And if they supported the bill, why would they not offer an affirmative vote? Naturally, abstentions may be the consequence of some sort of "competing principals" logic (Carey 2007). In either case, abstentions are hardly missing at random.

14.4. Alternative abstention-generating mechanisms

In IFE's Council-General, Councilors may choose to abstain for a variety of reasons. In this section, we explore three of these possibilities, which we refer to as *indifference*, *alienation*, and *abstention as disagreement*. The third of these mechanisms is a very likely candidate to account for most instances of abstention. Indeed, abstentions seem to express in most instances disagreement with a supermajority that is still not profound enough to justify an outright NAY vote. Instead, disagreement is expressed in the more muted form of an abstention, especially during IFE's earlier phases. This idiosyncratic behavior might be a consequence of built-in incentives to project an image of consensus and harmony in IFE's Council-General. In other occasions, as in the two noteworthy instances mentioned above, it is difficult to interpret abstentions as outright expressions of disapproval. In these cases, there is no immediately obvious rationale for abstention. We consider two alternative plausible mechanisms that generate random abstentions. Though these mechanisms satisfy the MAR assumption, they do not comply with the second necessary assumption about parameter distinctness. Consequently, these three mechanisms are examples of non-ignorable abstentions. We compare inferences about the ideological composition of the Carpizo Council based on different non-ignorable abstention mechanisms with those

---

[10] In other Councils, we have found striking split decisions in which one or two Councilors choose to abstain. A 2-4-4 (Y/N/A) split would be such an instance. On the one hand, it is unrealistic to stipulate that the four abstainers are expressing outright disapproval against a majority of YEA-sayers since they could have easily formed a majority coalition with the three NAY-sayers in the Council. On the other hand, the two abstainers may actually prefer the option voted by the four YEA-sayers but choose not to align with them on the record for unspecified (strategic) reasons.

that obtain from the assumption of ignorable abstentions that underlies the IRT model.

For the three non-ignorable abstention models, we base our inferences on information from two different sources. As in the IRT model, we consider a matrix of *recorded* votes with three possible entries for AYE, NAY, and abstention. The second source of information corresponds to an indicator matrix of *observed* votes, with 1's for every AYE or NAY entry in the *recorded* vote matrix and 0's elsewhere. The process that generates votes in favor or against a particular proposal is depicted in Equation (14.3). The process that generates abstentions can also be captured with an identical two-parameter IRT model, as in Equation (14.4):

$$P\,(m_{ij} = 1) = P\,(m^*_{ij} \geq 0) = \Phi\,(\delta_j\, z_i - \gamma_j) \tag{14.4}$$

In Equation (14.4), m^*_{ij} is a latent parameter that captures the propensity of Councilor i to abstain on vote j (m_{ij} is the 1/0 entry in the *recorded* vote matrix). Parameter m^*_{ij} is driven by an individual-specific abstention parameter z and item-specific parameters δ and γ, which are analogs of α and β in Equation (14.3).[11]

We adapt the basic setup of Equations (14.3) and (14.3) to correspond to each of the three non-ignorable missingness mechanisms. In the case of abstentions driven by *indifference*, we consider the possibility that Councilors may fail to register a preference if the utility differential between status quo and alternative is smaller than some individual-specific threshold. As in all the mechanisms that we consider, the theoretical logic that underlies vote preference (i.e., the choice of AYE or NAY) is minimal Euclidean distance. The *indifference* abstention-generating mechanism complies readily with the MAR assumption. To see this, consider that a Councilor's decision to abstain obtains after the Councilor corroborates that a particular bill will fail to change the status quo dramatically, and is no way driven by how the Councilor would have voted. Since the abstention-generating mechanism is MAR, we could easily ignore information contained in the *observed* vote matrix. However, the assumption of parameter distinctness is not entirely warranted, and it can be shown that the difficulty parameter for the abstention process (γ) and the discrimination parameter for the vote choice process (β) are *a priori* related (Rosas and Shomer 2008). Given the assumption that priors are non-distinct, the abstention-generating process is no longer ignorable. A similar case obtains in our second non-ignorable abstention-generating

---

[11] We relegate all details of model specification and estimation to Rosas and Shomer (2008).

mechanism. In this second case, we assume that abstentions are the product of alienation by ideological "extremists", an argument that finds a correlate in literature on mass voting behavior (Downs 1957; Plane and Gershtenson 2004; Riker and Ordeshook 1968). Councilors are still assumed to have individual-specific abstention propensities, but these are a function of their ideological propensities (i.e., $z = f(x)$). This mechanism still complies with the MAR assumption because though abstention propensities depend on ideal points, they do not necessarily depend on the Councilor's AYE or NAY vote choice. In other words, Councilors are assumed to abstain based on individual-specific propensities, regardless of the direction of their vote had they bothered to register one. However, based on the alienation assumption we know that a legislator that misses a heavy proportion of votes is more likely to have a less centrist ideological position. We incorporate this additional *a priori* information into a prior structure in which parameters z and x are sampled from a bivariate normal distribution.

As we mentioned before, we include inferences about ideal points based on these alternative abstention narratives for the purposes of comparison. From what we know about the informal rules that structured decision-making at IFE during the Carpizo Council, an incentive to deliver more or less consensual decisions was in place. This incentive flows from the fact that all Council-General decisions can in principle be appealed to an electoral High Court, the *Tribunal Federal Electoral*. In this context, a decision passed with one or two negative votes may, in case of appeal to the Tribunal, receive more scrutiny than a decision passed with one or two abstentions. If this interpretation is correct, we should consider *most* abstentions as NAY votes, i.e., as expressing disagreement with the majority's point of view. We insist that this treatment cannot be extended to all cases, for as we saw in the previous section there are some instances that cannot be rationalized according to this logic.

If this mechanism is indeed at play, then we can no longer assume that abstentions are MAR. A Councilor's decision not to register a vote is now driven by the vote choice itself, which violates this assumption. Parameters can still be assumed to be distinct, and therefore receive independent prior distributions, but we approximate the non-missing at random assumption by expanding the model in Equation (14.3) to also incorporate individual-specific abstention propensities. The extended model for vote choice appears in Equation (14.5):

$$P(y^*_{ij} \geq 0) = \Phi(\beta_j x_i + \gamma_j z_i - \alpha_j) \tag{14.5}$$

The abstention model remains as in Equation (14.3). With this specification, we recognize that vote choice may be driven by, among other factors, the propensity to abstain.[12]

As is well known, there are two sources of under-identification in item response models: scale invariance and rotational invariance (Jackman 2001). To solve the problem of scale invariance, the prior probability distributions of individual-specific ideal points are constrained to have unit variance. To solve the problem of rotational invariance, I constrain the ideal point of one Citizen Councilor sponsored by the leftist PRD to have most prior probability mass on the negative orthant, and the ideal point of one Citizen Councilor sponsored by the rightist PAN to have most prior probability mass on the positive orthant. Finally, the extended model of Equation (14.5) requires a further constraint on some parameters γ. We pick two votes where a single Councilor abstains, respectively, against a majority of nine affirmative votes and a majority of nine negative votes, and we constrain the γ parameters of these two votes to have opposite signs. We estimate all models in this section using WinBugs. In all cases, we run two chains started at overdispersed values and allow sufficiently long burn-in periods (five to ten thousand). Our inferences are based on four-hundred draws from the posterior distribution after apparent convergence; we monitored convergence through the Gelman-Rubin R statistic.[13]

14.5. Results

Our basic results regarding the ideological organization of the Carpizo Council are relayed graphically in graph 14.1, which displays point and interval estimates of the ideological location of Citizen Councilors (with symbol "X") and Legislative Councilors ("O") on a single ideological dimension (horizontal axis). The point estimate corresponds to the median of the posterior distribution of x; the interval estimate is the 50% highest posterior density interval of this distribution. For ease of comparison, Councilors are grouped according to the political party to which they belong (Legislators) or the political party that sponsored their candidacy to the Council (Citizens). Recall that the common wisdom about the Carpizo Council holds that its main political rift pitted Legislative Councilors,

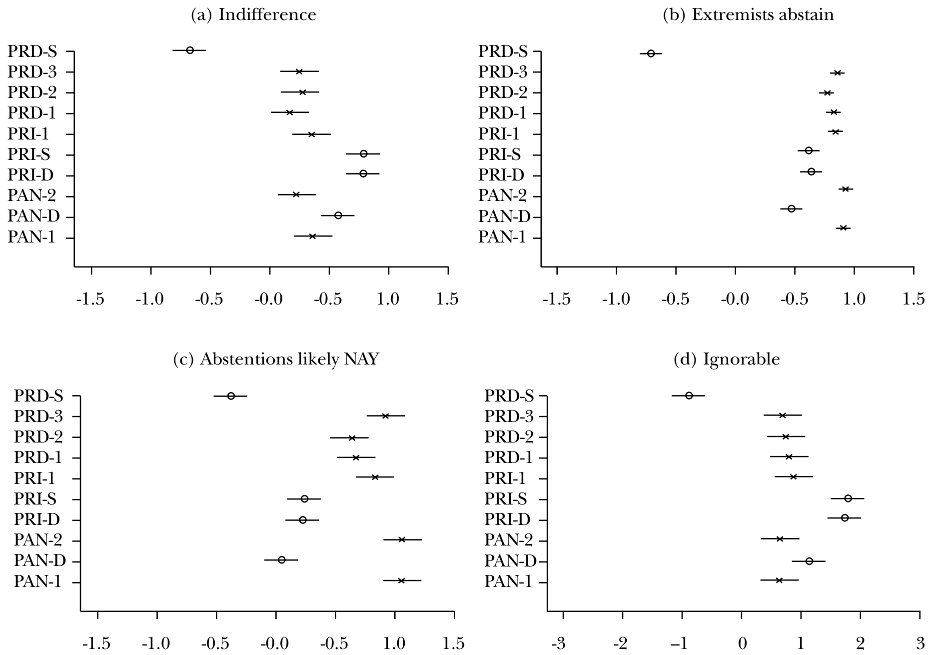
---

[12] In fact, one could proceed by introducing x in the abstention model, rather than z in the vote choice model, and inferences would remain by and large similar (Holman and Glas, 2005).

[13] WinBugs code and chains are available upon request.

which guarded the interests of the three main political parties, against six Citizen Councilors who despite being sponsored by specific political parties embodied technocratic efficiency and were assumed to remain above the fray of party politics. Be this as it may, Estévez, Magar and Rosas (2008) document that Citizen Councilors in Councils-General from 1996 to 2003 behaved according to a *party sponsorship* hypothesis, so that individuals sponsored by the same party tended to occupy the same ideological region in one-dimensional space.

GRAPH 14.1: Inferences about the ideological organization of Carpizo's IFE Council under alternative assumptions about missingness



The plots in graph 14.1 fail to substantiate an interpretation of the Carpizo Council as a mirror image of the Mexican Congress, as in no model do we see clear clusters of Councilors with common party membership or sponsorship. However, the plots also suggest that despite the mostly consensual behavior of IFE Councilors there are substantively important differences in their ideological positions. Most obviously, the PRD Senator's ideal point is clearly defined as out of line with the rest of the Councilors, which is not surprising given the voting record of this Councilor (out of 26 observed votes, he votes eleven times against the majority position). Because of the high number of abstentions produced by this Councilor (28), his position is relatively sensitive to alternative assumptions about the process that generates missing values. In particular, his position is pulled closer to that of the rest of the Councilors whenever we assume that missingn votes are mostly votes against the majority (Plot c) or when we assume that abstentions are ignorable (Plot d). Not surprisingly, the mechanism that assumes that extremist Councilors are more likely to abstain makes the PRD Senator more likely to appear far to the left (Plot b). Incidentally, this mechanism leads to estimation of relatively narrow credible intervals for all Councilors.

Aside from this Councilor's position, the main difference in the ideological configuration of this Council under alternative missingness assumptions concerns the relative position of Citizen Councilors vis-à-vis the ideal points of Legislative representatives. In Plots (b) and (c), Citizen Councilors appear to one side of the ideological space, whereas in Plots (a) and (d) these individuals, as a group, appear to be pivotal between the PRD Senator and the representatives of other congressional parties. This difference in the relative positions of Citizen and Legislative Councilors is extremely consequential; depending on the description that one considers, the Carpizo Council could be seen as an instance in which Citizen Councilors were ideologically opposed to Legislative Councilors (Plot c), an instance where ideological differences among all Councilors but the PRD Senate representative were largely trivial (Plot b), or a situation where Citizen Councilors played a pivotal role bridging the ideological gap between the PRD Senate representative's position, on the one hand, and the positions of the representatives of other congressional parties (Plots a and d). Still, an alternative way of analyzing these different results suggests more similarities than differences. In this regard, table 14.3 displays the estimated probability that each of the six Citizen Councilors could be the median voter in the Carpizo Council under the four different models.[14] The main similarity is that under no model specification are Legislator Councilors likely to have played a pivotal role; indeed, the probability that any of these four councilors could have been the median voter is trivial. The models that treat abstainers as extremists or abstentions

[14] To estimate these probabilities, we simply count the number of draws from the posterior distribution, out of 100, in which the ideal points of these Councilors are rank-ordered in fifth or sixth place, i.e., as the two medians in a set of ten.

as likely votes against the majority suggest that Councilors Zertuche and Granados, sponsored by the PRD, were the most likely candidates to occupy the two median positions. Under the other specifications, especially under the model that assumes that abstentions are ignorable, all Citizen Councilors have an approximately equal chance of being median voters.

TABLE 14.3: Likely median voters under alternative mechanisms

Councilor	Indifference	Extremists	Likely NAY	Ignorable
Zertuche (PRD)	0.068	0.223	0.278	0.186
Granados (PRD)	0.105	0.313	0.28	0.135
Ortiz (PRD)	0.128	0.111	0.063	0.151
Pozas (PRI)	0.164	0.159	0.11	0.188
Woldenberg (PAN)	0.107	0.037	0.016	0.13
Creel (PAN)	0.178	0.042	0.011	0.133

How could one arbitrate among the different conclusions supported by these models? One possibility that we pursue here is simply to compare the goodness of fit of the four alternative models. In this regard, the amount of consensus in the Carpizo Council makes it already relatively easy to predict how Councilors will vote. In fact, a trivial model predicting *all* Councilors to vote AYE on *all* votes would correctly account for 66% of data. The model that ignores abstentions predicts correctly 453 Councilor/item entries, which is equivalent to a proportional reduction of error of 0.28.[15] The model that assumes abstentions by indifference predicts 452 items correctly, whereas the model that assumes abstentions by extremists performs poorly with 445 correct predictions. The model that performs best—abstentions as disagreement—predicts 457 items correctly; though the improvement over the model that ignores abstentions is marginal, this would be the preferred model under the criterion of goodness of fit. Among the 76 Councilor/item cells that are missing, the model that ignores abstentions predicts that 25 of these will be NAY votes. In contrast, the model that treats abstentions as likely NAYs increases this count to 29.

[15] I predict that Councilor i votes AYE on item j if $y^*_{ij} > 0$, NAY otherwise.

14.6. Conclusion

We sought in this paper to explore some of the substantive effects of alternative assumptions about the process that generates abstentions in a small committee. We have explored some of these effects in a simulation setting elsewhere (Rosas and Shomer 2008). Here, we considered an actual decision-making body, Mexico's *Federal Electoral Institute*, an extremely important institution in that country's transition to democracy. Though this is a highly consensual body with a relatively low overall rate of abstentions, assumptions about the process that generates missing values lead to noticeably different conclusions about underlying latent traits.

We believe that most decision-making bodies are composed of politicians with the ability and incentive to act strategically. In many occasions, abstentions are likely the result of stratagems that politicians employ to keep their true voting choices under wraps. Even if abstentions are not the result of a stratagem, many non-purposeful abstention-generating mechanisms fail to comply with Rubin's necessary conditions and are therefore not ignorable under Bayesian inference. Consequently, the decision to ignore abstentions based on the assumptions of missing-at-random and parameter distinctness may be difficult to defend. It is incumbent upon the analyst to gather as much information as possible about a particular decision-making body in order to model the process that presumably generates abstentions as closely as possible. When this is not possible, the most honest path is to report conclusions based on a variety of model specifications, as we have done here. Given the rather noticeable effects that different models of abstention have on inferences about ideal points, reporting results from a single specification in the absence of sound theory is not enough.

References

Albert, J. H. and S. Chib. "Bayesian Analysis of Binary and Polychotomous Response Data." *Journal of the American Statistical Association* 88, no. 422 (1993): 669-679.

Carey, J. M. "Competing Principals, Political Institutions, and Party Unity in Legislative Voting." *American Journal of Political Science* 51, no. 1 (2007): 92-107.

Clinton, J., S. Jackman and D. Rivers. "The Statistical Analysis of Roll Call Data." *American Political Science Review* 98, no. 2 (2004): 355-370.

Downs, A. *An Economic Theory of Democracy*. Addison Wesley, 1957.

Estévez, F., E. Magar and G. Rosas. "Partisanship in non-partisan electoral agencies and democratic compliance: Evidence from Mexico's Federal Electoral Institute." *Journal of Electoral Studies* 27, no. 2 (2008): 257-271.

Holman, R. and C. A. W. Glas. "Modelling Non-Ignorable Missing-Data Mechanisms with Item Response Theory Models." *British Journal of Mathematical and Statistical Psychology* 58, no. 1 (2005): 1-17.

Jackman, S. "Multidimensional Analysis of Roll Call Data via Bayesian Simulation: Identification, Estimation, Inference, and Model Checking." *Political Analysis* 9, no. 3 (2001): 227-241.

Jones, M. P. and W. Hwang. "Party Government in Presidential Democracies: Extending Cartel Theory Beyond the U.S. Congress." *American Journal of Political Science* 49, no. 2 (2005): 267-282.

Londregan, J. *Legislative Institutions and Ideology in Chile's Democratic Transition.* New York, NY: Cambridge University Press, 2000.

Malo, V. and J. Pastor. "Autonomía e imparcialidad en el Consejo General del IFE, 1994-1995." Unpublished thesis, Departamento de Ciencia Política: Instituto Tecnológico Autónomo de México, 1996.

Martin, A. D. and K. M. Quinn. "Dynamic Ideal Point Estimation via Markov Chain Monte Carlo for the U.S. Supreme Court, 1953-1999." *Political Analysis* 10, no. 2 (2002): 134-153.

Mislevy, R. J. and P-K. Wu. *Missing Responses and IRT Ability Estimation: Omits, Choice, Time Limits, and Adaptive Testing.* Educational Testing Service, 1996.

Plane, D. L. and J. Gershtenson. "Candidates" ideological locations, abstention, and turnout in U.S. midterm Senate elections." *Political Behavior* 26, no. 1 (2004): 69-93.

Poole, K. T. and H. Rosenthal. "D-Nominate after 10 Years: A Comparative Update to *Congress: A Political-Economic History of Roll-Call Voting*." *Legislative Studies Quarterly* 26, no. 1 (2001): 5-29.

—. *Congress: A Political-Economic History of Roll-Call Voting.* Oxford University Press, 1997.

Riker, W. and P. Ordeshook. "A Theory of the Calculus of Voting." *American Political Science Review* 62 (1968): 25-42.

Rosas, Guillermo, and Yael Shomer. *Non-ignorable abstentions in roll-call analysis.* Working paper, 2008.

Rubin, D. B. "Inference and Missing Data." *Biometrika* 63, no. 3 (1976): 581-592.

Woldenberg, J. "Los consejeros ciudadanos del Consejo General del IFE: un primer acercamiento." In *Elecciones, diálogo y reforma: México 1994*, ed. Jorge Alcocer V. Vol. I Mexico City: Nuevo Horizonte, 1995.

15

A Problem in Conceptualizing Party Competition over Redistributive Taxation

John E. Roemer
Department of Political Science
Yale University

15.1. Introduction

In this paper, I will model the political economy of income-tax policy as the competition between two parties over the *post-fisc income function*, the function which assigns to each pre-tax income level, a post-tax-and-transfer income level. There will be very few restrictions on the policy space—the domain of such functions. This is intended to model the view that mathematical restrictions such as linearity have no justification other than rendering the problem tractable. Working on an infinite-dimensional space of possible policies models the view that the competition over tax policy is ruthless, with no holds being barred.

The equilibrium concept employed will be party-unanimity Nash equilibrium (PUNE[1]). Each party will consist of two factions, one which desires to maximize the average income of those citizens who vote for it (its *constituency*) and the other which wishes to maximize vote share, subject to at least tying the opposition. I call these factions the Guardians and the Opportunists.

I will characterize the PUNEs for this model, by making a simplifying assumption about the nature of stochastic voting. This will be an assumption that a certain function is concave. I will argue that the (unique) PUNE predicted by the model is not observed in reality, and therefore that the concavity assumption just referred to is a poor one. There is an independent argument that this function is not concave, as well.

But if this function is not concave, I will argue that the calculation of equilibrium is essentially impossible—more precisely, the problem for the Opportunists

[1] For the definition of PUNE, see Roemer (2001).

is a very difficult one—and this will justify the view that Opportunists use *rules of thumb* in the game of political competition. I will propose such a rule of thumb, and refer readers to another paper, where equilibria where Opportunists use the rule of thumb is studied.

Therefore, this paper has an essentially negative purpose: to rule out a fully 'rational' solution to the problem of political competition over tax policy, and to justify politicians' using certain rules of thumb.

15.2. The model

A. The economic environment and the policy space

The set of voter types is a pre-tax distribution of income, called h, distributed according to a probability measure F whose mean is μ. A policy is a mapping $X: \Re_+ \to \Re_+$ where $X(h)$ is the post-fisc income of a voter of type h. The restrictions on X are:

(T1) X is continuous (horizontal equity)

(T2) $\alpha \leq X'(h) \leq 1$, where $\alpha \in [0, 1]$, and the inequality holds at points where X is differentiable;

(T3) $\int X(h)\ dF(h) = \mu$.

Assumption (T3) says that the policy is purely redistributive. Formally, voters have no preference for leisure, so a voter of type h has utility given by $v(X; h) = X(h)$. Choosing $\alpha > 0$ is a simple way of attempting to recognize labor supply elasticity. The marginal tax rate is $1 - X'(h)$, so if $\alpha = 0.5$, then the parties agree not to raise marginal tax rates above 0.5, because, presumably, there would be severe labor-supply effects.

Denote the policy space by $\Im$.

B. Voting

A voter will face two policies X and Y. Voters of each type vote stochastically. Only a certain fraction of voters will vote for the party that is offering them the higher income. We postulate the existence of a function $S: \Re_+^2 \to [0, 1]$ with the following properties:

(S1) $S: \Re_+^2 \to [0, 1]$, S continuous

(S2) $S\,(x, y)$ is non-decreasing in x and non-increasing in y

(S3) $S(x, y) + S(y, x) = 1$
(S4) S is strictly concave in x;
(S5) $S_1(x, x)$ is decreasing in x, where $S_1(x, y) \equiv \frac{dS(x, y)}{dx}$.

The fraction of type-h voters voting for policy X when the choice is (X, Y) from the two parties will be $S(X(h), Y(h))$. Properties (S1), (S2), (S3), and (S5) are all reasonable. (S4) is not reasonable and it will have a powerful mathematical consequence. It is this assumption that I referred to in the introduction.

Let us take an example of a canonical stochastic voting process. Suppose that, offered incomes x and y from two parties, a member of a vote type h votes for x if and only if

$$x - y > \varepsilon$$

where ε is distributed according to a normal variate with mean zero. If this variate is drawn independently from this distribution for all voters of a type, then the fraction voting for x will be $S(x, y) = N(x - y)$, where N is the cdf of the normal variate. Notice that (S1), (S2), (S3), and (S5) hold for this process, but (S4) is false. In particular, the function S is convex if $x < y$ and concave if $x > y$.

C. Parties

The Guardians in a party will attempt to maximize the post-fisc income of those who vote for them. But those who vote for them comprise only a fraction of each voter type. The Opportunists attempt to choose a policy which maximizes vote share for the party, given what the other party is proposing. This motivates the following definition.

Definition 15.1. *A political equilibrium is a triple* $\{\theta(\cdot), X^A, X^B\}$, *where* $\theta: \Re_+ \to [0, 1]$ *and* $X^A, X^B \in \Im$. *Define the functions* $\Im \to \Re_+$ *by:*

$V^A(X) = \int \theta(h)\, X(h)\, dF(h)$, $V^B(X) = \int (1 - \theta(h))\, X(h)\, dF(h)$.

(P1) Given X^A, *there is no policy such that* $X \in \Im$ *such that:*

$V^A(X) \geq V^A(X^A)$ and $\int S(X(h), X^B(h))\, dF(h) \geq \int \theta(h)\, dF(h) \geq 1/2$

with at least one inequality strict;

(P2) Given X^B, *there is no policy such that* $X \in \Im$ *such that:*

$V^B(X) \geq V^B(X^B)$ and $\int S(X^A(h), X(h))\, dF(h) \geq \int 1 - \theta(h))\, dF(h) \geq 1/2$

with at least one inequality strict;

(P3) $\theta(h) = S(X^A(h), X^B(h))$.

(P3) says that the function θ gives the share of each type voting for party *A*. Thus, $V^A(X)$ gives the average income of *A*'s constituency were the policy to be *X*. Similarly, $V^B(X)$ is the average income of *B*'s constituency at *X*. (P1) says that, given the policy proposed by party *B*, there is no policy which would be unanimously preferred by both the Guardians and Opportunists of party *A*, and (P2) makes a similar statement for party *B*. Note that at an equilibrium, both parties receive exactly half the vote, because each receives at least one-half the vote.

Thus, $\{\theta(\cdot), X^A, X^B\}$ comprise exactly a party-unanimity Nash equilibrium where each party possesses a Guardian and Opportunist faction. The parties' constituencies are endogenous: that is, in equilibrium, each party takes its constituency to be those voters who vote for it. The notion of constituency is a statistical one here: in this way, we differ from the usual approach, in which constituencies are disjoint sets of voter types. Here, there are some voters of every income type in the constituency of each party. This conforms with reality.

We will be interested in a certain kind of equilibrium:

Definition 15.2. *A left-right political equilbrium is one where* $\theta(\cdot)$ *is a weakly decreasing function.*

In other words, in a left-right equilibrium, poorer voters tend to vote more for party *A* and richer voters for party *B*. We will hence call *A* and *B* the left and right parties, respectively.

15.3. The result

Suppose that θ is weakly decreasing and not constant. What is the solution of the problem?

$$\begin{array}{l} \max \int \theta(h)\ X(h)\ dF(h) \\ \text{s.t. } X \in \Im \end{array} . \quad \text{(program } L\text{)}$$

Note the space $\Im$ is convex. Program *L* is therefore a concave programming problem, because the objective function is linear in *X*. The solution to this problem is the following: *X* should give *as much as possible* to the lowest income type, i.e., to $h = 0$. This is the function $\hat{X}$ given by:

(1) $\hat{X}(h) = a + \alpha h$
(2) $a = (1 - \alpha)\mu$.

Condition (2) tells us that the function $\hat{X}$ integrates (dF) to μ; clearly it is impossible to give more to h=0, since the function increases at the slowest feasible rate.

The function $\hat{X}$ is the *ideal* policy of any left party (in a left-right equilibrium). The proof is found in Roemer (2008), but the intuition is simple. In program *L*, one wishes to maximize a linear combination where the biggest weights go on the earliest terms. Clearly the solution is to give as much as possible to the earliest terms. 'As much as possible' is restricted by the condition (T2). Hence, the solution.

Similarly, what is ideal policy of the right party, that is, the solution of:

$$\begin{aligned}&\max \int (1 - \theta\ (h))\ X(h)\ dF(h) \\ &\text{s.t. } X \in \Im\end{aligned} \quad \text{. (program } R\text{)}$$

It is the laissez-faire policy: $X^*\ (h) = h$. The proof is again found in Roemer (2008). The intuition is again simple. The right party wishes to give as much as possible to the type at infinity, which means to not waste anything on the poor (give them zero) and then to increase at a slope of one forever. (If one gave zero to an interval of poor voters, and then increased forever at a slope of one, the function would integrate to less than μ.)

We have:

Theorem 15.1. *If assumptions (S1)-(S5) hold, then there exists a unique left-right political equilibrium:* $\theta(h) \equiv 1/2$ *and* $X^A = X^B = \hat{X}$.

Thus, both parties propose the ideal policy of the poorest voter, the most extreme left policy. This result is extremely unrealistic, and I take it to impugn assumption (S4).

The theorem follows from the following fact:

Proposition 15.1. *If assumptions (S1)-(S5) are satisfied, then the policy* $\hat{X}$ *is a Condorcet winner in* $\Im$.

***Proof*:** See the Appendix.

This is surprising: for note the result is independent of the distribution *F*. In particular, it holds even if the median of *F* is greater than its mean. Intuitively, I believe it is the concavity of the vote-share function in x which drives this result. Suppose two policies X and Y have been proposed, and the vote share of the X policy is $\int S(X(h), Y(h))\ dF(h)$. Fix Y and consider deviations in X. It is to the advantage of the left party to shift resources down to the poorer voters (to increase vote share) since the function is concave in its first component. The proof of the proposition employs both (S4) and (S5).

Proof of Theorem 15.1:

1. We have noted that in any left-right equilibrium, $\hat{X}$ is the policy that maximizes the objective function of the Guardians of the left party. We first note that the triple $\{\frac{1}{2}, \hat{X}, \hat{X}\}$ is a left-right political equilibrium. Note that since $\theta(h) \equiv 1/2$, *all feasible policies give exactly the same value* for V^A and V^B, namely, $\mu/2$. There is no deviation that either party can make to increase its vote share, since $\hat{X}$ is a Condorcet winner. So neither party can profitably deviate.

2. Now let $\{\theta, X^L, X^R\}$ be any left-right political equilibrium. We know that each policy wins half the vote. If $X^L \neq \hat{X}$ then a left deviation to $\hat{X}$ will increase the utility of the Guardians in left, while not decreasing its vote share (since $\hat{X}$ defeats or ties X^R). Hence this is an acceptable deviation. Therefore we must have $X^L = \hat{X}$. But this means that $X^R = \hat{X}$, since the only policy that can defeat or tie $\hat{X}$ is $\hat{X}$. Hence $\theta\ (h) \equiv 1/2$ and the theorem is proved.

15.4. Discussion

I have modeled two parties competing over income tax policy, perhaps the fundamental domestic policy issue. The competition is over a very large space of functions, modeling the idea of ruthless competition. Each party contains a faction that represents constituents, in the sense of attempting to maximize their average post-fisc income, and a faction that wishes to maximize vote share, subject to at least tying the opposition party. It seems all this is eminently reasonable.

We depart from conventional practice both in working on an infinite-dimensional policy space (for which the PUNE concept is useful) and in modeling the constituency as a statistical concept. Since in actual elections, there is a significant fraction of voters at every income level that votes for each party, this seems to be a move in the direction of realism. Indeed, in the U.S., typically at least 30 percent of the bottom income quintile vote for Republicans and at least 30 percent of the top income quintile vote for Democrats.

We have modeled voting as stochastic, and have made certain strong assumptions on the nature of that process, which render the problem tractable. If $S(\cdot, y)$ were not a concave function, then solving for equilibrium would involve a non-concave programming problem in an infinite dimensional space: there are no dependable methods for solving such a problem. Probably the only method for computing political equilibria without such concavity is approximate, computational and complicated. But assuming concavity, we get a very unrealistic result: that both parties propose the extreme left policy in the unique equilibrium—and this, regardless of the distribution of voter types!

Furthermore, we have independent reason to challenge the concavity of S: the traditional microeconomic justification of stochastic voting (described in the paper) gives us a function S which is 's–shaped,' that is convex and then concave.

Solving the problem for the Opportunists given such a function is mathematically intractable. And this justifies the claim that the Opportunists must adopt some boundedly-rational approach to vote-share maximization if the policy space is really infinite-dimensional.

In Roemer (2008), I argue that one reasonable rule of thumb is for the Opportunists to focus upon winning the 'swing voters,' defined as those voter types who, historically, have split their votes evenly between the two parties. That paper characterizes political equilibrium under those rules.

Appendix

Proof of Proposition 15.1

1. We show that $\hat{X}$ maximizes vote share when competing against $\hat{X}$. This will show that $\hat{X}$ is a Condorcet winner: for if $\hat{X}$ maximizes vote share against $\hat{X}$, then any other policy against $\hat{X}$ gets at most 50% of the vote.

Suppose the claim were false. Then there exists a policy $X = (\hat{X} + g) \in \Im$ such that

$$\int S(\hat{X}(h) + g(h), \hat{X}(h))\ dF(h) > 0.5$$

To show this is impossible, we define the function:

$$\Delta(\varepsilon) = \int_0^\infty S(\hat{X}(h) + \varepsilon g(h), \hat{X}(h))\ dF(h) + \rho(\mu - \int_0^\infty (\hat{X}(h) + \varepsilon g(h))\ dF(h))$$
$$+ \int_0^\infty (\hat{X}'(h) + \varepsilon g'(h) - \alpha)\ r(h)\ dh$$

By premise (S4), Δ is a concave function. Note that $\Delta(0) = 1/2$. Suppose we can choose $\rho \geq 0$ and $r \geq 0$ such that $\Delta'(0) = 0$, then Δ is maximized at $\varepsilon = 0$. But notice that if $\rho \geq 0$ and $r \geq 0$ then $\Delta(1) = \int S(X(h), \hat{X}(h))\ dF(h)$ + non-negative terms. It will follow that $1/2 = \Delta(0) \geq \Delta(1) \geq \int S(X(h), \hat{X}(h))\ dF(h)$, a contradiction that will prove the theorem.

Compute, using integration by parts, that:

$$\Delta'(0) = \int S_1(\hat{X}(h), \hat{X}(h))\ g(h)\ dF(h) - \rho \int g(h)\ dF(h) + g(h)\ r(h)\Big|_0^\infty$$

$$- \int r'(h)\ g(h)\ dh = \int \Big[(S_1(\hat{X}(h), \hat{X}(h)) - \rho)\ f(h) - r'(h) \Big]\ g(h)\ dh + gr\Big|_0^\infty$$

Hence it is only necessary to choose $r(\cdot)$ and ρ so that:

$$r'(h) = (S_1(\hat{X}(h), \hat{X}(h)) - \rho)\ f(h) \text{ and } r(0) = 0\ r(\infty)$$

Choose $r(0) = 0$ and define $\rho = \int S_1(\hat{X}(h), \hat{X}(h))\, dF(h)$; then $r(\infty) = 0$ by integration.

Moreover, since $S_1(\hat{X}(h), \hat{X}(h))$ is a (weakly) decreasing function of h (see postulate (S5)), it follows that r' is initially non-negative and finally non-positive, so that, because of the end-point conditions, r is a non-negative function. This shows that $\hat{X}$ maximizes vote share against itself.

2. Furthermore, $\hat{X}$ is the unique vote-share maximizer against $\hat{X}$, since $S(x, y)$ is strictly concave in x. Hence any other policy running against $\hat{X}$ receives less than half the vote, proving that $\hat{X}$ is a strict Condorcet winner.

References

Roemer, J. E. *Political competition.* Cambridge, MA: Harvard University Press, 2001.

—. "A positive theory of income taxation when parties focus upon core and swing voter." *Cowles Foundation Discussion Paper,* no. 1637 (2008).

16

On the Formation of Voting Blocs

Jon X. Eguia
Department of Politics
New York University

16.1. Introduction

In legislative bodies such as lower or upper houses of representatives, senates, parliaments or national assemblies, legislators typically coalesce into cohesive political parties that discipline the voting behavior of their members. Other applications range from the formation of alliances of countries in international voting bodies, such as the European Union or the Arab League in the UN General Assembly, to the formation of factions that vote together in faculty meetings in an academic department. In all these applications, individual agents are able to communicate and coordinate with others. Specifically, agents can coalesce to form voting blocs.

A voting bloc is a set of agents who commit to vote together, aggregating their internal preferences into a common policy position that the whole bloc votes for, rolling internal dissent according to a voting rule internal to the bloc.

I study the formation of voting blocs by agents with ideological preferences defined over a two dimensional space. Agents form voting blocs for the purpose of affecting the policy outcome, moving it away from the status quo and Condorcet winning policy, toward their ideal policy. I show that there exist incentives to form voting blocs and that if one agent has a monopoly in the technology to coordinate voting blocs, then one voting bloc forms and the policy outcome moves away from the status quo and away from the Condorcet winning policy with positive probability. I also show that if any subset of agents can coordinate and form a voting bloc, at least two voting blocs form in equilibrium, and an equilibrium with two voting blocs exists.

The formation of voting blocs is an application in the field of political economy of the game-theoretic coalition formation literature surveyed by Carraro

(2003), Ray (2007) and Humphreys (2008). Traditional models of coalition formation assume that the utility of an agent depends solely on the coalition she belongs to, ignoring externalities from other agents. To account for externalities, the *partition function* approach pioneered by Thrall and Lucas (1963) recognizes that the utility of each individual depends on the whole partition of the set of agents into coalitions. Bloch (2003), Yi and Bloch and Gomes (2006) provide results for coalitions that generate either positive or negative externalities. Hyndman and Ray (2007) allow for both positive and negative externalities to occur simultaneously in a model with just three agents. Eguia (2007) studies the formation of a single voting bloc. The current application of the theory to the formation of voting blocs contains externalities that are both positive and negative, in a model with any finite number of agents can form any number of voting blocs.

Substantively, this theory on the formation of voting blocs is part of the formal literature on party formation. Snyder and Ting (2002) explain parties as informative labels; Levy (2004) as commitment devices for politicians; Morelli (2004) as coordination devices for voters; and Caillaud and Tirole (2002) as screening devices. All these theories explain party formation as a result of the interaction between candidates and voters. Baron (1989) and Jackson and Moselle (2002) note that members of a legislative body have incentives to form parties within the legislature, irrespective of the interaction with the voters, to allocate the pork available for distribution among only a subset of the legislators. My theory shows that legislators also have an incentive to form parties—voting blocs—in the absence of a distributive dimension, merely to influence the policy outcome over which they have an ideological preference. Cox and McCubbins (1993) find that legislators in the majority party in the U.S. Congress use the party as means to control the agenda and the committee assignments, and Aldrich (1995) explains that U.S. parties serve both to mobilize an electorate in favor of a candidate, and to coordinate a durable majority to reach a stable policy outcome avoiding the cycles created by shifting majorities. I prove that voting blocs form even if legislators do not control the agenda and there exist no cycles in the majority preference.

16.2. The theory

Let there be a legislative assembly $\mathcal{N}$ with N (odd) agents who make a policy decision, choosing an outcome in a two dimensional policy space. The status quo policy is $(0, 0)$. A policy proposal is put to a vote and if it gathers a simple majority of votes, it becomes the policy outcome, otherwise the status quo remains in place. Legislators have circular preferences around their ideal policy. For each legislator i, let p_i be the ideal policy of the legislator i and let p be the policy outcome. Let $||\cdot||$ be the Euclidean norm, then $u_i(p) = -||p - p_i||$.

The ideal policies of the legislators are distributed on a grid around the origin. Specifically, for some finite K, let the size of the grid be $2K \times 2K$ and for any $a, b \in \{-K, -K+1, \ldots, K-1, K\}$, let there be one agent with ideal policy (a, b). This corresponds to a discretized version of a uniform distribution of ideal policies in the policy space.

The size of the grid is arbitrary. If there is at least one legislator with an ideal policy at any given point on the grid, $K = 5$ generates an assembly larger than the U.S. Senate, and $K = 10$ larger than the U.S. House of Representatives. The distribution of ideal policies satisfies the radial symmetry condition detailed by Plott (1967), by which for any given agent with an ideal policy in some direction away from the status quo, there is another agent with an ideal policy in the exact opposite direction. With preferences that satisfy radial symmetry, the status quo policy (0, 0) is a median in all directions and a Condorcet winner, that is, the status quo defeats any other policy in pairwise comparisons. Therefore, if legislators vote their true preference in the assembly, any policy proposal fails.

I assume that, at a cost, legislators can make binding commitments to coordinate their votes. The timing is as follows:

1. Every agent can issue a proposal or invitation to any subset of other agents to form a voting bloc that includes the proposer. These proposals all become common knowledge as well.
2. Each agent who receives an invitation to form a voting bloc can accept at most one invitation, or she can reject them all. If every legislator who receives a proposal to form a given voting bloc accepts it, the bloc forms. Organizing this bloc is costly, and every member, including the legislator who first made the proposal, bears a cost $c > 0$. Agents join a bloc strategically, joining only if it makes them better off.
3. Nature chooses the agenda by independently drawing a point from distribution with uniform density in $[-1, 1]^2$. The chosen agenda becomes public and common knowledge.
4. Legislators who are members of a bloc meet on a caucus and they vote, choosing between the policy proposal detailed in the agenda or the status quo. Every voting bloc coordinates by simple majority: If a simple majority of its members votes in favor of the proposal in the caucus, the bloc as a

whole votes in favor in the assembly; if a simple majority votes against the proposal in the caucus, they all vote against the proposal in the assembly; and if they tie in the caucus, agents are free to vote as they wish in the assembly.

5. The assembly meets and votes, deciding by simple majority. Indepen-dent agents vote as they wish, while members of a bloc are bound by their commitment to follow the outcome of the caucus of their bloc.

It is a key assumption that legislators can make binding commitments to coordinate their votes within a bloc, and vote together in the assembly. The cost of orga-nizing a voting bloc captures the difficulty of making these commitments. If commitments are not feasible, the cost of organizing is effectively infinite. But the possibility of costlessly punishing defectors, if only by social sanctions, or the availability of bonds or deposits that legislators can put up front as guarantee that they will not defect from the bloc should suffice to enforce the commitments to vote together.

The strategy of each agent consists of three elements: The decision to issue invitations to form a voting bloc, the decision to accept one of these invitations, and the vote on the policy proposal.

The solution concept I use is Subgame Perfect Nash Equilibrium in iterated weakly undominated strategies.

Note that at the voting stages, once voting blocs have formed, only sincere voting survives the iterated elimination of weakly dominated strategies. Sincere voting in the assembly is weakly dominant for independents. Then by backward induction it is weakly dominant for members of a bloc to vote sincerely at the caucus meeting. Given that only sincere voting survives the iterative elimination of weakly dominated strategies, I assume that agents correctly anticipate sincere voting on the part of every other agent at all stages and all subgames, and I consider a reduced strategy space that deals only with the agenda and the decisions about forming voting blocs. I rule out abstention, assuming that agents who are indifferent (a non-generic event) vote in favor of the proposal.

The protocol to form a voting bloc is similar to Hart and Kurz's (1983) coalition game Γ. Since all the legislators in a voting bloc must agree to join in order for the bloc to form, it must be that the formation of a voting bloc benefits every member of the party.

The first result is a partial equilibrium result, solving the game in which only one legislator has the ability to send invitations to form a voting bloc. Let agent l have ideal policy $p_l = (x_l, y_l)$ such that $x_l \neq 0$ and $y_l = 0$.

Proposition 16.1. *Assume that only legislator l can issue invitations to form a voting bloc. An equilibrium exists. If the cost c is low enough, in every equilibrium a voting bloc forms and the policy proposal defeats the Condorcet winner and passes with positive probability.*

Proof. The game is finite, so existence follows directly from Nash's theorem (1950).

For any x, y, let $i_{x,y}$ denote the agent with ideal policy $p_i = (x, y)$. Without loss of generality, let $x_l > 0$. Let $A_1 = \{i_{x,y} : x > 0 \text{ and } x - 1 \leq |y| \leq x\}$. Consider the following strategy: Agent l proposes the formation of $A = l \cup A_1$. Let p_t be the exogenously given policy proposal. The bloc A favors the policy proposal p_t if and only if $i_{1,0}$ favors it. If $i_{1,0}$ favors it, l favors it and either all $i_{x,y} \in A$ with $y > 0$ or all $i_{x,y} \in A$ with $y < 0$ favor it as well, constituting a majority of the bloc in favor. If $i_{1,0}$ prefers the status quo, either all $i_{x,y} \in A$ with $y > 0$ or all $i_{x,y} \in A$ with $y < 0$ prefer the status quo as well. Given that A never votes as a bloc against the preference of $i_{1,0}$, no policy gathers a majority in the assembly if $i_{1,0}$ opposes it. Given a policy p_t, if $i_{1,0}$ and $i_{-1,1}$ and $i_{x,y}$ favor it for all x, y such that $x \geq 2$ and $y = -x + 2$, then p_t passes in the assembly. Similarly, p_t passes if $i_{1,0}$ and $i_{-1,-1}$ and $i_{x,y}$ favor it for all x, y such that $x \geq 2$ and $y = x - 2$. Since the slopes of the indifferent curves of $i_{1,0}$ and $i_{x,y}$ such that $x \geq 2$ and $y = -x + 2$ at (0, 0) are all greater than 1 and the indifference curve of $i_{-1,1}$ at (0, 0) is exactly 1, the set of policies that all these agents favor has a non empty interior. Given the symmetry of the distribution of preferences with respect to the horizontal axis, the area of policies that pass if A forms is divided into two areas, symmetric with respect to the horizontal axis.

I want to show that for any pair of policies (a, b) and $(a, -b)$ such that $i_{1,0}$ prefers one or the other of these policies to pass with equal probability and be implemented instead of (0, 0), every member of A prefers (a, b) and $(a, -b)$ as well. Since any such pair that makes $i_{1,0}$ strictly better off is better for any $j \in A$ than another pair with the same b, but with the first coordinate moved left to the point where $i_{1,0}$ is indifferent, it suffices to show that every $j \in A$ weakly prefers a pair of policies that make $i_{1,0}$ indifferent over the status quo implemented twice. A policy (a, b) makes $i_{1,0}$ indifferent if and only if

$$(a - 1)^2 + b^2 = 1 \tag{16.1}$$

$$a = 1 - \sqrt{1 - b^2} \tag{16.2}$$

Formally, I want to show that for any (x, y) such that $x > 0$ and $-x \leq y \leq x$, and any (a, b) such that $a = 1 - \sqrt{1 - b^2}$,

$$(x - a)^2 + (y - b)^2]^{1/2} + [(x - a)^2 + (y + b)^2]^{1/2} \leq 2[x^2 + y^2]^{1/2} \qquad (16.3)$$

Algebraic manipulations (omitted, but available from the author) of this expression yield

$$2(x^2 + y^2) + 2(x^2 + y^2)\ (x - 1)\ \sqrt{1 - b^2} \leq b^2\, y^2 + 2x^3 + 2xy^2 \qquad (16.4)$$

Since $0 \leq b \leq 1$ and $x \geq 1$, it suffices to check

$$2(x^2 + y^2)\ x \leq b^2\, y^2 + 2(x^2 + y^2)\,x \qquad (16.5)$$

which is always true for any y. Furthermore, if $b \neq 0$ the left hand side is strictly less than $2(x^2 + y^2)\,x$ and the last expression holds with strict inequality.

Given any point (a, b) that passes if A forms, $(a, -b)$ passes as well. Given the distribution on the exogenous agenda, (a, b) and $(a, -b)$ become the policy proposal and pass to become the policy outcome with equal probability. It follows that every $i \in A$ strictly benefits from the coordination of votes inside A. If c is low enough, the benefit outweighs c and it is a dominant strategy for every agent who receives an invitation to join the bloc. Then, agent l strictly gains ex ante from issuing this invitation.

Consider the subgame after l makes any other arbitrary invitation. These subgames are finite, hence they have a Nash equilibrium. Select the subgame and Nash equilibrium that generates the highest payoff to l. For c low enough, at least one subgame yields a payoff gain strictly higher than c. The maximum payoff gain then is also strictly greater than c. An outcome does not generate a strictly positive payoff gain unless the policy proposal defeats the status quo with positive probability, which occurs only if a voting bloc forms. It follows that the best response strategy of agent l must be to issue an invitation to form a voting bloc; in equilibrium this invitation is accepted, and the policy proposal defeats the status quo at each stage with positive probability. Ex ante all members of the bloc become better off.

The literature on the endogenous formation of parties in a legislative assembly has noted that parties form to distribute pork (Baron, 1989; and Jackson and Moselle, 2002), to control the agenda (Cox and McCubbins, 1993) or to eradicate cycles and solve the instability inherent to political competition in multiple dimensions (Aldrich, 1995). I show that legislators have incentives to coordinate their votes, coalescing into a voting bloc that exercises party discipline, purely for ideological gain, even if they have no control over the agenda, and even in the absence of majority cycles or stability. In Proposition 16.1 I show that a set of agents who coordinate their votes forming a voting bloc succeeds in defeating a Condorcet winning policy, and they are able to move the policy outcome in a way that is favorable to the bloc.

If the status quo policy is a Condorcet winner, standard theories of policy-making predict that the status quo will be the policy outcome. In Krehbiel's (1998) *pivotal politics* theory, the Condorcet winner (in one dimension, the median ideal policy) lies inside the *gridlock* area, where policies cannot be changed. The discretion of an agenda-setter with positive agenda control is proportional to the distance between the Condorcet winner (again, the median in one dimension) and the status quo in the seminal agenda-setter theory by Romer and Rosenthal (1978). Normative reasons as well indicate that a Condorcet winner status quo policy should not be changed: Any change benefits only a minority of agents, and is detrimental for a majority. If utilities are linear or concave in distance to the ideal policy, any deviation from the Condorcet-winning policy generates a loss in social welfare. Nevertheless, a group of legislators who share a common interest in one dimension of policy, but diverge in another dimension, can coalesce to coordinate their votes and win a majority to defeat the Condorcet winner and move the policy away from the status quo and toward their preference.

Members of a bloc exploit their common preference in one dimension, and they obviate their conflicting preferences on a second dimension, agreeing to vote for policies that bring a desired change in the dimension they agree upon. In this manner, they defeat the status quo policy with positive probability. Note that members of a bloc benefit in expectation. Ex post, there is a net aggregate gain for the bloc, but some members may be worse off.

The radial symmetry condition on the distribution of preferences does not drive the result. On the contrary, I impose the condition to stack the deck against the formation of a bloc, and to distinguish my argument from Aldrich's (1995) interpretation of parties as means to avoid instability. I show that assuming that forming a vote is costly, that the bloc cannot control the agenda and that there are no majority cycles to exploit, a disciplined voting bloc still manages to attain a net gain in expected utility by changing the policy outcome.

The formation of a single voting bloc is not an equilibrium of the complete game in which any legislator can invite others to form a voting bloc. In expectation, some non members become worse off. If they can form their own voting blocs, they too have incentives to coalesce. If legislators receive more than one invitation to join a bloc, coordination issues arise. For instance, if legislators i

and i' both issue invitations to legislators j and j' to form a three person voting bloc, a bloc forms if j and j' coordinate to accept the same invitation, but it fails to form otherwise. If legislators j and j' would benefit from forming either bloc but they fail to do so because they accept different invitations, they are in a coordination failure.

Definition 16.1. *Given the strategy of every $i \in A$, the strategy profile of a set of agents A is a coordination failure if*

(i) No $i \in A$ joins any voting bloc and

(ii) Every $i \in A$ would be strictly better of in expectation if A forms a voting bloc.

The definition of a coordination failure is contingent on the strategy profile of the other agents, so the strategies of a set of agents are a coordination failure only in view of what other agents do. The expectation is with respect to the realization of the agenda if it is exogenous, and the realization of mixed strategies by other agents. Note that this definition of coordination failure is very narrow. It excludes coordination failures with agents who join another voting bloc, even if these agents would prefer to leave their blocs and form a different bloc. The definition only applies to cases that we may deem as complete failures, where agents who would all benefit from forming a voting bloc, all end up being independent. Presumably, agents should be able to avoid these coordination failures. If so, in equilibrium, at least two voting blocs necessarily form.

Proposition 16.2. *Let any legislator be able to propose forming a voting bloc. If c is low enough, an equilibrium without coordination failures and with two voting blocs exists, and there is no equilibrium without coordination failures with less than two voting blocs.*

Proof. I first rule out equilibria without coordination failures and with no blocs. As shown in the proof of Proposition 16.1, $\exists A \subset \mathcal{N}$ such that every $i \in A$ is strictly better off if A forms a unique voting bloc. Hence an outcome with no blocs is a coordination failure by A.

I next rule out equilibria without coordination failures and with one bloc. Consider a strategy profile such that the set of agents A forms a voting bloc and no other bloc forms. In order for every $i \in A$ to be best responding, it must be that i strictly benefits from the formation of A. Pair all agents with $p_i \neq (0, 0)$ as follows: For any i with $p_i = (a, b)$, let $j(i)$: $\mathcal{N} \rightarrow \mathcal{N}$ be a one-to-one mapping such that $p_{j(i)} = (-a, -b)$. For any i and $j(i)$, if i strictly benefits from a change in policy from $(0, 0)$ to p, then $j(i)$ is hurt at least as much i benefits from this change. Let $B = \{j(i): i \in A\}$. If A and B each forms a bloc and no other agent forms another bloc, they cancel each other out and no policy proposal passes. Hence if A has an incentive to form a voting bloc, then the set of agents in B have an incentive to form a voting bloc as well, and the formation of a unique bloc by A is a coordination failure for B.

Third, I show existence of an equilibrium with two blocs. I consider two separate cases. Let $A_1 = \{i: p_i = (x, y) \text{ with } x > 0\}$, let $B_1 = \{j: p_j = (x, y) \text{ with } x < 0\}$, and let $C = \{i: p_i = (0, 0)\}$. Recall $i_{x,y}$ denotes the agent with ideal policy (x, y). I consider two separate cases.

One– Even K. Let $A_2 = \{i: p_i = (0, y) \text{ with } y \text{ odd}\}$ and let $B_2 = \{p_i = (0, y) \text{ with } y \neq 0 \text{ even}\}$. Let $A = A_1 \cup A_2$ and let $B = B_1 \cup B_2$. Let $C = \{i: p_i = (0, 0)\}$. Note that $\{A, B, C\}$ partitions the assembly and $|A| - |B| = 0$ and $|B| = |A| < N/2$. At the proposal stage, $i', i'' \in A$ propose the formation of A and $j', j'' \in B$ propose the formation of B. No other agent other than these four proposes forming a voting bloc. At the acceptance stage, given these four proposals, all $i \in A$ including i'' accept the invitation by i' and all $j \in B$ including j'' accept the invitation by j'. If i' and/or j' deviate at the proposal stage, agents accept the invitations by i'' and j'' instead. All agents in $A \cup B$ ignore any other individual deviation at the proposal stage and continue to accept the invitations by i' and j'. Following deviations by more than one agent at the individual stage, agents play an arbitrary undominated equilibrium of the continuation game. With these acceptance strategies, no agent has an incentive to individually deviate at the proposal stage, since the outcome does not change following the deviation. At the acceptance stage, assuming that the acceptance strategies for the equilibrium proposals are indeed a best response, after a deviation at the proposal stage that proposes the formation of a voting bloc D with at least two members of $A \cup B$, it is a mutual best response for every invited agent to reject this invitation to join D, because given that the other agent(s) reject(s) the invitation to form D, D is not going to form. Accepting an invitation to form a bloc D that only includes one member $i \in A \cup B$ is not a best response for i because all agents outside $A \cup B$ have the same ideal policy, so only i has her vote reversed by party discipline in D. Given that the distribution of ideal policies inside A and B are symmetric with respect to the x axis, the set of policies that pass if either A or B forms is symmetric with respect to the x axis: If only A forms, only policies to the right of the origin pass; if only B forms, only policies to the left of the origin pass. Given the symmetry along the vertical dimension, if only A forms, every $j \in B$ is hurt, and if only B forms, every $i \in A$ is hurt. It follows, if c is small enough, that all members of A and B are better off joining their voting blocs to keep the policy outcome at the origin.

Two– Odd K. At the proposal stage, $i', i'' \in A_1$ propose the formation of A_1, j', $j'' \in B_1$ propose the formation of B_1, and $i_{0,0}$ proposes the formation of $C_1 = \mathcal{N} \setminus \{A \cup B\} = \{i: p_i = (x, y) \text{ with } x = 0\}$. No other agent other than these five proposes forming a voting bloc. At the acceptance stage, given these four proposals, all $i \in A_1$ including i'' accept the invitation by i', and all $j \in B_1$ including j'' accept the invitation by j' and all members of C_1 reject the invitation to form C_1. If i' and/or j' deviate at the proposal stage, agents accept the invitations by i'' and j'' instead. All agents in $A_1 \cup B_1$ ignore any other individual deviation at the proposal stage and continue to accept the invitations by i' and j'. Agents in C_1 ignore deviations at the proposal stage that do not involve members of C_1 and continue to reject the invitation to form C_1. These agents also ignore proposals that invite members of C_1, as long as ignoring them is not undominated. If they receive an invitation that weakly dominates rejection of all invitations, then all $i \in C_1$ except $i_{0,1}$ accept the invitation to join C_1, and $i_{0,1}$ accepts the new invitation. With these acceptance strategies, no agent has an incentive to individually deviate at the proposal stage, since the outcome does not change following the deviation. It remains to be shown that these acceptance strategies are best responses. By the same argument as in case one, if A_1 forms and c is low enough, every $i \in B$ is better off if B_1 forms, hence accepting the invitation to form B_1 is a best response, and same for A_1. Given that A_1, B_1 form, the policy outcome is $(0, 0)$ regardless of whether C_1 forms or not, hence it is a best response not to form it. As before, following a deviation at the proposal stage that proposes the formation of a voting bloc D with at least two members of $A_1 \cup B_1$, it is a mutual best response for all the invited members who belong to $A_1 \cup B_1$ to reject the invitation, since D is not going to form given that the other agent(s) reject(s) the invitation. Following a deviation that proposes the formation of D with only one member of $A_1 \cup B_1$ and this invitation is such that it makes ignoring all invitations weakly dominated, given that only one member of C_1 accepts this invitation, D is not going to form, so the member of $A_1 \cup B_1$ is better off rejecting D to form A_1 and B_1 instead. For the members of C_1, rejecting both the invitation to form C_1 and the invitation to form D is weakly dominated, but accepting either, if c is low enough, is undominated. Given that the member of $A_1 \cup B_1$ invited to D rejects D and prefers to have A_1 and B_1 formed instead, any acceptance strategies such that C_1 does not form is a mutual best response for the members of C_1, in particular, $i_{0,1}$ accepting D and all others accepting C_1 is a mutual undominated best response. Hence the equilibrium is complete.

References

ALDRICH, J. H. *Why Parties? The Origin and Transformation of Political Parties in America.* Chicago: University of Chicago Press, 1995.

BARON, D. "A Noncooperative Theory of Legislative Coalitions." *American Journal of Political Science* 33, no. 4 (1989): 1048-1084.

BLOCH, F. "Non-Cooperative Models of Coalition Formation in Games with Spillovers." In C. Carraro ed. *The Endogenous Formation of Economic Coalitions.* Edward Elgar Publishing, 2003.

BLOCH, F. and A. GOMES. "Contracting with Externalities an Outside Options." *Journal of Economic Theory* 127, no. 1 (2006):171-201.

CAILLAUD, B. and J. TIROLE. "Parties as Political Intermediaries." *Quarterly Journal of Economics* 117, no. 4 (2002): 1453-1489.

CARRARO, C. *The Endogenous Formation of Economic Coalitions.* Edward Elgar Publishing, 2003.

COX, G. W. and M. D. MCCUBBINS. *Legislative Leviathan.* Berkeley: University of California Press, 1993.

EGUIA, J. X. "United We Vote." *Journal of Public Economic Theory* 9, no. 4 (2007): 1-33.

HART, S. and M. KURZ. "Endogenous Formation of Coalitions." *Econometrica*, 51, no. 1 (1983): 1047-1064.

HUMPHREYS, M. "Theory of Coalitions and its Applications." *Annual Review of Political Science* 11 (2008): 351-386.

HYNDMAN, K. and D. RAY. 2007. "Coalition Formation with Binding Agreements." *Review of Economic Studies* 74(4): 1125-1147.

JACKSON, M. and B. MOSELLE. "Coalition and Party Formation in a Legislative Voting Game." *Journal of Economic Theory* 103, no. 1 (2002): 49-87.

KREHBIEL, K. *Pivotal Politics: A Theory of US Lawmaking.* University of Chicago Press, 1998.

LEVY, G. "A Model of Political Parties." *Journal of Economic Theory* 115, no. 2 (2004): 250-277.

MORELLI, M. "Party Formation and Policy Outcomes under Different Electoral Systems." *Review of Economic Studies* 71, no. 3 (2004): 829-853.

NASH, J. "Equilibrium Points in N-Person Games." *Proceedings of the National Academy of Sciences* 36 (1950): 48-49.

PLOTT, C. R. "A Notion of Equilibrium and its Possibility under Majority Rule." *American Economic Review* 57, no. 4 (1967): 331-347.

RAY, D. *A Game-Theoretic Perspective on Coalition Formation.* Oxford University Press, 2007.

ROMER, T. and H. ROSENTHAL. "Political Resource Allocation, Controlled Agenda, and the Status Quo." *Public Choice* 33, no. 4 (1978): 27-44.

SNYDER, J. and M. TING. "An Informational Rationale for Political Parties." *American Journal of Political Science* 46 (2002): 90-110.

THRALL, R. M. and W. F. LUCAS. "N-Person Games in Partition Form." *Naval Research Logistics Quarterly* 10 (1963): 281-293.

YI, S. "Endogenous Formation of Economic Coalitions: A Survey of the Partition Function Approach." In C. Carraro ed. *The Endogenous Formation of Economic Coalitions.* Edward Elgar Publishing, 2003.

17

An Automated Model of Government Formation

Enriqueta Aragonès
Institute of Economic Analysis
Spanish National Research Council (CSIC)

Pilar Dellunde
Department of Philosophy
Autonomous University of Barcelona

> *There can be no doubt at all that the government formation process, which begins with a particular election result that leaves open many coalition possibilities and ends with the formation of a government comprising a particular combination of parties, is one of the fundamental processes of European parliamentary democracy. Understanding how a given election result leads to a given government is, when all is said and done, simply one of the most important substantive projects in political science.*
>
> LAVER AND SCHOFIELD (1990, 89)

17.1. Introduction

The aim of this paper is to propose a suitable procedure to analyze the process of government formation in a parliamentary democracy. In particular, we would like to characterize a large class of bargaining strategies suitable for the agents involved in this process and analyze their performance.

The process of government formation in parliamentary democracies is characterized by many institutional features that lie in the definition of the process itself: the choice of the formateur (the party in charge of initiating the negotiations), the support needed to form a government (the proportion of parliamentary votes required for the investiture of the executive), the existence of a limited amount of time for the negotiation after which either a new formateur might be selected, or a new election is called, among others.

There are other institutional features that indirectly may affect this process such as: the support needed for policy implementation (the proportion of parliamentary votes required to pass a law), the power of the executive in terms of policy implementation (relative to the power of the legislative), the existence of different governmental levels, the requirements to call a vote of confidence, the possibility of calling early elections,... The centrality of the process of government formation is such that almost all institutional features might be thought relevant for it.

Other features might have an effect on the result of the parties' negotiation for government formation, such as the current characteristics of the party system. The number of parties that have parliamentary representation as well as their ideologies and the distribution of parliament seats among them are specific features of great relevance to determine the process of government formation.

The number and type of issues that appear as relevant during the bargaining process might also have an important effect on the negotiation process and its outcome. We might distinguish between quantitative issues, that refer to the distribution of the value of holding office (more specifically, they might refer to the distribution of executive positions among party members, for instance, the government presidency, the parliament presidency, the cabinet ministers,...) and qualitative issues, that refer to issues that involve an ideological dimension (for instance, the amount of public service to be provided, the level of taxation,...).

In addition there are intangible issues such as: getting the deal done, making voters happy, standing by one's principles, being fair, beating the competitors, looking good to the constituency, preserving one's reputation, setting a precedent,... These issues might also play a central role during the process of government formation, however they will be treated in a different way since they mostly affect the way in which parties make decisions.

Finally, the specific way in which the negotiations take place might also have an important effect on the process and its outcome. In particular, the negotiation can be performed sequentially, given an order of the issues, simultaneously by subsets of issues,... And the different possible coalitions might also bargain sequentially or simultaneously.

Hence, when studying the process of government formation in parliamentary democracies there are a lot of features that have to be taken into account. The existing theories of bargaining in economics, including Rubinstein's (1982) seminal paper and all its extensions[1], offer a lot of insights that have been used by the emerging literatures on legislative bargaining and government formation.

---

[1] Extensions of Rubinstein's model include: Sutton (1986) and Osborne and Rubinstein (1990) that discuss the effects of assuming unanimity; Krishna and Serrano (1996) that generalize it to any number of players; Jun (1987) and Chae and Young (1988 and 1994) introduce bilateral negotiations, and Chaterjee, Dutta, Ray, Sengupta (1993) introduce transferable utility.

The legislative bargaining literature includes Baron and Ferejohn's (1989) basic model of legislative bargaining; Baron's (1991a) application to pork barrel; Romer and Rosenthal (1978 and 1979) assume an exogenous status quo; Eraslan (2002) shows the uniqueness of payoffs for stationary equilibria; Banks and Duggan (2000 and 2002) extend it to bargaining over policy; Calvert (1989) analyzes legislative reciprocity, and Jackson and Moselle (2002) combine bargaining over policy with distributive bargaining.

The literature on government formation includes Baron and Ferejohn (1991b) which extends their previous work to explain government formation; Baron (1993) assumes endogenous parties; Diermeier and Myerson (1994) introduce a veto player; Laver and Schofield (1990) and Laver and Shepsle (1996) analyze the role of the formateur; Austen-Smith and Banks (1988) introduce strategic voting; Merlo (1997) studies the effects of deadlines and delays; Diermeier and Merlo (2000) consider dynamic features; Baron and Diermeier (2001) analyze a multidimensional space; Diermeier, Eraslan and Merlo (2003) study the effects of the investiture vote and the no-confidence vote; Aragones (2007a and 2007b) analyzes a two dimensional model and applies it to a real world case (Catalan Government).

Most of these works focus on the effect of particular features of the process of government formation. We provide a procedure that allows to consider a large number of features and to compare their effects. For this purpose, we describe a formal model of government formation and combine it with an automated and tractable negotiation mechanism for autonomous agents. Our multilateral bargaining model is based on the bilateral automated negotiation by Faratin, Sierra and Jennings (1998) and extends it to a multi-agent automated negotiation.

The rest of the paper is organized as follows: the next section presents the formal model. Section 17.3 describes the negotiation process. Section 17.4 describes the automated negotiation mechanism. In section 17.5 we introduce the basic hypothesis to be tested in future experimental research. Finally section 17.6 offers some concluding remarks.

17.2. The formal model

We consider a finite set of political parties $\mathbf{P} = \{1, 2,..., P\}$ with parliamentary representation. We assume that an election has already taken place, and the proportion of parliament seats that party p has obtained is given by v_p. We assume that $\sum_{p\in\mathbf{P}} v_p = 1$ and $0 \leq v_p < 1/2$ for all $p \in \mathbf{P}$. We do not consider the possibility that a party has a majority of the seats, that is, $v_p > 1/2$ for some party p, since the analysis of this case would lead to trivial results[2].

We assume that the issues that parties care about are of two kinds: qualitative issues and quantitative issues. Let $\mathbf{Q} = \{1, 2,..., Q\}$ denote the set of qualitative issues, and let $\mathbf{S} = \{1, 2,..., S\}$ denote the set of quantitative issues. We assume that both Q and S are natural numbers. Let q and s denote a generic elements of sets $\mathbf{Q}$ and $\mathbf{S}$ respectively.

We represent an issue $q \in \mathbf{Q}$ by a real interval $[0, \bar{q}] \subset \Re$. Each $x \in [0, \bar{q}]$ represents a specific policy position on issue q. We assume that each political party has a most preferred policy over each one of the qualitative issues, derived from the party's ideological principles. Let $q_p \in [0, \bar{q}]$ denote the most preferred policy position by party p on issue q. The elements of $\mathbf{Q}$ can be thought of as representing the ideological issues over which political parties have preferences. We assume that the parties' preferences on this space as such that different political parties have different ideal points.

A quantitative issue, $s \in \mathbf{S}$ can be thought of as a set of executive positions that correspond to the same rank. For instance, the government presidency, the parliament presidency, the cabinet ministers,... We represent an issue $s \in \mathbf{S}$ by an interval $[0, \bar{s}] \subset \Re$. Each quantitative issue s represents a specific rank of executive positions and $\bar{s}$ represents the total amount of seat of rank s available. We assume that *ceteris paribus* the payoffs of all political parties increase with the number of executive positions that they obtain and with the rank of the positions that they obtain. An allocation of executive positions among parties will be given by a specific allocation of positions of each rank.

Thus, parties have to bargain over a multidimensional policy space of $Q + S$ dimensions, where each dimension represents a different issue.

We assume that parties are mainly concerned about holding office, that is, they care about being members of the governing coalition and about obtaining executive positions on the quantitative issues. On the other hand, the policy implemented on the qualitative issues may affect the vote support of parties that are members of the governing coalition in future elections, and therefore their future payoff. Naturally, parties that are not members of the governing coalition are perceived by voters as detached from the implemented policies, because voters do not consider them responsible. Therefore, it is reasonable to think that their payoffs are not affected by the government's policy choices. It is normally

[2] We rule out the possibility that $v_p = 1/2$ because it does not represent a real situation since most parliaments have an odd number of seats.

the case that parties that are not members of the governing coalition are not allocated any executive positions. This implies that their payoffs are not affected by the government choice on the quantitative issues either. Thus, the payoffs of parties that are not members of the governing coalition would not be affected by any of the government's choices. We normalize the utility of a party that is not a member of the governing coalition to zero, and we represent the utility that party p obtains if it becomes a member of the governing coalition by $U_p\ (\mathbf{q}, \mathbf{s})$ where $(\mathbf{q}, \mathbf{s})$ represents the vector of policies implemented by the governing coalition.

Therefore, the payoff function of party p can be defined as follows:

$$V_p\ (G, (\mathbf{q}, \mathbf{s})) = \begin{cases} 0 & \textit{if} \quad p \notin G \\ U_p\ (\mathbf{q}, \mathbf{s}) & \textit{if} \quad p \in \mathrm{G} \end{cases} \tag{17.1}$$

Where G denotes a governing coalition. A governing coalition must be decisive in terms of policy choices, that is, it must be supported by at least a majority of seats of the parliament. Since we assume that no party obtains a majority of the seats, parties are supposed to form coalitions that have the support of at least a majority of the parliament in order to form a decisive coalition. Within the proposed governing coalition, the members will have to negotiate a policy compromise.

$\mathbf{q} = (x_1,\ldots, x_Q) \in \Re^{|\mathbf{Q}|}$ is a Q-dimensional vector and each component represents a policy compromise on each qualitative or ideological issue. $\mathrm{s} \in \Re^{|\mathbf{S}\times\mathbf{P}|}$ represents the allocation of executive positions among the members of a coalition. $\mathbf{s}$ has $|\mathbf{S}|$ components, one for each quantitative issue[3]. Each component of $\mathrm{s} = (s_1,\ldots, s_S)$ is represented by a vector of dimension $\mathbf{P}$, $s_j = \left(s_j^1,\ldots, s_j^P\right)$ where $s_j^p \geq 0$ represents the number of executive positions of rank j allocated to party p. We order the different ranks in a decreasing manner, that is, we assume that higher ranked positions are represented by lower values of j. Therefore, the dimension of a vector of policy proposals $(\mathbf{q}, \mathbf{s})$ is $r = |\mathbf{Q}|+|\mathbf{S}\times\mathbf{P}|$.

Only parties that are members of the governing coalition may obtain a positive allocation of executive positions of any rank. The sum of the components of the vector corresponding to issue s_j cannot be larger than $\overline{s}_j$, thus for all $\mathbf{s} \in \mathbf{S}$ and for all j we must have $\sum_{p\in\mathbf{P}} s_j^P = \sum_{p\in G} s_j^P \leq \overline{s}_j$. Let $\mathbf{s}^p$ denote the $\mathbf{S}$ -dimensional

[3] We will assume that the set of positions to be allocated is exogenous, that is, the governing coalition can neither create new positions to be allocated, nor can increase the number of positions at any level, even though this is not what happens in real situations. This will be left for future work.

vector whose components represent the allocations of executive positions of each rank to party *p*.

We assume that the utility that party *p* obtains if it becomes a member of the governing coalition when the implemented policy is $(\mathbf{q}, \mathbf{s})$ is given by

$$U_p(\mathbf{q}, \mathbf{s}) = Q_p(\mathbf{q}) + K_p S_p(\mathbf{s}) \tag{17.2}$$

where

$$Q_p(\mathbf{q}) = -\sum_{q \in \mathbf{Q}} \pi_p(q)\,(x_q - q_p)^2 \tag{17.3}$$

represents the payoff that party *p* derives from the compromise reached on the qualitative issues[4]; *q* represents the policy compromise on issue *q* by the governing coalition, and $\pi_p(q)$ represents the weight that party *p* assigns to issue *q*, such that $\pi_p(q) > 0$ and $\sum_{q \in \mathbf{Q}} \pi_p(q) = 1$.

According to this utility function, parties' preferences over policies are single peaked and convex. The parameters $\pi_p(q)$ represent the relative importance of the qualitative issues in the ideology of party *p*. If $\pi_p(q) = \pi_p$ for all $q \in \mathbf{Q}$ we have that all qualitative issues have the same effect on the utility of party *p*, thus all qualitative issues are as important in the ideology of party *p*. If $\pi_p(q) > \pi_p(q')$ then issue *q* is regarded as more important than issue q' by party *p*. Since $\sum_{q \in \mathbf{Q}} \pi_p(q) = 1$, we have that as the value of $\pi_p(q)$ increases, issue *q* becomes more important for party *p*, and therefore party *p* requires a more favorable compromise on the other issues for a given deal on *q*.

Similarly, $S_p(\mathbf{s})$ represents the payoff that party *p* derives from the compromise reached on the quantitative issues. In general one can assume that the payoff that party *p* derives from the quantitative issues depends on the distribution of executive positions to all coalition members. With this formulation we could represent instances in which a party might care about the difference between the executive positions he obtains in each rank and the positions that some other coalition members obtain in each rank.

However it seems natural to assume that a party only cares about the number of executive positions allocated to himself. In particular, without much loss of

[4] Since the utility function assumed on the qualitative issues is separable, we are assuming that the perception of the parties is that the ideological issues are not interrelated.

generality we could assume that $S_p(\mathbf{s}) = \sum_{s_j \in \mathbf{S}} \mu_p(s_j) \cdot s_j^p$ where $\mu_p(s)$ represents the weight that party p assigns to issue s, and it is such that $\mu_p(s) > 0$ and $\sum_{s \in \mathbf{S}} \mu_p(s) = 1$; and since parties derive a larger utility from higher ranked executive positions, we assume that $\mu_p(s_j) > \mu_p(s_{j'})$ for $j < j'$. Finally, $K_p > 0$ represents the relative importance that party p assigns to the quantitative issues with respect to the qualitative issues.

Notice that the payoff that party p obtains from the quantitative issues, $S_p(\mathbf{s})$, is always positive while the payoff that he obtains from the qualitative issues, $Q_p(\mathbf{q})$, is always negative. This implies that $K_p S_p(\mathbf{s})$ may be thought of as a reservation value: a party will never accept to become a member of a governing coalition if it has to support a policy compromise on the qualitative issues that gives him a (dis)utility larger than the value that he obtains from the quantitative issues.

This observation allows us to define an Individual Rationality constraint for each party. Formally, the set of policies from which party p derives a utility of zero, $\{(\mathbf{q}, \mathbf{s}): U_p(\mathbf{q}, \mathbf{s}) = 0\}$, defines the boundary of the set of policies that are Individually Rational for party p. The size of this set depends on the magnitude of $S_p(\mathbf{s})$: the larger the payoff that party p derives from the quantitative issues, the larger the set of policies that party p is willing to support in a given governing coalition. That is, the more a party values to be a member of the governing coalition the more flexible he will be in terms of trading-off policy.

17.3. Negotiation structure and tactics

The negotiation process is initiated with the selection of a formateur: a party that is in charge of making the first offers and it can also be responsible for building up a governing coalition. There is a time limit for the negotiation that is set up by institutional regulations. We assume that this time limit is exogenously given and denoted by $t_{\max}$. After the time limit is reached either we will assume that the game is over or that the game restarts from the beginning. In the first case, the negotiation terminates and if there has not been an agreement within a majoritarian coalition everyone obtains the payoff corresponding to a failure.

The negotiation process among the agents consists of a succession of offers and counter offers of values for $(\mathbf{q}, \mathbf{s})$ that continues until an offer is accepted by all the members of a decisive coalition within the maximal time limit or until the time limit is reached. If an offer is accepted by all the members of a decisive coalition within the time limit, the government forms and the policy compromise is implemented. The parties within the governing coalition receive the payoffs corresponding to the implemented policy and all other parties receive a zero payoff. If no decisive coalition reaches an agreement before the time limit, the negotiation is over. In this case we assume that either a new formateur is chosen and the whole process starts again with discounted payoffs or that the game is over and all parties receive a zero payoff.

A sequence of offers and counter-offers is called a negotiation thread. A tactic is a function that generates decision and uses as input a given single criterion. In our case a tactic may generate either an offer in terms of a policy vector, or a decision over which parties to invite to join in a coalition. When generating policy vectors, tactics might be based on criteria such as the amount of time remaining before the maximal time limit of the negotiation, the best offer that a party has received so far, the history of the strategies used by the different parties, expectations on the other parties' behavior, among others. When generating decisions over which parties to invite to join in a coalition, tactics might be based on criteria such as properties of the coalition in terms of size, properties of the coalition in terms of the ideologies of the members, etc.

A strategy for a party at a given moment of the negotiation has two main components: the decision over which parties to extend the offer (to invite to join in a coalition) and the kind of offer in terms of a policy choice. We will assume that the strategies of the parties are generated by linear combinations of tactics. The different weights assigned to the different tactics (or criteria) in a given negotiation strategy indicate their relative importance. Since each tactic is based on a specific criterion, the different weights assigned to each tactic represent the relevance or importance assigned to the corresponding criterion. In order to achieve flexibility in negotiation the parties may wish to change their ratings of the different criteria over time. For example, at the beginning of a negotiation thread it may be more important to take into account the competitors' behavior than the remaining time, in which case the tactics that emphasize the behavior of other parties will be given greater weights than those based on the amount of time remaining.

We will now describe different protocols to select the formateur, different procedures that define the negotiation threads, different parties' types when selecting a partner, and different ways to construct a sequence of offers. They correspond to particular examples of tactics that can be used.

17.3.1. Selecting the formateur

We will consider different protocols that select the formateur: the party who starts the negotiation.

Protocol 1: the formateur is the party with the largest share of seats.

Protocol 2: the formateur is chosen by a lottery and each party is selected with a probability proportional to his share of seats.

Protocol 3: the formateur is chosen by a lottery and each party is selected with equal probability.

Formally, the protocol to decide the formateur is given by a lottery $(f_1,.., f_P)$ such that $f_p \geq 0$, $\sum_{p \in \mathbf{P}} f_p = 1$ and

Protocol 1: $f_p = 1$ for p such that $v_p > v_p{}'$ for all $p' \neq p$.
Protocol 2: $f_p = v_p$ for all p.
Protocol 3: $f_p = \frac{1}{P}$ for all p.

We will also consider the possibility of selecting more than one formateur, that is, several parties might be able to start different negotiation threads that would take place simultaneously. In this case the different formateurs may be selected using a random device that, as before, could depend on the parties' seat shares in the parliament.

17.3.2. Choosing a coalition

We may consider three different negotiation procedures for parties to engage in a negotiation: bargaining among parties, bargaining within coalitions, and simultaneous bargaining within coalitions. In each the party that is in charge of engaging the negotiation has to decide which coalition of parties he makes an offer to.

Bargaining among parties: The party selected by the protocol becomes the formateur. The formateur has to choose a coalition of parties C and makes an offer to the parties that are members of a coalition C.

We will assume an exogenously given ordering of the parties represented by a permutation of the elements of **P**. The ordering within a given coalition is determined by restricting the application of the ordering on **P** to the members of the coalition. We will consider different permutations and analyze their effect on the results. A permutation that orders the parties according to the proportion of seats obtained by each party is of particular relevance in our case.

Parties in C respond sequentially according to the exogenously given ordering by accepting or rejecting the offer. If no party in C rejects the offer then the game is over and the offer is implemented: the coalition C forms the government and the proposed policy is implemented. Otherwise, the first party that rejects the offer becomes the new proposer. He has to choose a coalition of parties C' and make an offer to the parties that are members of coalition C'. If no party in C' rejects the offer then the game is over and the offer is implemented: the coalition C' forms the government and the proposed policy is implemented. And so on.

When deciding to whom to make an offer, parties may be of one of three types:

— A party is an **explorer** if he never makes an offer to a party or coalition that made him the last offer.
— A party is a **replier** if he always makes an offer to a party or coalition that made him the last offer.
— A party is of the **mixed-type**, if he makes an offer to a party that made him the last offer only if the last offer that he obtained is good enough, otherwise he makes the offer to a different party.

We say that a party considers that an offer is good enough if the utility level that he obtains with that offer is close enough to the utility level that he would obtain with the counter offer he would make. Let $\Psi_p \in \Re^+$ denote the threshold used by the party to determine whether the last offer he obtained is good enough.

Bargaining within coalitions: The party selected by the protocol becomes the formateur. The formateur has to choose a coalition of parties C and makes an offer to the parties that are members of a coalition C.

Parties in C respond sequentially according to the ordering exogenously given, as described before, by accepting the offer, making counter-offers or rejecting the offer. As long as all parties are either accepting offers or making counter-offers the negotiation proceeds. When all parties accept a given offer the negotiation ends, the coalition becomes the governing coalition and the set of policies proposed in the accepted offer are implemented. If a party rejects a coalition (this implies that he rejects the offers that he has received and he chooses not to make a counter-offer) the formateur has to choose a new coalition and start a new negotiation thread.

Simultaneous bargaining within coalitions: In this case we assume that at the first stage of the game several formateurs are selected. Each formateur will behave as in the case of bargaining within coalitions. Thus, the negotiation threads described above will proceed in parallel, and a given party may be involved in different negotiation threads at the same point in time. The first negotiation thread that ends successfully is the one that forms the governing coalition.

We can also assume additional properties on the type of coalitions that are called during the negotiation process derived from the existing theories of coalition formation. In particular, parties could consider coalitions based on policy blind theories. In this line we find the Minimal Winning Coalition Theory by von Neumann and Morgenstern (1953), the Minimum Winning Coalition Theory by Riker (1962), and Leiserson's (1966) refinement of the Minimal Winning Coalition Theory using the 'smallest number of parties' bargaining principle. We could also consider coalitions based on theories that assume that policy choice plays a role in the parties' payoffs, such as the Minimal Connected Winning Coalition Theory by Axelrod (1970) and its refinement based on the smallest ideological range by de Swaan (1973).

In addition, we could either restrict parties to negotiate only over minimal winning coalitions or we could allow them to form surplus coalitions. We would need a much more complex framework in order to allow for minority coalitions to form the government.

Finally, we could use the same model and procedure to analyze the effect of the requirement of a q- rule with $q > 1/2$ (supermajorites) over the vote of the parliament in order to form a government.

17.3.3. Offers and counteroffers

An offer from party p at time t is represented by O_p^t where t is an integer that denotes the time at which the proposal is offered, $p \in \mathbf{P}$ denotes the party that offers the proposal. The offer O_p^t denotes a particular value of the vector $(\mathbf{q}, \mathbf{s})$ described before and it represents the policy proposal that this party makes.

When a party receives an offer it has to evaluate it and decide whether to accept it or reject it. If the utility that the party recipient of the offer, p' derives from an offer O_p^t proposed to him is larger than the utility that he would derive from the counter offer that he is ready to send, $O_{p'}^{t+1}$, then party p' accepts the offer. Otherwise he rejects it. After a party has rejected an offer he becomes the new proposer, and therefore, he chooses a coalition and a policy and starts a new negotiation thread.

Formally, for $t \leq t_{\max}$

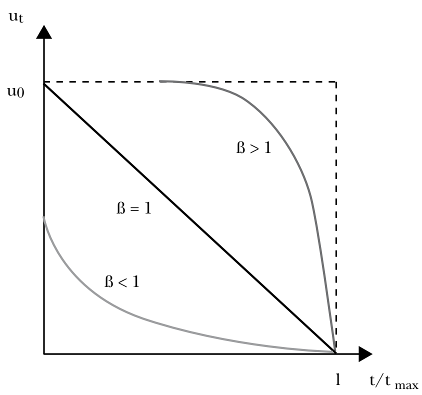
$$I_{p'}(t, p, p', O_p^t) = \begin{cases} reject & if\, U_{p'}(O_p^t) < U_{p'}\left(O_{p'}^{t+1}\right) \\ accept & if\, U_{p'}(O_p^t) \geq U_{p'}\left(O_{p'}^{t+1}\right) \end{cases} \tag{17.4}$$

For each party we construct a sequence of offers, one for each period t, that will be used by the party only when either it receives an offer or it is selected to be a proposer. It is natural to assume that the offers that a party sends out are more and more generous for the party's competitors over time, as the party becomes more and more impatient to reach an agreement. Therefore we assume that the offers send out by a party over time are such that the utility levels obtained by the party from his own offers are declining, starting at an exogenously given initial utility level (u^0) at time $t = 0$ until they reach his reservation value as t approaches t_{max}.

An exogenously given functional form will be used to construct the sequence of offers for each one of the parties. Specific features and parameters values of the functional form used will be used in order to indicate the rate of patience or impatience of a party, the speed at which a party is willing to concede, etc. We will use two different families of functions that exhibit this type of behavior: polynomial and exponential.

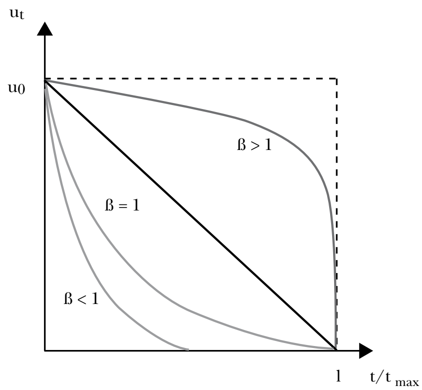
In the polynomial case a functional form that computes the decreasing level of utility over time is given by $u^t = u^0 - u^0\left(\frac{t}{t_{max}}\right)^{\frac{1}{\beta}}$ where $\beta > 0$ is a parameter that determines its degree of convexity. When $\beta < 1$ (concave function) the level of utility goes rapidly close to the reservation value and then keeps conceding slowly. These tactics are called 'opening up'. When $\beta = 1$ (linear function) the level of utility moves linearly with time to the reservation value. These are called linear tactics. Finally, when $\beta > 1$ (convex function) the initial level of utility is maintained until the time is almost exhausted, and then it decreases rapidly. These tactics are called 'holding back'. See figure 17.1.

FIGURE 17.1: Polynomial case: β>1 represents a 'holding back' strategy, β<1 represents an 'opening up' strategy, and β=1 represents a linear strategy



Similarly, in the exponential case a functional form that computes the decreasing level of utility over time is given by $u^t = 1 + u^0 - \exp\left\{\left(\frac{t}{t_{max}}\right)^{\beta} \ln(1 + u^0)\right\}$, where β > 0 is a parameter that determines its degree of convexity. When β < 1 (concave function), the level of utility goes rapidly to close to the reservation value. These tactics are called 'opening up'. When β > 1 (convex function) the initial level of utility decreases very slowly until the time is almost exhausted, and then it decreases rapidly. These tactics are called 'holding back'. When β = 1 (convex function) the tactics are 'holding back' but the level of utility moves a bit slower than with higher values of β. See figure 17.2.

FIGURE 17.2: Exponential case: β>1 represents a 'holding back' strategy and β<1 represents an 'opening up' strategy



Comparing the two families of functions we have that on the one hand in both cases values of β larger than 1 imply 'holding back' tactics, and values of β smaller than 1 imply 'opening up' tactics. On the other hand, for values of β larger than 1 the exponential function concedes faster at the beginning than the polynomial one, and for values of β smaller than 1 the polynomial function concedes faster at the beginning than the exponential one.

These two families of functions provide an infinite set of tactics (for all possible values of and thus they will allow us to model concession in very different ways).

17.4. Automated negotiation model

Artificial intelligence's objective when applied to the negotiation framework is to present a formal model with an automated and tractable negotiation mechanism for autonomous agents, although the outcomes might not be optimal. A multi-agent system (MAS) is a system composed of multiple interacting intelligent agents. The agents in a multi-agent system have several important characteristics:

— Autonomy: the agents are at least partially autonomous
— Local views: no agent has a full global view of the system, or the system is too complex for an agent to make practical use of such knowledge
— Decentralization: there is no one controlling agent (or the system is effectively reduced to a monolithic system)

Typically multi-agent systems research refers to software agents. However, the agents in a multi-agent system could equally well be robots, humans or human teams. A multi-agent system may contain combined human-agent teams. Multi-agent systems can manifest self-organization and complex behaviors even when the individual strategies of all their agents are simple.

In the AI literature we can find different examples of protocols for many-to-many negotiations. In Kraus, Wilkenfeld and Zlotkin (1995) the authors introduce a strategic model of negotiation that takes the passage of time during the negotiation process into account. A distributed negotiation mechanism is introduced that is simple, efficient and stable. Using this negotiation mechanism autonomous agents have strategies that result in efficient agreements without delays. In their model they consider the problem where agreements involve all the agents, but they don't deal with situations in which agents are free to form any coalition that includes some of the agents while excluding others. In Dang and Huhns (2005) and in Nguyen and Jennings (2004) the authors introduce two approaches that differ from ours because they consider concurrent negotiations that are either multiple one-to-many or many-to-many bilateral.

In the present section we sketch an algorithm for a many-to-many multilateral negotiation protocol. We assume that the delivery time is negligible compared to the time interval of each negotiation round. First of all, for the sake of clarity we introduce the following distinctions among sets of parties, in relation with their autorization to start negotiations, their desire to continue a negotiation thread or the fact that they have sent a message to a certain agent.

A_p^t as the set of parties with whom party p is authorized to lead a negotiation at time t

S_p^t as the set of parties with whom party p wants to negotiate at time t

Q_p^t as the set of parties from whom party p receives messages at time t

The agents communicate and compromise to reach mutually beneficial agreements. We will use the following notation for representing the negotiation messages:

$O_{p \to q}^t$ as the proposal that party p offers to party q at time t

$M_{p \to q}^t$ as the message that party p sends to party q at time t

$M_{p \to q}^t \in \{O_{p \to q}^t, Accept, Reject, Pre - Accept, Over\}$

TABLE 17.1: Negotiation messages

Accept	An agent formalizes the pre-accepted offer ending the negotiation
Pre-Accept	An agent pre-accepts a previous offer
Reject	An agent rejects a previous offer
Over	An agent ends a negotiation thread

Automated negotiation is a key form of interaction in systems that are composed of multiple autonomous agents. The objective of these interactions is to reach an agreement through an iterative process of making offers. The content of the proposals is a function of the strategy of the agents. The sketch of algorithm we present here enables software agents to generate offers during the negotiation. In the formalization of the algorithm we distinguish, at any given time t, two types of agents, one agent that leads the negotiation at time t, the formateur (that we identify with the property $A_p^t \neq \emptyset$) and the rest of agents that either answer their proposals or remain silent, because the formateur does not negotiatie with them.

A Reject Message has as a consequence that a new negotiation thread starts, involving new agents, and according to the protocol chosen, a new leader of the negotiation taking the responsibility of getting to an agreement. An Accept Message finishes the negotiation while a Pre-Accept Message is a provisional acceptance of an offer, submitted to further negotiation if one of the agents does not pre-accept the offer made by the formateur. Since there is an institutional maximum time limit, the protocol has the termination property, that is, guarantees that any negotiation process following it will eventually terminate. For the sake of clarity we don't include here the instructions for time $t = t_{max}$.

Initialization

If agent p is a formateur (that is, $A_p^{t_0} \neq \emptyset$) **then**
sends $O_{p \to q}^{t_0}$ to all $q \in S_p^{t_0} \cap A_p^{t_0}$

Negotiation

while $t < t_{max}$ **do**
if $A_p^{t+1} \neq \emptyset$ **then** in case that, for some $q_0 \in Q_p^t$, $q_0 \notin S_p^{t+1}$
sends *Reject* to every $q \in A_p^{t+1} \cap Q_p^t$
in case that, for some $q_0 \in Q_p^t$, $M_{q_0 \to p}^t = Reject$
sends *Over* to every $q \in S_p^{t+1} \cap A_p^{t+1}$ otherwise
in case that, for every $q_0 \in Q_p^t$, $M_{q_0 \to p}^t = Pre - Accept$
sends *Accept* to every $q \in S_p^{t+1} \cap A_p^{t+1}$ and **then End.**
in case that, for some $q_0 \in Q_p^t$, $M_{q_0 \to p}^t = O_{q_0 \to p}^t$
if $U(O_{q_0 \to p}^t) \geq \max\ (U\, O_{p \to q_0}^{t+1}), \max\ \{U(O_{r \to p}^t : r \in S_p^{t+1} \cap Q_p^t\}$
sends *Pre – Accept* $(O_{q_0 \to p}^t)$ to q_0.
and for every $q \in S_p^{t+1} \cap A_p^{t+1}$, $q \neq q_0$, **sends** $O_{p \to q}^{t+1} = O_{q_0 \to p}^t$
otherwise send $O_{p \to q}^{t+1}$ to every $q \in S_p^{t+1} \cap A_p^{t+1}$
in case that, for some $q_0 \in Q_p^t$, $M_{q_0 \to p}^t = Over$
sends $O_{p \to q}^{t+1}$ to all $q \in S_p^{t+1} \cap A_p^{t+1}$
if $A_p^{t+1} = \emptyset$ **then**
for every $q \in Q_p^t$ but $q \notin S_p^{t+1}$ **sends** *Reject*
and for every $q \in S_p^{t+1} \cap Q_p^t$ **then**
case $M_{q \to p}^t = O_{q \to p}^t$ **then**
if $U(O_{q \to p}^t) \geq U(O_{p \to q}^{t+1})$

send *Pre – Accept* ($O^{t}_{q \to p}$)
otherwise send $O^{t+1}_{p \to q}$
case $M^{t}_{q \to p}$ = *Over* **then** don't send any message.
case $M^{t}_{q \to p}$ = *Reject* **then** don't send any message.
case $M^{t}_{q \to p}$ = *Pre – Accept* **then sends** *Pre – Accept.*

17.5. Negotiation strategies

We will analyze the outcomes obtained with different negotiation strategies, that is, different linear combinations of tactics for different and relevant values of the parameters.

The simplest set up would be obtained if we assume that a negotiation strategy consists of a single tactic that is kept constant over time, that is, all weights of the linear combination are equal to zero except for one. Increasing the sophistication level we may consider that the weights of the linear combination are constant.

A more complex set up would have these weights changing according to the history of the negotiation, that is, according to the results obtained in the previous negotiation rounds. Finally, we could reach a higher level of sophistication by considering that the weights change according to the expectations that the parties build upon the behavior of their competitors as the negotiation process evolves.

We want to evaluate the performance of the strategies according to: the utility obtained by the parties, the number of deals made, and the net payoffs obtained by the parties computing as costs the number of negotiations rounds needed to obtain a deal. When analyzing the effect of the different protocols to select the formateur we have to compare the three protocols defined above for different values of the parameter v_p representing the vote share. Regarding the three types of behavior assumed when bargaining among parties, we need to compare the effect of each one being used against any combination of types of the competitors and in addition we will have to consider the effect induced by assuming different values of the threshold Ψ_p.

Finally, we have to analyze the effect of the different ways considered to construct an offer. In this case we have to distinguish between: a polynomial versus an exponential function; 'holding back' versus 'opening up' tactics, for different values of the parameter β; different values of the initial level of utility u^0, and different values of the time limit t_{max}. And we also have to analyze the effect of each one of these tactics when played against any combination of types for the competitors. In addition we will have to consider the effect of different values for the parameters of the payoff functions of the parties such as: the parties' ideal points and weights on the qualitative issues, the parties' weights on the quantitative issues, and the relative weight that parties' assign to quantitative issues with respect to qualitative issues.

From the combination of the formal model of government formation and the automated negotiation protocol described before, we should be able to test some hypotheses based on expected results. We list some of them here:

1. **Protocols:** a larger probability of being selected as a formateur, f_p, implies a clear advantage in terms of a larger probability of making a deal and a larger utility.
2. **Types of partners:** when most parties are repliers the formateur has an advantage. In particular, if there is a deal, he is always in it independently of his type. The number of possible coalitions increases with the proportion of explorers, and when all parties are explorers any coalition is possible. Large values of Ψ_p imply a behavior replier-like and small values of Ψ_p imply a behavior explorer-like. Thus the effect of different values for this parameter should follow from the ones described above for the different types. Figure 17.3 to figure 17.8 illustrate the results for the case of three parties.

FIGURE 17.3: All parties are repliers

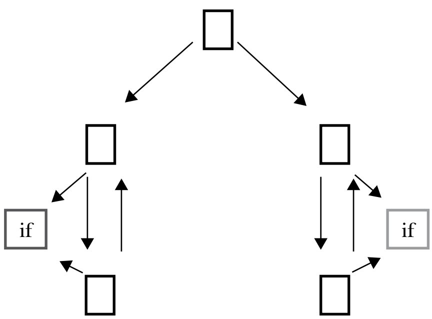


FIGURE 17.4: All parties are explorers

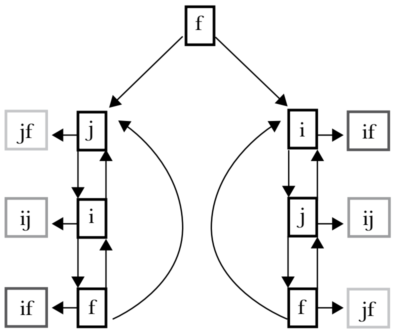


FIGURE 17.5: The formateur (f) is an explorer and the others (i and j) are repliers

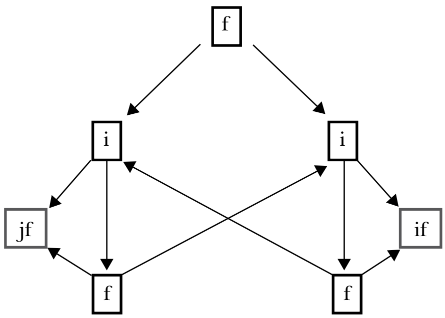


FIGURE 17.6: The formateur (f) an one of the parties (i) are repliers and the other party (j) is and explorer

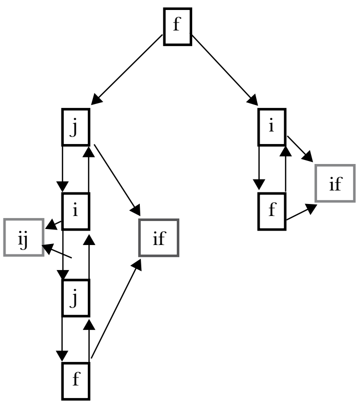


FIGURE 17.7: The formateur (f) is a replier and the others (i and j) are explorers

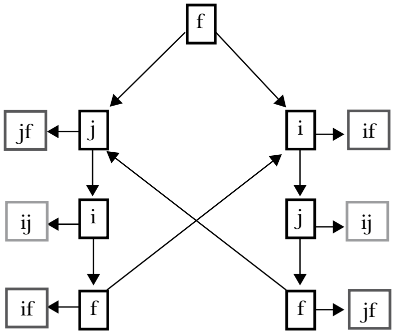
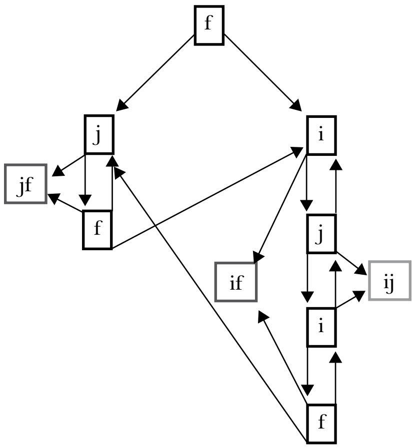


FIGURE 17.8: The formateur (f) and one of the parties (i) are explorers and the other party (j) is a replier



3. **Types of tactics:** the 'holding back' tactics imply:
   — smaller number of deals, and this effect is worse in the polynomial case when t_{max} is small.
   — a larger utility, given that there is a deal.
   — smaller number of deals and larger utility when t_{max} is large
   — small number of deals

'opening up' tactics imply:

- — smaller utility and this effect is worse in the exponential case when t_{max} is large.
- — larger utility for small and smaller utility for large t_{max}
- — larger number of deals for small t_{max}
- — smallest utility given a deal.

4. **Maximal time:** larger t_{max} implies a larger number of offers, therefore 'opening up' tactics imply larger net payoffs for small t_{max}.

Most of the empirical work on the politics of coalition in parliamentary democracies seeks to account for the coalitions that actually form. A comprehensive survey can be found in Laver and Schofield (1990). Martin and Stevenson (2001) provide a list of the properties mostly observed in governing coalitions such as: evidence of minimal winning coalitions forming as opposed to surplus or minority coalitions, coalitions with fewer parties, coalitions that contain the party with the largest proportion of seats, coalitions with smaller ideological divisions are most likely. However, Laver and Schofield (1990) show that most governments are either minority or surplus governments. Finally, Diermeier and Merlo (2004) show that in most cases the largest party is not selected as the formateur. We expect that the results obtained from our proposal would offer new explanations to the existing empirical findings, and would shed some light on the contradicting ones.

17.6. Conclusion

The combination of a formal model of government formation with an automated negotiation mechanism for autonomous agents described in this paper should provide some new insights on how to develop tractable formal models of government formation that could help us understand how a given election result leads to a given government. Furthermore, the results that could be obtained from this combination might be the source of new explanations to some of the existing empirical findings. These are the two main academic goals of this project in the subfield of government formation.

In addition this project will produce a contribution to the literature of artificial intelligence, since the algorithm for a many-to-many multilateral negotiation protocol extends the existing automated negotiation models mostly based on either bilateral or one-to-many negotiations.

Finally, the ultimate goal of this project is to provide some recommendations regarding bargaining behavior to agents that engage in real bargaining for government formation situations. We aim to obtain an evaluation of the performance of a large class of bargaining strategies. Having a characterization of the performance of a given strategy in a number of qualitative different environments, would allow for specific recommendations of given strategies on particular environments.

References

ARAGONÈS, E. "Government Formation with a Two Dimensional Policy Space." *International Journal of Game Theory* 35, no. 2 (2007a): 151-184.

—. "The Key Party in the Catalan Government." *Spanish Economic Review* 9, no. 4 (2007b): 249-271.

AUSTEN-SMITH, D. and J. BANKS. "Elections, coalitions, and legislative outcomes." *American Political Science Review* 82 (1988): 405-422.

AXELROD, R. *Conflict of Interest*, Chicago: Markham, 1970.

BANKS, J. and J. DUGGAN. "A bargaining model of collective choice." *American Political Science Review* 94 (2000): 733-788.

—. *A bargaining model of legislative policy making*. Working Paper, Department of Political Science, University of Rochester, 2002.

BARON, D. "Majoritarian incentives, pork barrel programs, and procedural control." *American Journal of Political Science* 34 (1991a): 57-90.

—. "A spatial bargaining theory of government formation in a parliamentary system." *American Political Science Review* 85 (1991b): 137-164.

—. "Government formation and endogenous parties." *American Political Science Review* 87 (1993): 34-47.

BARON, D. and D. DIERMEIER. "Elections, governments and parliaments in proportional representation systems." *Quarterly Journal of Economics* 116 (2001): 933-967.

BARON, D. and J. FEREJOHN. "Bargaining in legislatures." *American Political Science Review* 83 (1989): 1181-1206.

CHAE, S. and J. A. YANG. "The unique perfect equilibrium of an N-person bargaining game." *Economics Letters* 28 (1988): 221-223.

—. "An N-person pure bargaining game." *Journal of Economic Theory* 62 (1994): 86-102.

CHATERJEE, K., B. DUTTA, D. RAY and K. SENGUPTA. "A noncooperative theory of coalitional bargaining." *Review of Economic Studies* 60 (1993): 463-477.

DANG, J. and M. N. HUHNS. "An extended protocol for multiple-issue concurrent negotiation." *Proceedings, The Twentieth National Conference on Artificial Intelligence and the Seventeenth Innovative Applications of Artificial Intelligence Conference*. Pittsburgh, Pennsylvania, USA, July 9-13, 2005: 65-70.

DIERMEIER, D., H. ERASLAN and A. MERLO. "A structural model of government formation." *Econometrica* 71, no. 1 (2003): 27-70.

DIERMEIER, D. and A. MERLO. "Government turnover in parliamentary democracies." *Journal of Economic Theory*, 94 (2000): 46-79.

—. "An Empirical Investigation of Coalitional Bargaining Procedures." *Journal of Public Economics*, 88 (2004): 783-797.

DIERMEIER, D. and R. MYERSON. *Bargaining, veto power, and legislative committees*. Working paper No. 1089, CMSEMS, Northwestern University, 2002.

ERASLAN, H. "Uniqueness of stationary equilibrium payoffs in the Baron-Ferejohn model." *Journal of Economic Theory* 103(2002): 11-30.

FARATIN, P., C. SIERRA and N. R. JENNINGS. "Negotiation decision functions for autonomous agents." *Robotics and Autonomous Systems* 24 (1998): 159-182.

JACKSON, M. and B. MOSELLE. "Coalition and party formation in a legislative voting game." *Journal of Economic Theory* 103 (2002): 49-87.

JUN, B. H. "A structural consideration on 3-person bargaining." Chapter III in *Essays on Topics in Economic Theory*, Ph. D. Thesis, Department of Economics, University of Pennsylvania, 1987.

KRAUS, S., J. WILKENFELD and G. ZLOTKIN. "Multiagent Negotiation under Time Constraints." *Artificial Intelligence* 75 (1995): 297–345.

KRISHNA, V. and R. SERRANO. "Multilateral Bargaining." *Review of Economic Studies* 63 (1996): 61-80.

LAVER, M. and N. SCHOFIELD. *Multiparty Government: the politics of coalition in Europe.* Oxford: Oxford University Press, 1990.

LAVER, M. and K. SHEPSLE. *Making and breaking governments: cabinets and legislatures in parliamentary democracies.* Cambridge: Cambridge University Press, 1996.

LEISERSON, M. "Coalitions in Politics." Ph.D. Thesis, Yale University, 1966.

MARTIN, L. W. and R. T. STEVENSON. "Government formation in parliamentary democracies." *American Journal of Political Science* 45, no. 1 (2001): 33-50.

MERLO, A. "Bargaining over governments in a stochastic environment." *Journal of Political Economy* 105 (1997): 101-131.

NGUYEN, D. and N. R. JENNINGS. "Coordinating multiple concurrent negotiations." *Proceedings, Third International Joint Conference on Autonomous Agents and MultiAgent Systems.* New York, USA, 2004: 1056-1063.

OSBORNE, M. and A. RUBINSTEIN. *Bargaining and Markets.* Academic Press, 1990.

RIKER, W. *The Theory of Political Coalitions.* New Haven: Yale University Press, 1962.

ROMER, T. and H. ROSENTHAL. "Political resource allocation, controlled agendas, and the status quo." *Public Choice* 33 (1978): 27-44.

—. "Bureaucrats versus voters: on the political economy of resource allocation by direct democracy." *Quarterly Journal of Economics* 93 (1979): 563-587.

RUBINSTEIN, A. "Perfect equilibrium in a bargaining model." *Econometrica* 50 (1982): 97-108.

SUTTON, J. "Non-cooperative bargaining theory." *Review of Economic Studies* 53, no. 5 (1986): 709-724.

Von NEUMANN, J. and O. MORGENSTERN. *Theory of Games and Economic Behavior.* Princeton University Press, 1953.

18

Bargaining One-Dimensional Policies and the Stability of Super Majority Rules

Daniel Cardona
Department of Applied Economics
University of the Balearic Islands

Clara Ponsati
Institute of Economic Analysis
Spanish National Research Council (CSIC)

18.1. Introduction

We address negotiations where a policy must be selected from a continuous one-dimensional set of alternatives, and decisions are taken by the approval of a (super) majority. For these environments we examine the stability of (super) majority rules. The one-dimensional setup is a classical formulation in the social choice literature, which applies to many examples. For instance, the location of a facility, the election of a public official, the choice of tax rates or minimum wages, or the budget allocated to a specific project.

We consider groups of individuals with single-peaked and strictly concave utilities that are heterogeneous only in the locations of their most preferred alternative, their peaks. We assume that decisions must be negotiated over time and that the approval of a majority of the group is required for an agreement. Naturally, the predicted outcomes of these negotiations depend on the majority rule required to settle a choice. Therefore different majority rules generate different individual and collective benefits. We discuss what outcomes prevail for each majority rule, and we show that a precise unique prediction arises naturally under each majority. Then we apply this prediction to address the question of what majority rules are stable.

Cardona and Ponsati (2007, 2008) examine this class of bargaining games and provide a complete description of stationary subgame perfect equilibrium outcomes for a rather general family of one-dimensional environments. In Cardona and Ponsati (2007) we assume that the proposer is selected by a fixed protocol. Here, as in Cardona and Ponsati (2008), we follow the approach of Banks and Duggan (2000) and we assume that negotiations follow the standard random proposers protocol: at the beginning of each round, an agent is selected at random to make a proposal which is approved if it obtains the favorable vote of a (super) majority. Upon approval, the selected alternative is implemented and the game ends. If the proposal is not approved, a new round of bargaining begins in the following period.

Cardona and Ponsati (2008) provide the complete description of the equilibria of this game. In the present paper we build on this characterization (that we review here for the reader's convenience) to explore the stability of majority rules. A very precise prediction prevails for bargaining under each majority rule, which naturally induces individual preferences on majority rules. For each profile of peaks and each majority requirement, we explicitly characterize the subgame perfect equilibrium in stationary strategies, and its existence and uniqueness are established. The unique equilibrium is fully described by the approval set, the (unique) subinterval of alternatives that are accepted by the required majority. The size of this majority matters a lot in determining the approval set. We also identify the limit equilibrium outcome as players become infinitely patient. We establish that in the limit the approval set shrinks to a unique alternative, and we supply the explicit formula that determines this alternative. When we turn attention to the stability properties of majority rules that are implied by the unique equilibrium, very strong results apply for populations with a symmetric distribution of peaks. In these environments, weakening the majority requirement spreads the range of equilibrium alternatives while preserving the mean. When utilities are strictly concave, this implies that all individuals have a strict preference for unanimity over any other majority rule. The conclusion is that in these populations unanimity is the unique Pareto efficient majority requirement. For general populations the unanimity rule is always stable, but other results are not as clear cut. Any majority can be stable in small groups. However, in a large population stability demands a super-majority.

The present paper contributes to the literature that addresses multilateral bargaining over social choices (See Baron and Ferejohn (1989) and Banks and Duggan (2000, 2006). We also add to the literature on the endogenous emergence, efficiency and stability of majority rules. The general analysis of social choices over social choice rules is a classical problem. Its modern formalization starts with the discussion of the distinctive role of unanimous consent by Buchanan and Tullock (1963). Barberà and Jackson (2004) address the question of stability of collective choices under majoritarian regimes and address the endogenous emergence and stability of majoritarian regimes. We refer the reader to Cardona and Ponsati (2007, 2008) for a more complete discussion of the literature.

The remainder of the paper is organized as follows. Section 18.2 presents the environment and the bargaining game. Section 18.3 reviews the main results of Cardona and Ponsati (2008) on the characterization, existence and uniqueness of the stationary subgame perfect equilibrium, and on the asymptotic equilibrium outcomes. The stability of majority rules is discussed in section 18.4. Proofs omitted in the main text are in the Appendix.

18.2. The model

A population of n individuals $I = \{i_1, \ldots i_n\}$ (n is assumed odd) must collectively select an alternative in $[0, 1]$. With some abuse of notation $i \in I$ denotes both a generic individual player and the location of her peak—the (unique) alternative that she likes best. A collective decision is the result of a negotiation that proceeds over discrete time, $t = 0, 1, 2\ldots$ and where an agreement requires the support of a majority $q \in Q = \{k/n\colon k = (n+1)/2, \ldots, n\}$ of the players. For some of the results that refer to large populations we will take the set of individuals to be $I = [0, 1]$ and the description of the population will be given by a positive density over $(0, 1)$ and the set of majority rules will be all $Q = [1/2, 1]$. In either setup the cumulative distribution function of peaks is denoted by F and the proportion (measure) of agents in a set $S \subseteq I$ is denoted by $\mu(S)$.

Negotiations begin at $t = 0$ and proceed as follows. At each $t \geq 0$ a player is selected at random (all with equal probability) to make a proposal. Then, she chooses an alternative in $[0, 1]$ and all other players (sequentially in the natural order) reply with acceptance or rejection.[1] The proposal is approved if the subset of players that accept it is a subset S, with $\mu(S) \geq q$.

Upon approval, the proposed alternative is implemented and the game ends. Otherwise, the game moves to $t + 1$, a new proposer is selected, and so on. If the game ends with approval of alternative x at date t, player i obtains utility $\delta^t u(x, i)$,

$$u(x, i) = 1 - c(|i - x|)$$

[1] When there is a continuum of players weakly dominated strategies at the voting stage are excluded.

where $c(0) = 0$, $c'(z) > 0$, $c''(z) \geq 0$ and $\delta \in (0, 1)$ is the common discount rate. Perpetual disagreement yields zero payoffs.

A *strategy* for a given player specifies her actions—a proposal, and an acceptance/rejection rule—for each subgame. At a *stationary strategy* a player makes the same proposal whenever she is selected and always accepts proposals that are no further away from her peak than some given threshold. A *stationary subgame perfect equilibrium* (henceforth an *equilibrium*) is a profile of stationary strategies that are mutually best responses at each subgame.

18.3. Equilibrium outcomes

Next, we describe the equilibrium outcomes that arise in the present setup. The detailed proofs of the results that follow are in Cardona and Ponsati (2008). We will argue that for each majority q there is a unique equilibrium, and that this equilibrium is fully characterized by the pair of alternatives, $\underline{x}(q)$ and $\overline{x}(q)$, that set the bounds of the *approval set:*[2]

$$A(q) = \left[\underline{x}(q), \overline{x}(q)\right]$$

Fix an equilibrium and denote by x_i the (time independent) proposal of a typical player i. Then, given the random protocol and the distribution of peaks in the population, proposals arise as draws $x_i \sim [\underline{x}, \overline{x}]$. Let U_i denote the time invariant expected utility of player i (prior to appointing the proposer) and let x^e denote the expected equilibrium alternative. From Banks and Duggan (2000), we know that in our framework, all stationary subgame perfect equilibria must be no-delay pure strategies equilibria; thus, x^e is well defined. Moreover, note that $U_i \leq 1 - c(|x^e - i|) \leq u(i, i) = 1$. Let a_i solve $c(a_i) = 1 - \delta U_i$. Then, player i accepts a proposal x if and only if $x \in A_i = [i - a_i, i + a_i] \cap [0, 1] = [\underline{x}_i, \overline{x}_i]$ and $x^e \in A_i$ for all $i \in I$.

Given the individual acceptance set, we define the approval set of coalition $S \subset I$, as

$$A_S = \cap_{i \in S} A_i$$

Note that it is non-empty and connected. That is, it takes the form $A_S = [\underline{x}_S, \overline{x}_S]$ where

---

[2] These bounds depend also on δ.

$$\underline{x}_S = \max\{\underline{x}_i : i \in S\} \text{ and } \overline{x}_S = \min\{\overline{x}_i : i \in S\}$$

The equilibrium proposals x_i must be approved by a majority q; i.e. it must lie in the *approval set*

$$A(q) = \{z \in [0, 1]: z \in A_S \text{ for some } S \text{ such that } \mu\,(S) \geq q\}$$

In any equilibrium all the proposals in the approval set, and only these, receive the support of the necessary majority. Since for any $S \subseteq I$, A_S is connected and $x^e \in A_S$ it can be shown (see Cardona and Ponsati, 2008) that A(q) is also connected. Thus, to characterize the equilibrium it sufficies to provide a precise description of the (connected) approval set. We do this next.

The following notation will ease the exposition. For an arbitrary non-empty interval of alternatives $[x, y] \subseteq [0, 1]$ the expected utility of agent i is given by

$$U_i\,[x, y] \equiv F(x)\;u(x, i) + \int_x^y u(z, i)\;dF(z) + (1 - F(y))\;u(y, i)$$

Definition 18.1. *For any* $(x, i) \in [0, 1] \times I$*, let* $\underline{b}\,(x, i)$ *and* $\overline{b}\,(x, i)$ *be defined such that*

$$u(\underline{b}, i) = \delta U_i\,[\underline{b}, x]$$

$$u(\overline{b}, i) = \delta U_i\,[\overline{b}, x]$$

Definition 18.2. *For any* $(x, i) \in [0, 1] \times I$*, define the lower and upper bounds* $\underline{\zeta}\,(x, i)$ *and* $\overline{\zeta}\,(x, i)$ *as*

$$\underline{\zeta}\,(x, i) = \max\{\underline{b}(x, i), 0\}$$

$$\overline{\zeta}\,(x, i) = \min\{\overline{b}(x, i), 1\}$$

Roughly speaking, the upper (lower) bound indicate how much is willing to concede an agent i to her right (left) when the lower (upper) bound of the (connected) approval set is x.

It can be shown (see Lemma 18.4 in the Appendix) that $\underline{\zeta}_i\,(x, i) \geq 0$, $\overline{\zeta}\,(x, i) \geq 0$, with strict inequalities when the functions attain some value in (0, 1). Thus, the following Lemma holds.

Lemma 18.1. *Fix an equilibrium. Consider two individuals* $i, j \in I$*, such that* $j > i$*. Then* $\underline{x}_j \geq \underline{x}_i$*, and* $\overline{x}_j \leq \overline{x}_i$*, with strict inequalities for interior acceptance thresholds.*

Since in any equilibrium the bounds of the individual acceptance sets are ordered, we may now derive the necessary and sufficient conditions that determine equilibrium approval set: the limits of the approval set are determined exclusively by two (pivotal) agents that depend of the required quota. Given q let= $l(q)$ and r= (q) be the individuals that have $1 - q$ individuals with peaks to their left and right, respectively. By Lemma 18.1 a proposal $x < \underline{x}_r$ is $x < \underline{x}_i$ for all $i \geq r$, and therefore cannot be approved under majority rule q. Similarly a proposal $x > \overline{x}_l$ is $x > \overline{x}_i$ for all $i \leq l$, and therefore cannot be approved under majority rule q. Furthermore, any proposal in $[\underline{x}_r, \overline{x}_l]$ lies in the acceptance set of at least q individuals. Hence $A(q) = [\underline{x}(q), \overline{x}(q)] = [\underline{x}_r, \overline{x}_l]$ where $\underline{x}_r = \underline{\zeta}(\overline{x}_l, r)$ and $\overline{x}_l = \overline{\zeta}(\underline{x}_r, l)$.[3]

Whenever no confusion arises we skip q and write the approval set as $[\underline{x}, \overline{x}]$. In an equilibrium with approval set $[\underline{x}, \overline{x}]$ players $i < \underline{x}$ must propose $x_i = \underline{x}$, players $i > \overline{x}$ propose $x_i = \overline{x}$ and players $i \in [\underline{x}, \overline{x}]$ must propose their peak. Hence, the expected utility of a typical player i is $U_i = U_i [\underline{x}, \overline{x}]$ that will be written as $U_i (q)$ when the dependence on q is relevant, and as U_i whenever no confusion arises.

We are now ready to supply the complete description of the equilibrium outcomes for each q.

Proposition 18.1. *(Equilibrium Outcomes) There is a unique equilibrium for each majority requirement q. For each q, in the unique equilibrium, a proposal x is approved if and only if* $x \in A(q) = [\underline{x}(q), \overline{x}(q)]$, *and individuals play the following strategy:*

1. Proposals: whenever player i is selected she proposes
$$x_i = \begin{cases} \underline{x}_r & \text{if } i < \underline{x}_r \\ \text{i} & \text{if } i \in [\underline{x}_r, \overline{x}_l], \\ \overline{x}_l & \text{if } i > \overline{x}_l \end{cases}$$
2. Acceptance-Rejection: when the proposal is x, player i accepts it if and only if $u\,(x; i) \geq \delta\ U_i\,[\underline{x}_r, \overline{x}_l]$.

Next, we describe the convergence of equilibrium outcomes as players become arbitrarily patient. Proposition 18.2, establishes that for each q, the approval set converges to a single alternative, and provides a straightforward closed form characterization of this limit for each q and F.

Proposition 18.2. *(Unique asymptotic outcome). Fix a majority rule q and a sequence of discount factors such that* $\delta_k \to 1$. *As* $\delta_k \to 1$ *the approval set converges to a unique limit equilibrium alternative*

---

[3] Notice that $\underline{x}_r \leq x^c \leq \overline{x}_l$.

$$\lim_{\delta_k \to 1} \underline{x}(q) = \lim_{\delta_k \to 1} \overline{x}(q) = x(q)$$

1. If there exists an $x^* \in [l(q), r(q)]$ solving

$$K(x) \equiv F(x) \frac{u_x^+(x, l(q))}{u(x, l(q))} + [1 - F(x)] \frac{u_x^-(x, r(q))}{u(x, r(q))} = 0^4$$

then x^* is unique and $x(q) = x^*$.

2. Otherwise, there exists a unique $i^* \in I \cap [l(q), r(q)]$ such that $K(x) > 0$ for $x \in [l(q), i^*)$ and $K(x) < 0$ for $x \in [i^*, r(q)]$, and $x(q) = i^*$.

18.4. Stable (super)majority rules

As the unique equilibrium clearly depends on what majority rule applies, the (expected) payoffs at each majority rule naturally determine individuals' preferences over majority rules. Thus, the preferences of individual i over $q \in Q$ are given by $U_i(q) = U_i[\underline{x}(q), \overline{x}(q)]$. With these individual preferences well specified for all $i \in I$, we are ready to asses the stability of majority rules.

A minimal requirement of stability is Pareto optimality.

PARETO OPTIMALITY: We will say that q is *a Pareto optimal majority rule* if there is no $q' \in Q$, $q' \neq q$ such that $U_i[\underline{x}(q'), \overline{x}(q')] \geq U_i[\underline{x}(q), \overline{x}(q)]$ for all $i \in I$, with strict inequality for a subset $S \subset I$, $\mu(S) > 0$.

Next, we show that in the special class of symmetric populations the comparison of the different q via the Pareto criterion is sufficient to deliver unambiguously strong results.

SYMMETRIC POPULATIONS: A population is *symmetric* if for every individual $i \in [0, 1/2)] \cap I$ there is an individual $j = 1 - i \in I$, i.e. $F(i) = 1 - F(1 - i) + \mu(i)$ for all $i \in I$.

Under a symmetric distribution of peaks, weakening q induces a mean preserving spread of the acceptance set. The proof of this appears in Cardona and Ponsati (2008).

---

[4] $u_x^+(x, i)$ and $u_x^-(x, i)$ denote the right and left derivative of $u(x, i)$ with respect to i. These derivatives may not coincide when $u(x, i)$ is not differentiable at $x = i$.

Proposition 18.3. *(Weakening q induces a mean preserving spread of the outcome distribution). Let the population be symmetric and consider any pair* q, $q' \in Q$, $q < q'$, *then* $\underline{x}(q) < \underline{x}(q') < \overline{x}(q') < \overline{x}(q)$, *and the distribution of equilibrium outcomes under q is a mean preserving spread of the distribution under* q'.

It is well known that, for any strictly concave utility, a mean preserving spread induces a decrease in expected utility. Hence when $c'' < 0$ all $i \in I$ prefer q' over q. When $c'' = 0$, the players with peaks in $[\underline{x}(q), \overline{x}(q)]$ still prefer a mean preserving contraction of the set, while for other players the utility is linear in the relevant range and thus a mean preserving contraction of the set leaves them indifferent. Hence, the following holds.

Proposition 18.4. *For a symmetric population unanimity* $q = 1$ *is the unique Pareto optimal rule.*

Let us now examine the stability of (super) majority rules under more general populations. As the Pareto criterion is presently too weak to discriminate among different rules, we will examine what rules satisfy the following notion of stability, which is closely related to the self-stability notion in Barberà and Jackson (2004).

STABILITY: We say that q is *a stable majority rule* if there is no $q' \in Q$, $q' \neq q$ and S with $\mu(S) \geq q$ such that $U_i\,[\underline{x}(q'), \overline{x}(q')] \geq U_i\,[\underline{x}(q), \overline{x}(q)]$ for all $i \in S$, with strict inequality for a subset $S' \subset S$, $\mu(S') > 0$.

When a rule q is stable, no attempt to revise the majority requirement to an alternative q' can receive the support of a q majority. This property is always satisfied by the unanimity rule $q = 1$. It suffices to check that any weakening of the unanimity requirement necessarily hurts either the player(s) with peak (close to) 0 or the player(s) with peak (close to) 1.

Proposition 18.5. *The set of stable rules is non-empty and it contains at least the unanimity rule* $q = 1$.

Proof. To see that $q = 1$ is stable we check that a change to any $q' \in Q$, $q' < 1$ hurts a subset of players of positive measure; either the players close to $l = 0$ or the players close to $r = 1$ are worse off. By Lemma 18.3 for any $q' < 1$, one of the following four cases applies: (i) $\overline{x}(q') \leq \overline{x}(1)$ and $\underline{x}(q') < \underline{x}(1)$, (ii) $\overline{x}(q') > \overline{x}(1)$ and $\underline{x}(q') \geq \underline{x}(1)$ (iii) $\overline{x}(q') > \overline{x}(1)$ and $\underline{x}(q') < \underline{x}(1)$ or (iv) $\overline{x}(q') = \overline{x}(1)$ and $\underline{x}(q') = \underline{x}(1)$. It is immediate that in case (i) players close to 1 are worse off, while in case (ii), players close to 0 are worse off. In case (iii), the spread of possible outcomes increases. This means that players close to 0 (resp. 1) are better off if and only if the mean alternative approaches 0. But this implies that players close 1 are strictly worse off. Hence, when $q = 1$ there is no $q' < 1$ at which all players are better off.

Checking the stability of rules $q' < 1$ is not as straightforward as it is for $q = 1$. Nevertheless, conditions to rule out stability can be rather direct. For large populations, where $\underline{x}(q')$, $\overline{x}(q)$ and $U_i(q)$ are differentiable with respect to q, we can rely on marginal conditions. Lemma 18.2 provides sufficient conditions assuring that $U_i'(q) > 0$, for coalition of individuals of measure greater than q, hence q is not stable.

Lemma 18.2. *Consider a large population. Assume* $\underline{x}'(q) \geq 0$, $\overline{x}'(q) \leq 0$, *with at least one strict inequality, and* $l(q), r(q) \in [\underline{x}(q), \overline{x}(q)]$. *Then, there is a coalition S with* $\mu(S) \geq q$ *such that* $U_i'(q) \geq 0$ *for all* $i \in S$ *and* $U_i'(q) > 0$ *for all* $j \in S'$, *where* $S' \subseteq S$ *with* $\mu(S') > 0$.

It is easy to check that the conditions of Lemma 18.2 apply at $q = 1/2$. Therefore for large populations a stable rule must be a super-majority.

Proposition 18.6. *For a large population a stable rule must be a super-majority.*

Proof. At $q = 1/2$, $l = r$, so that the conditions of Lemma 18.5 hold (by single peakedness), and therefore $\underline{x}'(q) \geq 0$ and $\overline{x}'(q) \leq 0$ for every F. Hence it is an immediate consequence of Lemma 18.2 that at $q = 1/2$, $U_i'(1/2) > 0$ for a coalition of players of measure greater that 1/2.

The requirement that the population is large is important. For finite populations every majority, including the simple majority, can be stable. We make this point with an example.

Example 18.1. *(All majorities are stable, and the simple majority maximizes total surplus). Let* $I = \{0, 0.1, 0.25, 0.7, 1\}$, $u(x; i) = 1 - (x - i)^2$ *and* $\delta = 0.999$. *The bounds of the approval set, individual expected utility and total surplus are as follows:*

q	3/5	4/5	1
$\underline{x}$	0.17943	0.34814	0.43883
$\overline{x}$	0.3205	0.35127	0.44036
U_0	0.93353	0.87792	0.80689
$U_{0.1}$	0.97353	0.9378	0.88478
$U_{0.25}$	0.99602	0.99012	0.96411
$U_{0.7}$	0.79349	0.87707	0.93211
U_1	0.43348	0.57671	0.68577

It is easily checked that players 0, 0.1 *and* 0.25, *are better off at* $q = 3/5$ *than at any other quota. Thus, the simple majority is stable. The two super majorities,* $q = 4/5, 1$ *are also stable. In this example, the acceptance bounds change monotonically in* q.

Furthermore, the set of stable rules is not necessarily connected. In the following example the simple majority and unanimity are stable, but the intermediate super majority is not.

Example 18.2. *(Simple majority and unanimity are stable, but and intermediate super majority is not).*

Let $I = \{0, i, 0.3, j, 1\}$, *where* $i = 0.16304$ *and* $j = 0.83884$. $u_i(x) = 1 - (x - i)^2$ *and* $\delta = 0.99$. *Even though the values of i and j have been selected such that the expected outcome under* $q = 4/5$ *and* $q = 1$ *is the same (implying that* $q = 1$ *dominates* $q = 4/5$ *in the Pareto sense), the example is generic.*

The bounds of the approval set and individual expected payoffs are as follows:

q	**3/5**	**4/5**	**1**
$\underline{x}$	0.11621	0.49	0.49374
$\overline{x}$	0.48379	0.51188	0.50626
U_0	0.88036	0.75113	0.75121
U_i	0.95466	0.88718	0.88726
$U_{0.3}$	0.97598	0.96038	0.96046
U_j	0.69573	0.88423	0.8843
U_1	0.49909	0.74864	0.74871

The simple majority $q = 3/5$ *is stable (players* 0, *i and 0.3 are better off at* $q = 3/5$ *than at any other quota). The super majority,* $q = 4/5$ *is not stable since it is Pareto dominated by* $q = 1$. *Unanimity* $q = 1$ *is stable.* Note that acceptance bounds change non-monotonically in q.

To conclude we remark that stability is not a concern in the limit as $\delta \to 1$. Recall that (by Proposition 18.2) for each $q \in Q$ the equilibrium outcomes converge to the single alternative $x(q)$ Thus each player expects gains $u(x(q), i)$.

Consider the natural extension of stability to the limit as $\delta \to 1$.

ASYMPTOTIC STABILITY: We say that q is *an asymptotically stable majority rule* if there is no rule $q' \in Q$, $q' \neq q$, and $S \in W(q)$ such that $u(x(q'), i) \geq u(x(q), i)$ for all $i \in S$, with strict inequality for an $S' \subseteq S$ such that $\mu(S') > 0$.

This requirement turns out to be satisfied by all majority rules.

Proposition 18.7. *Every $q \in Q$ is asymptotically stable.*

Proof. Take any two rules $q \neq q'$. Recall that $x(q) \in [l(q), r(q)]$. If $x(q) > x'(q)$, a change from q to q' decreases the benefits of all individuals $i \geq r(q)$, a coalition with mass $1 - q$. If $x(q) < x'(q)$ then all individuals $i \leq l(q)$ are worse off at q', and again this coalition has mass $1 - q$. Hence, given q no change to an alternative q' can have the support of a winning coalition S with $\mu(S) \geq q$.

Proposition 18.6 is based on the fact that for simple majority the approval set cointains some outcomes y with $y < l(q)$ and some x with $x > r(q)$, which is similar to the condition derived in Barberà and Jackson (2004) for a majority rule not being self-stable. I.e., when the outcome does not belong to the core of the underlying cooperative game, which happens whenever $x \notin [l(q), r(q)]$. Since the asymptotic outcome satisfies $x(q) \in [l(q), r(q)]$ for all q, the result follows.[5]

Appendix

Proof of Lemma 18.2. We will check that the expected gains are increasing in q either for all $i \in [\underline{x}, 1]$ or for all $i \in [0, \overline{x}]$. For any player i,

$$U_i'(q) = F(\underline{x}(q))\ u_x(\underline{x}(q), i)\ \underline{x}'(q) + (1 - F(\overline{x}(q)))\ u_x(\overline{x}(q), i)\ \overline{x}'(q)$$

Now $\underline{x}'(q) > 0$ and $\overline{x}'(q) < 0$ imply the following:

1. $U_i'(q) > 0$ for all $i \in [\underline{x}(q), \overline{x}(q)]$ since $u_x(\underline{x}(q), i) \geq 0$ and $u_x(\overline{x}(q), i) \leq 0$ with at least one strict inequality.
2. $U_i'(q) > 0$ for all i $\in (\overline{x}(q), 1]$ whenever $F(\underline{x}(q))\ \underline{x}'(q) \geq -(1 - F(\overline{x}(q)))\ \overline{x}'(q) > 0$, since for these players $u_x(\underline{x}(q), i) > u_x(\overline{x}(q), i) > 0$.
3. $U_i'(q) > 0$ for all $i \in [0, \underline{x}(q))$ whenever $0 < F(\underline{x}(q))\ \underline{x}'(q) \leq -(1 - F(\overline{x}(q)))\ \overline{x}'(q)$, since for these players $0 > u_x(\underline{x}(q), i) > u_x(\overline{x}(q), i)$.

Hence, there is at least a fraction q of the players that increase their expected utility upon a marginal increase in the majority requirement.

[5] Similarly, the unanimity case (see Proposition 18.5) is such that $0 = l(1) \leq \underline{x}(1) < \overline{x}(1) \leq r(1) = 1$.

Lemma 18.3. *For every* $q < 1$, $\bar{x}(q) \geq \bar{x}(1)$ *or* $\underline{x}(q) \leq \underline{x}(1)$.

Proof. Let $[\underline{x}(q), \bar{x}(q)]$. By Lemma 18.4 $\bar{\zeta}(\underline{x}, l)$ is increasing in l and $\underline{\zeta}(\bar{x}, r)$ is increasing in r. Thus, the pair $(\underline{x}, \bar{x})$ must lie in the set

$$Z = \{(\underline{x}, \bar{x}) \in [0,1]^2 : \bar{x} \geq \bar{\zeta}(\underline{x}, 0)\} \cap \\ \cap \{(\underline{x}, \bar{x}) \in [0,1]^2 : \underline{x} \leq \underline{\zeta}(\bar{x}, 1)\}.$$

Assume that $[\underline{x}, \bar{x}] \subseteq [\underline{x}(1), \bar{x}(1)]$. Then, as $(\underline{x}, \bar{x}) \in Z$, we have that

$$\underline{\zeta}(\bar{x}, 1) \geq \underline{x} \geq \underline{x}(1) = \underline{\zeta}(\bar{x}(1), 1)$$

$$\bar{\zeta}(\underline{x}, 1) \leq \bar{x} \leq \bar{x}(1) = \bar{\zeta}(\underline{x}, (1), 1)$$

Moreover, since $\underline{\zeta}_x(x, 0) \geq 0$ and $\underline{\zeta}_x(x, 1) \geq 0$ this is possible only if $[\underline{x}, \bar{x}] = [\underline{x}(1), \bar{x}(1)]$.

Lemma 18.4. $\bar{\zeta}(\underline{x}, l)$ *is increasing in* l, *and* $\underline{\zeta}(\bar{x}, r)$ *is increasing in* r.

Proof. We prove the statement for $\bar{\zeta}(\underline{x}, l)$; a similar argument is valid for $\underline{\zeta}(\bar{x}, r)$. Fix $\underline{x}$ and consider $\bar{\zeta}(\underline{x}, l)$ at two possible values of $l = l_1, l_2$ with $l_1 < l_2$. There are two possibilities:

1. $\bar{\zeta}(\underline{x}, l_1) = \bar{b}(\underline{x}, l_1)$, implying $u(\bar{\zeta}(\underline{x}, l_1), l_1) = \delta U_{l_1}$. In this case, if $\bar{\zeta}(\underline{x}, l_2) = 1$ the result follows directly. Otherwise, $\bar{\zeta}(\underline{x}, l_2) = \bar{b}(\underline{x}, l_2)$. Moreover, by making some calculations it can be shown that $u(\bar{b}(\underline{x}, l_2), l_1) < \delta U_{l_1}$. Thus, $u(\bar{\zeta}(\underline{x}, l_2), l_1) < u(\bar{\zeta}(\underline{x}, l_1), l_1)$. Moreover, as $\bar{\zeta}(\underline{x}, l_2) > l_2 > l_1$ we have that $\bar{\zeta}(\underline{x}, l_1) < \bar{\zeta}(\underline{x}, l_2)$.
2. In case that $\bar{x} = \bar{\zeta}(\underline{x}, l_1) = 1$, it must be that $u(1, l_1) \geq \delta U_{l_1}$. Again, direct calculations show that this implies $u(1, l_2) > \delta U_{l_2}$ so that $\bar{\zeta}(\underline{x}, l_2) = 1$.

Lemma 18.5. *Consider a large population. If* q *and* F *are such that* $u_x(\bar{x}; l) < 0$, $u_x(\underline{x}; l) > 0$, $u_x(\bar{x}; r) < 0$, *and* $u_x(\underline{x}; r) > 0$, *then* $\underline{x}'(q) \geq 0$ *and* $\bar{x}'(q) \leq 0$.

Proof. Differentiating $\bar{x}(q) - \bar{\zeta}(\underline{x}(q), l(q))$ and $\underline{x}(q) - \underline{\zeta}(\bar{x}(q), r(q))$ yields

$$\bar{x}'(q) - \bar{\zeta}_x(\underline{x}(q), l(q))\ \underline{x}'(q) - \bar{\zeta}_i(\underline{x}(q), l)\ l'(q) = 0$$
$$\underline{x}'(q) - \underline{\zeta}_x(\bar{x}(q), r(q))\ \bar{x}'(q) - \underline{\zeta}_i(\bar{x}(q), r(q))\ r'(q) = 0$$

By Lemma 18.4, $\overline{\zeta}_i(\underline{x}(q), l(q)) \geq 0$ and $\underline{\zeta}_i(\overline{x}(q), r(q)) \geq 0$. Moreover, since $l'(q) < 0$ and $r'(q) > 0$, using the previous equations we get

$$\overline{x}'(q) \leq \overline{\zeta}_x(\underline{x}(q), l(q))\ \underline{x}'(q)$$
$$\underline{x}'(q) \geq \underline{\zeta}_x(\overline{x}(q), r(q))\ \overline{x}'(q)$$

Moreover, $u_x(\overline{x}; l) < 0$ and $u_x(\underline{x}; l) > 0$ imply that $\overline{\zeta}_x(\underline{x}(q), l(q)) \in (-1, 0]$, and $u_x(\overline{x}; r) < 0$ and $u_x(\underline{x}; r) > 0$ imply that $\underline{\zeta}_x(\overline{x}(q), r(q)) \in (-1, 0]$. Hence, the first inequality rules out $\underline{x}'(q) > 0$ and $\overline{x}'(q) > 0$, while the second rules out $\underline{x}'(q) < 0$ and $\overline{x}'(q) < 0$.

The possibility that $\underline{x}'(q) < 0$ and $\overline{x}'(q) > 0$ is ruled out by Lemma 18.6. Hence, it must be that $\underline{x}'(q) \geq 0$ and $\overline{x}'(q) \leq 0$.

Lemma 18.6. *For a large population, if $\overline{x}'(q) > 0$ then $\underline{x}'(q) \geq 0$.*

Proof. Differentiating $\overline{x} - \overline{\zeta}(\underline{x}, l)$ with respect to q yields

$$\overline{x}'(q) - \overline{\zeta}_x(\underline{x}, l)\ \underline{x}'(q) - \overline{\zeta}_i(\underline{x}, l)\ l'(q) = 0$$

Moreover, it can be shown that $|\overline{\zeta}_x(\underline{x}, l)| < 1$ so that since $l'(q) < 0$, assuming $\underline{x}'(q) < 0$ implies

$$\overline{x}'(q) + \underline{x}'(q) < \overline{\zeta}_i(\underline{x}, l)\ l'(q) \leq 0$$

Similarly, differentiating $\underline{x} - \underline{\zeta}(\overline{x}, r)$ with respect to q yields

$$\underline{x}'(q) - \overline{\zeta}_x(\underline{x}, l)\ \overline{x}'(q) - \overline{\zeta}_i(\underline{x}, l)\ r'(q) = 0$$

Since $|\overline{\zeta}_x(\underline{x}, r)| < 1$ and $r'(q) > 0$, assuming $\overline{x}'(q) > 0$ implies

$$\underline{x}'(q) + \overline{x}'(q) > \overline{\zeta}_i(\underline{x}, l)\ r'(q) \geq 0$$

which is a contradiction.

References

BANKS, J. S. and J. A. DUGGAN. "Bargaining Model of Collective Choice." *American Political Science Review* 94 (2000): 73-88.

—. General Bargaining Model of Legislative Policy-making." *Quarterly Journal of Political Science* 1 (2006): 49-85.

BARBERÀ, S. and M. O. JACKSON. "Choosing How to Choose: Self-Stable Majority Rules and Constitutions." *Quarterly Journal of Economics* (2004): 1011-1048.

BARON, D. and J. FEREJOHN. "Bargaining in Legislatures." *American Political Science Review* 83 (1989): 1181-1206.

BUCHANAN, J. and R. TULLOCK. *The Calculus of Consent: Logical Foundations of Constitutional Democracy.* Ann Arbor: University of Michigan Press, 1962.

CARDONA, D. and C. PONSATI. "Bargaining one-dimensional social choices." *Journal of Economic Theory* 137 (2007): 627-651.

—. *Efficient majority rules for one-dimensional bargaining.* Mimeo, 2008.

19

War or Peace

Carmen Beviá
Department of Economics and Economic History and CODE
Autonomous University of Barcelona

Luis C. Corchón
Department of Economics
Carlos III University of Madrid

19.1. Introduction

Economics it is the study of how agents distribute limited means in order to achieve certain goals. Economics usually assumes that agents are rational and selfish, i.e. they know how to cash in any possible advantage and their objectives are basically related with self satisfaction. The main insight of economics is that, under certain conditions, the unrestricted interaction of rational and selfish players produces an outcome that is socially optimal. Moreover, selfishness is some kind of necessary condition for social optimality in the following sense: only when agents work for the (anonymous) market and not to satisfy the needs of their beloved ones is possible to achieve the division of labor that is necessary for social optimality. In other words, the market is a device to bring good (social optimality) from the bad nature of men (selfishness). This story was basically cooked by Adam Smith (1776) and formalized through the past 230 years by results like the two fundamental theorems of welfare economics, the existence of a competitive equilibrium, the Ricardian model of trade, so on and so forth.

The previous story has to be qualified in many respects. 1) Agents are not always rational: sometimes their objectives are ill-defined or they fail to forecast the consequences of their acts. 2) Unrestricted interaction of agents may yield suboptimal allocations when markets are incomplete or imperfectly competitive or information is asymmetric or when there are effects that do not work through the market (externalities). 3) The market does not produce just allocations. To call socially optimal the allocations produced by the market is not good naming.

"Efficient Allocations" is a more apt description. It must be remarked that none of these points modify radically the Adam Smith story: The two fundamental theorems of welfare economics and the existence of competitive equilibrium can be proved under incomplete or intransitive preferences and in many cases the market achieves restricted efficiency (i.e. relative to a set of constraints) or it gets close to efficiency, when the number of agents is large.

Starting with the work of Tullock (1967), Krueger (1974) and Becker (1983), economists began to look at situations far beyond of those envisioned by Adam Smith and his followers. Suddenly, instead of an orderly world where the unrestricted interaction of self-interested agents produces efficiency we have a world where the unrestricted interaction of self-interested agents produces... contests. This is not the place to describe the main insights achieved by this new branch of economics. The interested reader may read the surveys of Corchón (2007) or Konrad (2007) to gauge the range of potential applications: Advertising, political competition, litigation, lobbying, arms races, sports events, R&D competition, contract theory, insurrections and conflicts, rent-seeking and rent-defending contests, etc. Venerable results like the welfare loss due to monopoly or to transaction costs were given a new light when it was shown that the cost of the underlying contest for the rents accruing to a monopolist or to the owner of a resource were far greater than the costs spotted by the traditional theory. And the theory of contests became a tool to analyze the supreme contest of all: *war*.

The systematic study of war in the modern era owes a lot to the work of a Prussian officer, Carl von Clausewitz (1780-1831). He was involved in the Napoleonic wars, in which the Prussian army was defeated again and again by its Corsican nemesis and formed part of a select group of high-rank officers who designed the reform of the Prussian army.[1] It was quite an advanced reform by its time and one that was misunderstood by other nations but yielded an unquestionable German military superiority in the next hundred years. It was based on two pillars: Instructions for the officers—later extended to low level command—and the abolition of physical punishments for the soldiers.[2] As a part of the former, Clausewitz began to write a textbook where he would lay the foundations of the study of war and its political and philosophical implications. Unfortunately, Clausewitz died before his book was in read-able form. His wife, the Prussian

[1] This reform runs parallel to the reform of the university undertaken by Wilhelm von Humboldt that had such a profound influence in modern universities.

[2] Contrarily to what was claimed by the Anglo-French propaganda the Prussian military system was not more based on authority than their own systems. And certainly the Prussian army was much more decentralized than those of its two rivals thanks to the superior education of their officers.

aristocrat Marie von Brühl, edited the book, finally published in 1832, and wrote a preface to it. Many of the modern ideas on war, including a sentence in which he acknowledges the game-theoretical nature of war ("war is akin to a card game", Chapter 1) owe their origin to this book.

During many years, the study of the two most important mechanisms of resource allocation, the market and the war, followed separate paths. It was only when game theory began to take a central position in the education and the values of economists that both studies began to converge. A survey of the early efforts of game theorists to understand war was written by O'Neill in 1990. At this time other developments were taking place. Hirshleifer (1991) and Skaperdas (1992) provided game theoretical studies of war based on the Contest Success Function (CSF), an analytical tool provided by the early work of Tullock. Hirshleifer came back three years later with a catchy title: "The Dark Side of the Force". This was what economists had forgotten in the last two hundred years. That destruction, violence, death and massacres all come from the same root as the market. All chants to the beauty of self-interest began to wither away.

By now, models of war are no longer an analytical novelty, so perhaps we may think about the opposite question: How is that countries come up to a peaceful coexistence when war is an option.[3] There are simple answers to this question: Firstly, some countries would have difficulties if they wanted to start a fight, think of Chile and Switzerland.[4] Secondly, there are circumstances in which a third party, say the United Nations, can safeguard the borders making the conflict unlikely. Finally, peace treaties might be signed, but the immediate question is: Why is such an agreement observed? There may be cases where asymmetric information or infinite lifetimes may enforce these agreements. They may also be enforced by an external agent like NATO or the UN. But in general, it is not clear why players have incentives to stick to the agreement. In fact, it is commonly argued that when the agreement involves a transfer of resources from one country to another, this transfer may encourage the recipient to ask for more.

In this paper we study the validity of transfers to stop war when agents can not commit to any action post-transfer and when the time horizon is finite and information is symmetric. We show that in this case, transfers may, indeed, stop war. Why? Because once the transfer has been made, the donor now looks like a less appealing target and the recipient risks more than before. Thus, the transfer mechanism embeds incentives to fulfill a peace treatment without any need for

[3] See Garfinkel and Skaperdas (2007) for a survey of the modern theories of war.

[4] But Spain and Chile fought a war in 1865!

external enforcement. The goal of this paper is to study the strength of these incentives in a simple model of war and peace.

Our framework is that of a non-cooperative game with four stages. In the first stage transfers are made. In the second stage players decide simultaneously if they declare war or not. If one of them declares war, war occurs. In the third stage, if there is war, each player decides the war effort. In the last stage the outcome of the war is determined and the winner takes all. We assume that the probability of winning war is a function of war efforts and the responsiveness of the probability of winning war to war efforts. The latter is an inverse measure of the role of chance in war. For simplicity, we assume that all war efforts are lost in the war. Players are endowed with a resource that can be devoted to war effort or consumed. The resources of Player 1 are larger than those of Player 2 so the first (respect to the second) player will be called the rich (resp to the poor) player. Thus, in our model there are two parameters: the role of chance in war and the inequality of resources between players. In a companion paper (Beviá and Corchón 2008) we study a more general model where part of the war effort can be recovered by the winner and a country may enjoy a military superiority on the other.

Our first result (Proposition 19.1) is that in absence of transfers, when both players are unconstrained (i.e. when each player spends in war less than its resources) and the probability of winning a war is proportional to war efforts, peace is the equilibrium outcome. However, when the probability of winning a war is less responsive to efforts than in the proportional case, war is declared by the poor player when the ratio of its resources to aggregate resources falls into a certain interval: This is explained by the fact that for war to be a good option for the poor player, she cannot be very poor, because in this case her chances to win the war are slim, neither too rich because in this case war is too risky a strategy. This interval grows when the probability of winning war is made less sensitive to war efforts, i.e. when war is more random. Thus we may say that, in this case, wars are caused because the randomness of the outcome makes war a good investment from the point of view of the poor player.

Our second result (Proposition 19.2) is that when players are unconstrained, there is always a transfer that brings peace in equilibrium and such that both players are better off than under war. The interpretation of this result is that when wars are caused by the randomness of the outcome, transfers stop the war since they avoid the destruction of the latter and make the poor player a peaceful one because she has more to loose.

We move to analyze other cases. Suppose that the rich player is unconstrained but the poor player is constrained (notice that the case where the rich player is constrained and the poor player is unconstrained is impossible). Then, we show in Proposition 19.3 that in the absence of transfers, war is declared by the poor player. In this case war is caused by the inequality of resources between players.

In our last result (Proposition 19.4) we show the following. Suppose that the rich player is unconstrained but the poor player is constrained and that the probability of winning a war is proportional to war efforts. Then, unless the ratio of the resources in the hands of the poor player to aggregate resources is less than, approximately, 1.8%, there is a transfer that brings peace in equilibrium and makes both players better off than under war. After the transfer, the poor player owns a quarter of aggregate resources. Thus, when wars are caused by resource inequality, transfers work very well, unless inequality is so large that the conflict is non-solvable. The last case reminds us of the famous sentence by Karl Marx: «The proletarians have nothing to lose but their chains».

Summing up, the message of this paper is that the transfer mechanism works well in a number of cases. In particular when wars are caused by the randomness of the outcome, wars can always be stopped by this mechanism. And wars caused by resource inequality can also be prevented by transfers, unless inequality is really very large. The companion paper (Beviá and Corchón 2008), presents several historical examples of transfers that were successful in ending or preventing wars. While we do not claim that our findings explain such facts, it is reassuring to find that the transfer mechanism has been used, with success, throughout history. The companion paper also analyzes extensively the case were the probability of winning war is proportional to war efforts and also considers the role of differential efficiency waging war.

The rest of the paper goes as follows. In Section 19.2 we spell the model. Section 19.3 gathers our results. Finally, in Section 19.4, we discuss other mechanisms of transfers that have been used in the literature.

19.2. The model

There are two players with resources V_1 and V_2. W.l.o.g. we will assume that $V_1 > V_2$. They play the following game.

In the first stage, each player may transfer part of his resources to the other player.[5]

[5] We describe payoff-relevant events only. But the first stage can be envisioned as a summit between the leaders of the two countries where they sign a non-aggression treaty and, under the table, one of the leaders gets the transfer.

In the second stage, each of them decides whether to declare war on the other player or not. If one of them declares war, war occurs. If both abstain from declaring war, peace results.

In the third stage, if there is peace, the game ends. Payoff to player i is his resource V_i, $i = 1, 2$. If there is a war, each player has to commit part of his resources to the war effort, denoted by e_i, $i = 1, 2$. It is assumed that there is no outside credit and therefore no player can use in the war more than his available resources.

In the fourth stage, war is waged. The outcome is partially determined by nature and partially determined by war efforts.

If p_i is the probability that player $i = 1, 2$ wins the war, we assume that

$$p_i = \frac{e_1^\gamma}{e_1^\gamma + e_2^\gamma},\ 0 \leq \gamma \leq 1. \tag{19.1}$$

The functions in (19.1) are called *contest success functions (CSF).* The parameter γ measures the sensitivity of the probability of winning war to the efforts. When $\gamma = 0$, the outcome of war is purely random. When $\gamma = 1$, we will say that the CSF are *proportional.*

A motivation for this functional form is that it seems reasonable to require that the CSF is homogeneous of degree zero, so winning probabilities do not depend on how resources are measured (pounds or francs, number or thousands of soldiers, etc.). This is the CSF proposed by Tullock (1980) that has been ubiquitously used in the literature.

We will assume that there is a winner who takes all, i.e. the war does not end in a stalemate, and that war effort cannot be recovered.

For simplicity we assume, as it is customary in the literature, that players are risk-neutral.[6] Thus, the payoff of, say Player 1, if he wins the contest is $V_1 + V_2 - (e_1 + e_2)$ and zero otherwise. Let $V \equiv V_1 + V_2$. Expected payoff of player i denoted by $E\pi_i$, is

$$E\pi_i = p_i\,(V - (e_1 + e_2)). \tag{19.2}$$

Finally we assume that information is complete and that the equilibrium concept is subgame perfection.

---

[6] Clearly, transfers would work even better if players were risk averse.

19.3. The results

We solve the game backwards. Since no player has to move in the fourth stage, let us begin by analyzing the third stage.

Expected payoffs for player i assuming that war has been declared are

$$E\pi_i = \frac{e_1^\gamma}{e_1^\gamma + e_2^\gamma}\,(V-(e_1+e_2)). \tag{19.3}$$

Given that $p_2 = 1 - p_1$ we have that

$$\frac{\partial E\pi_1}{\partial e_1} = \frac{e_2^\gamma e_1^{\gamma-1}\gamma}{(e_1^\gamma + e_2^\gamma)^2}\,(V-(e_1+e_2)) - p_1 k; \tag{19.4}$$

$$\frac{\partial E\pi_2}{\partial e_2} = \frac{e_1^\gamma e_2^{\gamma-1}\gamma}{(e_1^\gamma + e_2^\gamma)^2}\,(V-(e_1+e_2)) - (1-p_1)k. \tag{19.5}$$

Setting $\frac{\partial E\pi_1}{\partial e_1} = 0$, $i = 1,2$, and dividing (19.4) by (19.5) equation reference goes here we obtain that $e_1^{\gamma+1} = e_2^{\gamma+1}$ which implies that $e_1 = e_2$. Now (19.4) reads

$$\frac{\partial E\pi_1}{\partial e_1} = \frac{\gamma e_i^{2\gamma-1}}{4e_i^{2\gamma}}\,(V-2e_1) - \frac{1}{2} = 0,\ i = 1,2. \tag{19.6}$$

The solution to (19.6) is,

$$e_1^* = e_2^* = \frac{\gamma V}{2(\gamma+1)} \text{ and } E\pi_1^* = E\pi_2^* = \frac{V}{2(\gamma+1)}. \tag{19.7}$$

Since resources are limited for each player, the equilibrium effort in case of war will depend on players being or not being constrained. We analyze all the possible situations.

19.3.1. Both players are unconstrained

This case arises if $e_1^* \leq V_i$ for all i, that is,

$$V_1 \geq \frac{\gamma V}{2(\gamma+1)} \text{ and } V_2 \geq \frac{\gamma V}{2(\gamma+1)}. \tag{19.8}$$

Since $V_1 > V_2$, if the second inequality holds, the first inequality also holds. In this case, equilibrium efforts and expected payoff if war occurs are given by (19.8).

For war to be a rational option, we need the following:

$$E\pi_1^* = \frac{V}{2(\gamma+1)} > V_1 \text{ or } E\pi_2^* = \frac{V}{2(\gamma+1)} > V_2. \qquad (19.9)$$

We first notice that it is impossible that both inequalities occur, because adding them up we get $\frac{V}{\gamma+1} > V$ which is impossible. This implies that Player 1 has no incentive to go to war because if it had, Player 2 would also have incentives to declare war (since $V_1 > V_2$). Thus, we are left with the case where only the second inequality in (19.9) holds, so the second country has an incentive to go to war. Occurrence of war is equivalent to

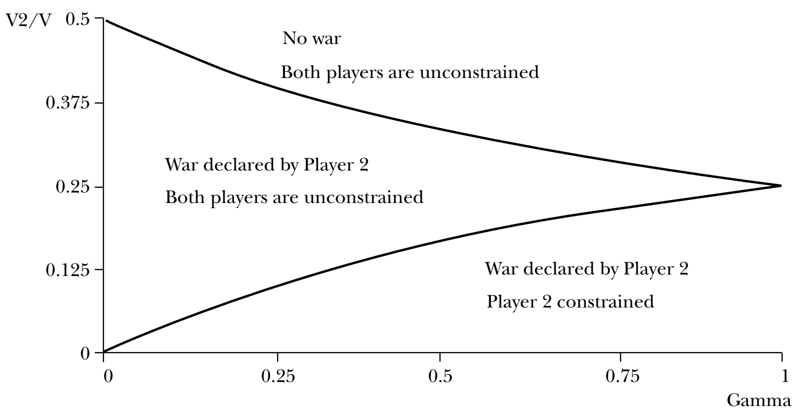
$$V_2 \geq \frac{\gamma V}{2(\gamma+1)} \text{ and } \frac{V}{2(\gamma+1)} > V_2. \qquad (19.10)$$

which is possible whenever $\gamma < 1$. Our next result summarizes this discussion.

Proposition 19.1. *If both players are unconstrained, in the absence of transfers, no war is declared if* $\gamma = 1$. *When* $\gamma < 1$, *war is declared by Player 2 for any value of* $V_2/V \in \left(\frac{\gamma}{2(\gamma+1)}, \frac{1}{2(\gamma+1)}\right)$.

In figure 19.1 we show the area where war is possible in the case where both players are unconstrained. Above the increasing line both players are unconstrained. Below the decreasing line war is declared by Player 2.

FIGURE 19.1: War occurrence



We see that the occurrence of war depends on several factors. First, notice that equilibrium efforts are increasing with γ and expected payoffs are decreasing with γ. Therefore, when γ increases the area where war occurs decreases. For a small γ the poor player has a chance of winning the war without much effort which implies a sizeable loot should war be won. Second, the ratio of the resources of Player 2 with respect to those of Player 1 should not be too high,—because otherwise Player 2 risks a lot—nor too low, because in this case Player 2 is constrained.

An important consequence of this result is that when $\gamma \to 0$ war is possible for any value of V_2/V. In other words, here war is possible even in the absence of inequality. This is because when the success of war is not very sensitive to war efforts, both players use only a small part of their resources in war and the loot of the winner is considerable.

Let us study equilibrium in the first stage of the game. A transfer from the rich player to the poor player will stop war if:

1. Both players are better off than if they had a war.
2. No one has now incentives to declare war.

   If such a transfer exists we will say that a *peace agreement is feasible.*

   The following proposition shows that a peace agreement is feasible in this case.

Proposition 19.2. *If both players are unconstrained, a peace agreement is feasible. The minimal transfer that avoids war is such that it makes Player 2 be indifferent between war and peace, that is* $V_2 + \hat{T} = \dfrac{V}{2(\gamma+1)}$.

Proof. Notice first that $\hat{T}$ is always smaller than V_1, if it were not, $(V/2(\gamma+1)) - V_2 > V_1$, or $V > 2(\gamma+1)\ V$ which is impossible. So it is feasible for Player 1 to make the transfer.

Consider the second stage of the game where war would be declared when no transfers are made, but that a transfer $\hat{T}$ such that $V_2 + \hat{T} = V/2(1+\gamma)$ has been made. After the transfer, payoff for Player 1 in case of peace is:

$$V_1 - \hat{T} = V_1 - \frac{V}{2(\gamma+1)} + V_2 = V - \frac{V}{2(\gamma+1)}. \qquad (19.11)$$

Assuming that this payoff is less than the one in case of a war with no transfer,

$$V - \frac{V}{2(\gamma+1)} < \frac{V}{2(\gamma+1)} \Leftrightarrow V < \frac{V}{\gamma+1}. \qquad (19.12)$$

which is impossible. Thus, $\hat{T}$ is such that both players are better off than if they had had a war. It is only left to show that after the transfer no one has incentives to declare war, which in this case is equivalent to showing that Player 1 is still unconstrained, that is:

$$V_1 - \hat{T} \geq \frac{\gamma V}{2(\gamma+1)} \Leftrightarrow V - \frac{V}{2(\gamma+1)} \geq \frac{\gamma V}{2(\gamma+1)} \Leftrightarrow 1 \geq \frac{\gamma}{2\gamma+1}. \quad (19.13)$$

By the previous proposition, if the relatively poor player has an incentive to go to war before the transfer, $\gamma < 1$. But then, $1 > \gamma > \frac{\gamma}{2\gamma+1}$, as desired. ■ The interpretation of this result is that, as we saw before, war is a rational option for Player 2 when it is a kind of lottery, i.e. the outcome of the war does not depend much on war efforts. But in this case a transfer acts as a costless lottery that leaves both players better off.

19.3.2. Both players are constrained

This case arises iff $\frac{\partial E\pi_1 (V_1, V_2)}{\partial e_1} > 0$. But this is impossible. From (19.4) and (19.5),

$$\frac{\partial E\pi_1 (V_1, V_2)}{\partial e_1} = -p_i \leq 0. \quad (19.14)$$

19.3.3. Player 1 is unconstrained and Player 2 is constrained

This case arises if $V_2 < e_2^*$ and $\frac{\partial E\pi_1 (V_1, V_2)}{\partial e_1} < 0$. Given that the second condition always holds, this is equivalent to:

$$\frac{V_2}{V} < \frac{\gamma}{2\gamma+1}. \quad (19.15)$$

Graphically, this situation corresponds to the area below the increasing line in figure 19.1.

The optimal effort of Player 1 in case of war, $\overline{e}_1$, is the solution of

$$\frac{V_2^\gamma \gamma}{(e_1^\gamma + V_2^\gamma)} (V - (e_1 + V_2)) = e_1. \quad (19.16)$$

The expected payoff of Player 1 is

$$E\pi_1 = \frac{\bar{e}_1^{\gamma}}{\bar{e}_1^{\gamma} + V_2^{\gamma}} \left(V - (\bar{e}_1 + V_2)\right). \tag{19.17}$$

Since $\bar{e}_1 < V_1$,

$$\frac{\bar{e}_1^{\gamma}}{(\bar{e}_1^{\gamma} + V_2^{\gamma})} (V - (\bar{e}_1 + V_2)k) \le \frac{V_1}{(V_1^{\gamma} + V_2^{\gamma})} (V - (\bar{e}_1 + V_2)k). \tag{19.18}$$

Since $V_1^{\gamma} / (V_1^{\gamma} + V_2^{\gamma})$ is increasing with γ,

$$\frac{V_1^{\gamma}}{(V_1^{\gamma} + V_2^{\gamma})} (V - (\bar{e}_1 + V_2)k) \le \frac{V_1^{\gamma}}{(V_1 + V_2)} (V - (\bar{e}_1 + V_2)k) < V_1. \tag{19.19}$$

Thus, Player 1 has no incentive to declare war.
In this case, only Player 2 declared war in the absence of transfer as we prove in the following proposition.

Proposition 19.3. *If Player* 1 *is unconstrained and Player* 2 *is constrained, in the absence of transfers, war is declared by Player* 2.

Proof. The expected value for Player 2 in case of war is given by

$$E\pi_2 = \frac{V_2^{\gamma}}{(\bar{e}_1)^{\gamma} + V_2^{\gamma}} \left(V - (\bar{e}_1 + V_2)\right). \tag{19.20}$$

Since $\bar{e}_1$ is the solution of equation (19.16),

$$\frac{V_2^{\gamma}}{(\bar{e}_1)^{\gamma} + V_2^{\gamma}} \left(V - (\bar{e}_1 + V_2)\right) = \frac{\bar{e}_1}{\gamma}. \tag{19.21}$$

Thus, Player 2 will have incentives to declare war whenever $\bar{e}_1 / \gamma > V_2$.

Let us see that $\bar{e}_1 > V_2$.

Consider the following function:

$$F(e_1) = \frac{V_2^{\gamma}\ \gamma}{(e_1^{\gamma} + V_2^{\gamma})} \left(V - (\bar{e}_1) + V_2)\right) - e_1. \tag{19.22}$$

This function is decreasing in e_1 and in the optimal effort of Player 1, $\bar{e}_1$, $F(\bar{e}_1) = 0$. Let us evaluate the function for $e_1 = V_2$.

$$F(V_2) = \frac{V_2^{\gamma}\ \gamma}{(V_2)^{\gamma} + V_2^{\gamma}} \left(V - (V_2 + V_2)\right) - V_2 =$$

$$= \frac{\gamma}{2}(V - 2V_2) - V_2$$

$$= \frac{\gamma}{2} V - V_2 (\gamma + 1).$$

Since Player 2 is constrained, $V_2 < (\gamma / 2(\gamma + 1))V$, thus $F(V_2) > 0$. Given that $F(e_1)$ is decreasing in e_1 and $F(\overline{e}_1) = 0$, $\overline{e}_1 > V_2$. Therefore, $\overline{e}_1 > \gamma V_2$ which implies that Player 2 always has incentives to declare war. ■

The general analysis of transfers in this case becomes more difficult than in the case where no player was constrained. But by studding the particular case $\gamma = 1$, we can conclude that, now, not always a transfer avoids war.

If $\gamma = 1$, the dividing point between being constrained or not for Player 2 is 1/4. Player 2 is constrained if and only if $V_2 / V < 1/4$. In this case, it is easy to compute the equilibrium effort for Player 1, $\overline{e}_1$. Equilibrium efforts are given by:

$$\overline{e}_1 = \sqrt{VV_2} - V_2 \text{ and } \overline{e}_2 = V_2. \tag{19.23}$$

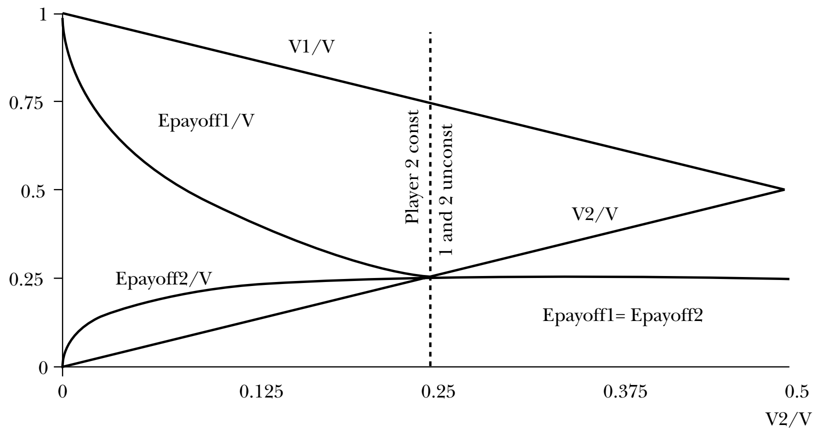
Payoffs amount to

$$E\pi_1^* = V + V_2 - 2\sqrt{VV_2} \text{ and } E\pi_2^* = \sqrt{VV_2} - V_2. \tag{19.24}$$

Figure 19.2 shows $E\pi_1^* / V$ as a function of the relative wealth of Player 2 in case of war. The expected payoff is decreasing with V_2 / V for Player 1 and increasing for Player 2. When $V_2 / V \geq 1/4$ both agents are unconstrained and the expected payoff is constant and equal to $V / 4$. The increasing straight line corresponds to V_2 / V, and the decreasing straight line corresponds to V_1 / V. It is clear from the picture that Player 1 never has incentive to declare war, his expected payoff in case of war is always less that his initial wealth. So war is too costly for Player 1. However, the expected payoff for Player 2 is bigger than his initial wealth whenever $V_2 / V < 0.25$. Thus, a relative wealth of 0.25 for Player 2 determines the decision between war and peace in the absence of transfers when $\gamma = 1$.

Now we are ready for the analysis of the first stage of the game. In this case, a peace agreement is feasible if there is a transfer from Player 1 to Player 2, T, such that:

FIGURE 19.2: Expected payoffs as a function of the relative wealth of player 2



1. After the transfer, both players are better off than if there had been a war. That is,

$$E\pi_1^* = V + V_2 - 2\sqrt{VV_2} \leq V_1 - T; \tag{19.25}$$

$$E\pi_2^* = \sqrt{VV_2} - V_2 \leq V_2 + T. \tag{19.26}$$

Or equivalently:

$$-\frac{V_2}{V} + \sqrt{\frac{V_2}{V}} \leq \frac{V_2 + T}{V} \leq -\frac{V_2}{V} + \sqrt{\frac{V_2}{V}}\,. \tag{19.27}$$

2. After the transfer, peace is an equilibrium outcome. That is, both players are unconstrained:

$$\frac{1}{4} \leq \frac{V_2 + T}{V}\,. \tag{19.28}$$

Summing up, we need to study under what conditions there is a transfer T such that

$$\max\left(\frac{1}{4}, -\frac{V_2}{V} + \sqrt{\frac{V_2}{V}}\right) \leq \frac{V_2 + T}{V} \leq -\frac{V_2}{V} + 2\sqrt{\frac{V_2}{V}}\,. \tag{19.29}$$

Let $f(\frac{V_1}{V}) = -\frac{V_2}{V} + \sqrt{\frac{V_2}{V}}$. Notice that since $\frac{V_2}{V} < \frac{1}{4}$ and $f(\frac{V_2}{V})$ is increasing in $\frac{V_2}{V}$, $f(\frac{V_2}{V}) < \frac{1}{4}$. Therefore, inequality (19.29) can be rewritten as:

$$\frac{1}{4} \leq \frac{V_2 + T}{V} \leq -\frac{V_2}{V} + 2\sqrt{\frac{V_2}{V}}. \qquad (19.30)$$

For this inequality to be well defined, we need that

$$\frac{1}{4} < -\frac{V_2}{V} + 2\sqrt{\frac{V_2}{V}}. \qquad (19.31)$$

Otherwise, it will be impossible to get a transfer that avoids war. In the following Lemma we prove under which values of V_2 / V the inequality (19.31) holds.

Lemma 19.1. *Let* $g(\frac{V_2}{V}) = -\frac{V_2}{V} + 2\sqrt{\frac{V_2}{V}}$. *For all* V_2 / V *such that* $V_2 / V > 1.7949 \times 10^{-2}$, $g(\frac{V_2}{V}) > 1/4$.

Proof. Notice first that $g(\frac{V_2}{V})$ is increasing for all $V_2 / V \in [0, 0.5]$. It is easy to see that 1.7949×10^{-2} is the solution of $g(\frac{V_2}{V}) = 1/4$ in the interval $[0, 0.5]$. It follows directly that $g(\frac{V_2}{V}) > 1/4$ for all $V_2 / V > 1.7949 \times 10^{-2}$. ■

Now, we are ready to state the main result for this case.

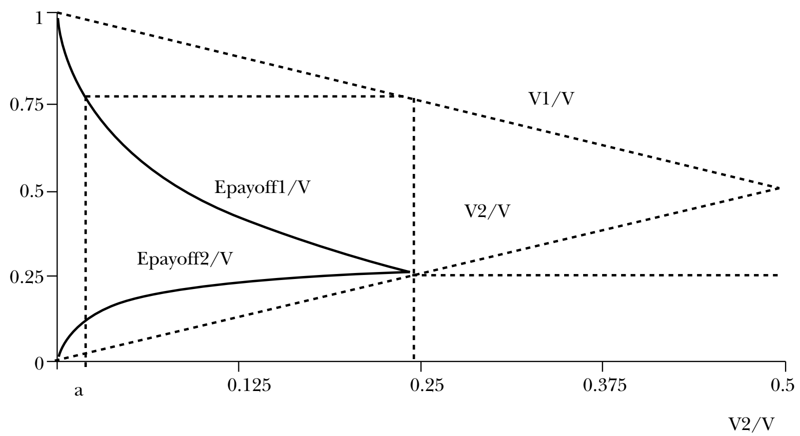
Proposition 19.4. *If* $\gamma = 1$ *and Player 2 is constrained, a peace agreement is feasible if and only if* $\frac{V_2}{V} > 1.7949 \times 10^{-2}$. *The minimal transfer,* $\hat{T}$, *that avoids war is such that it makes Player 2 be indifferent between war and peace, that is,* $\frac{V_2 + \hat{T}}{V} = \frac{1}{4}$.

Proof. By Lemma 19.1, if $\frac{V_2}{V} \leq 1.7949 \times 10^{-2}$, then $-\frac{V_2}{V} + 2\sqrt{\frac{V_2}{V}} \leq \frac{1}{4}$. Thus, it will be impossible to find T such that condition (19.30) holds. Thus, a peace agreement is feasible if and only if $\frac{V_2}{V} > 1.7949 \times 10^{-2}$. Let $\hat{T}$ be such that $\frac{V_2 + \hat{T}}{V} = \frac{1}{4}$. First notice that $\hat{T}$ exists because $\frac{V_2}{V} < \frac{1}{4}$ and for all $\frac{V_2}{V} > 1.7949 \times 10^{-2}$, $\frac{1}{4} < -\frac{V_2}{V} + 2\sqrt{\frac{V_2}{V}}$. Since $\frac{V_2 + \hat{T}}{V} = \frac{1}{4}$, both players are unconstrained now, the expected payoff for Player 2 is $\frac{V}{4}$, thus Player 2 is indifferent between war and peace. That $\hat{T}$ is the minimal transfer need is trivial, since a smaller transfer will still make Player 2 constrained and therefore ready to declare war. ■

If resource inequality is very large, negotiations cannot avoid war because the minimal transfer that will stop Player 2 from declaring war is so expensive for Player 1 that it makes this agent worse off than if war were waged.

We illustrate this last result in figure 19.3. The point a correspond to $V_2 / V =$ 1.7949×10^{-2} notice that if this is the relative wealth of Player 2, after the transfer $((V_2 + T) / V = 0.25)$ the relative wealth of Player 1 is exactly equal to his expected payoff before the transfer if war occurs. Thus, is $V_2 / V < a$, Player 1 will prefer war to pay the transfer.

FIGURE 19.3: Feasibility of an agreement



20.4. Conclusion

In this paper we presented a model of war where players are rational, information is complete and there are no binding agreements. We have shown that war can be avoided by transferring resources from one player to another in a variety of cases.

To end the paper, we discuss other mechanisms of altering initial conditions to the advantage of one or several players, that have been used in other parts of the literature. The review of the literature on war proper is made in the companion paper (Beviá and Corchón 2008).

1: Burning money. In some games, the outcome in equilibrium is affected by the capability of a player to destroy her own resources (van Damme 1989; Ben-Porath and Dekel 1992). This resembles what happens here but the mechanism by which the destruction affects the outcome is very different. In "burning money" games, it is a signal that one of the players is going after a certain payoff in a subgame. In our case, it is a way of reaching a certain subgame.

2: Transfer/Destruction of Endowments. In General Equilibrium models it is sometimes good for a country to transfer goods to another country. This is the so-called "Transfer Paradox" in International Trade (Leontief 1937; Samuelson 1952, 1954; Gale 1974). The paradox arises because by making a transfer (or even by destroying one's resources, as in Aumann and Peleg 1974) agents affect relative prices. Again, our case is different because in our model there is only one good, so relative prices play no role whatsoever. What happens in our case is that transfers affect both the opportunity cost and the expected revenues of war: i.e., once the potential aggressor has been loaded with money she risks too much and can gain very little by going to war.

3: Economic Diplomacy. Ponsati (2004) studies bilateral conflicts that affect the welfare of a third party. The conflict takes the form of a war of attrition, and intervention is modelled as the possibility that the stakeholder aids the agreement with transfers to the contenders. In this case, the source of money is external to the conflict.

4: Patents. Gallini (1984) has shown that an incumbent firm may license its production technology to reduce the incentive of a potential entrant to develop its own, possibly better, technology. If the licensing contract leaves the potential entrant with its expected return from further research, this firm has no incentive to engage in further R&D activity. This is akin to the idea that the rich country pays the poor country in order to deter it from attacking. But the model is different from ours since we have the problem of preventing *either* country from attacking the other.

References

Aumann, R. and B. Peleg. "A Note on Gale's Example." *Journal of Mathematical Economics* 1, no. 2 (1974): 209-211.

Becker, G. "A Theory of Competition Among Pressure Groups for Political Influence." *The Quarterly Journal of Economics* 98 (1983): 371-400.

Ben-Porath, E. and E. Dekel. "Coordination and the potential for self-sacrifice." *Journal of Economic Theory* 57 (1992): 36-51.

Beviá, C. and L. Corchón. "Peace Agreements without Commitments." Working Paper 081508, Universidad Carlos III, 2008.

Clausewitz, Carl von. *On War.* Spanish edition translated by Carlos Fortea (1832). La Esfera de los Libros, Madrid, 2005.

Corchón, L. "The theory of Contest. A Survey." *Review of Economic Design* 11, no. 2, (2007): 69-100.

DAMME, E. van. "Stable equilibria and forward induction." *Journal of Economic Theory* 48 (1989): 476-496.

GALE, D. "Exchange Equilibrium and Coalitions: an Example." *Journal of Mathematical Economics* 1 (1974): 63-66.

GALLINI, N. T. "Deterrence by Market Sharing: A Strategic Incentive for Licensing." *American Economic Review* 74, no. 5 (1984): 931-941.

GARFINKEL, M. and S. SKAPERDAS. "Economics of Conflict: An Overview." In T. Sandler and K. Hartley, eds., *Handbook of Defense Economics*, Amsterdam: North Holland, 2007.

HIRSHLEIFER, J. "The Paradox of Power." *Economics & Politics* 3 (1991): 177-200.

—. "The Dark Side of the Force: Western economic Presidential Address." *Economic Inquiry* 32, no. 1, (1994): 1-10.

KONRAD, K. "Strategy in Contests. An Introduction." WZB-Markets and Politics Working Paper No. SP II 2007-01, 2007.

KRUEGER, A. "The Political Economy of the Rent-Seeking Society." *American Economic Review* 64 (1974): 291-303.

LEONTIEF, W. "Note on the Pure Theory of Capital Transfer." In *Exploration in Economics*, Taussig Festschrift, 1937: 84-91.

O'NEILL, B. "A Survey of Game Theory Models on Peace and War." In R. Aumann and S. Hart, eds. *Handbook of Game Theory*. North Holland, 1990.

PONSATI, C. "Economic Diplomacy." *Journal of Public Economic Theory* 6, no. 5(2004): 675-691.

SAMUELSON, P. A. "The transfer problem and transport costs: The terms of trade when impediments are absent." *Economic Journal* 62, no. 246 (1952): 278-304.

—. "The transfer problem and transport costs II: Analysis of effects of trade impediments." *Economic Journal* 64, no. 254 (1954): 264-289.

SKAPERDAS, S. "Cooperation, conflict, and power in the absence of property rights." *American Economic Review* 82, no. 4 (1992): 720-739.

SMITH, A. *An Inquiry into the Nature and Causes of the Wealth of Nations*. First Edition. London: Strahan and Cadell, 1776.

TULLOCK, G. "The Welfare Cost of Tariffs, Monopolies and Theft." *Western Economic Journal* 5 (1967): 224-232.

20

Arrow's Theorem on Single-Peaked Domains

Sean Gailmard
Department of Political Science
University of California – Berkeley

John W. Patty
Department of Government and Institute for Quantitative Social Science
Harvard University

Elizabeth Maggie Penn
Department of Government and Institute for Quantitative Social Science
Harvard University

20.1. Introduction

For any weakly Paretian preference aggregation rule defined on the domain of all single-peaked preferences over a finite set of at least three alternatives, satisfaction of independence of irrelevant alternatives implies that the preference aggregation rule is neutral (i.e., it does not depend upon the labels of the alternatives). The result is briefly related to the study of political institutions by pointing out several institutional features that violate neutrality, including bicameralism, gatekeeping powers, supermajority requirements, and veto power.

20.2. Collective rationality and neutrality

It is not too strong to argue that the following theorem, Arrow's Possibility Theorem, is the foundation of the modern analytical study of political institutions.

Theorem 20.1 (Arrow 1951): *If there are three or more alternatives and at least two individuals, each of whom may have any preference over the alternatives, then the only Pareto efficient preference aggregation rule that satisfies independence of irrelevant alternatives is dictatorial.*

Black's median voter theorem is as well-known as Arrow's Theorem.

Theorem 20.2 (Black 1948): *If individual preferences are single-peaked, then majority rule is a non-dictatorial preference aggregation rule that satisfies independence of irrelevant alternatives.*

Obviously, the difference between the theorems of Arrow and Black is the degree of heterogeneity that individual preferences may exhibit. Of course, Black's theorem does *not* state that all preference aggregation rules make sense when preferences are known to be single-peaked. Furthermore, Black's theorem as an existence and characterization result does not go particularly far, as majority rule is a precise institutional form.[1] In this paper, we show that any weakly Paretian preference aggregation rule that is independent of irrelevant alternatives must be neutral even when preferences are known to be single-peaked. In other words, even in instances in which there is a well-defined, transitive majority preference relation, neutrality is required for collective choice to be simultaneously weakly Paretian and independent of irrelevant alternatives. The key to the result is that single-peakedness is not "enough" information about the alternatives. To know with certainty that the alternatives "can be ordered'" (by the voters' preferences) is not equivalent to knowing *how* the alternatives will be ordered.

Given the generality and power of Arrow's Theorem, the novelty of our results lies in their application to *the* canonical setting for models of political institutions: the unidimensional spatial model.[2] The next Section defines the (canonical) theoretical framework and proves the paper's main result. In Section 0, we offer a brief discussion of the connections between our results and the analysis of political institutions.

20.2.1. Notation and definitions

There is a finite collection of K *alternatives* (or *policies*), X, and a finite collection of n *individuals* (or *voters*) N. We assume that $K \geq 3$ and $n \geq 2$. Individual i's preferences are represented by a reflexive, transitive and complete binary relation R_i. The notation xR_iy implies that i weakly prefers x to y, xP_iy implies that i strictly prefers x

[1] For example, May (1952) famously demonstrated that majority rule is equivalent to neutrality, anonymity, and a monotonicity requirement. See also Moulin (1980).

[2] A smattering of examples to justify the term "canonical" might include Downs(1957), Davis et al. (1970), McCubbins et al. (1994), Poole and Rosenthal (1997), and Krehbiel (1998) among, of course, many others.

to y, and $xI_i y$ implies i is indifferent between x and y. We write $M(R_i)$ to denote the maximal element(s) of R_i: $x \in M(R_i) \Leftrightarrow xR_i y, \forall y \in X$. Without any interesting loss of generality, any element of $M(R_i)$ is referred to as i's *most-preferred policy* or *ideal point*.

Throughout, $\rho = (R_i, \ldots, R_n)$ denotes an n-dimensional preference profile describing the preferences of all individuals: the notation R^n represents the collection of all n-dimensional profiles of weak orders on X. Any nonempty set $\mathrm{D} \subseteq R^n$ is referred to as a *preference domain*, and with strict inclusion, D is referred to as a *restricted domain*. We will come back to restricted domains in more detail in Section 0. And $\mathrm{P}^n \subset \mathrm{R}^n$ denotes the collection of all n-dimensional profiles of linear (*i.e.*, strict) orders on X. For any preference profile $\rho \in \mathrm{R}^n$, $\rho|_S$ denotes the restriction of ρ to the set of alternatives $S \subseteq X$. Similarly, for any individual preference $R \in \mathrm{R}$, $R_i|_S$ denotes the restriction of i's preference relation to the set S. For any preference profile $\rho \in \mathrm{R}^n$ and pair of alternatives $(x, y) \in X^2$, the notation $P(x, y; \rho) \equiv \{i \in N: xP_i y\}$ denotes the set of individuals who strictly prefer x to y under ρ, and $R(x, y; \rho) = \{i \in N: xR_i y\}$ denotes the set of individuals who do not strictly prefer y to x.

20.2.2. Preference aggregation rules

A *preference aggregation rule* is any function, F: $\mathrm{D} \to \mathrm{R}$, that maps preference profiles into weak orders over X. The notation $xR_F(\rho)y$ denotes weak social preference under F at profile $\rho \in \mathrm{D}$ and $xP_F(\rho)y$ denotes strict social preference. The following definitions characterize several properties of preference aggregation rules.

Definition 20.1 (Weakly Paretian): *A preference aggregation rule F is weakly Paretian if for all $\rho \in \mathrm{R}^n$ and all $(x, y) \in X^2$,*

$$P(x, y; \rho) = N \Rightarrow xP_F(\rho)y$$

Definition 20.2 (Independent of Irrelevant Alternatives (IIA)): *A preference aggregation rule F is independent of irrelevant alternatives (IIA) if, for all $(x, y) \in X^2$ and all $(\rho, \rho') \in \mathrm{D}^2$,*

$$\rho|_{\{x,y\}} = \rho'|_{\{x,y\}} \Rightarrow F(\rho)|_{\{x,y\}} = F(\rho')|_{\{x,y\}}$$

Definition 20.3 (Neutrality): *A preference aggregation rule F is neutral if for every permutation σ: $X \to X$, and every profile $\rho \in \mathrm{P} \cap \mathrm{D}$,*[3]

[3] As with collective choice functions, we define neutrality with respect to strict preference profiles.

$$xR_F(\rho)y \Leftrightarrow \sigma(x)\ R_F(\sigma(\rho))\ \sigma(y)$$

20.2.3. Restricted domains: single-peakedness and free triples

In this section we define two restricted preference domains: single-peaked preferences and the two-free triple domain. Restricted domains have attracted the interest of many scholars because they may lead to the existence of non-dictatorial Arrovian preference aggregation rules. Our interest, as we discuss briefly in the conclusion, is less about the existence, and more about the characterization, of such preference aggregation rules.

Single-peaked preferences. The domain of *single-peaked preferences* is the set of all profiles of preferences such that there exists a function q: $X \rightarrow \{1, 2, ..., K\}$ such that q is a bijection and every individual's preferences are consistent with a quasi-concave utility function of $\{q(x): x \in X\}$. This preference domain is denoted by $S^n \subset R^n$. While this preference restriction is widely utilized and intuitively quite simple, Ballester and Haeringer (2007) prove that the set S^n is completely characterized by two conditions, worst-restriction (Sen 1966; Sen and Pattanaik 1969) and α-restriction, defined below.

Definition 20.4 (Worst-restriction): *A profile ρ is worst-restricted if, for every triple of alternatives,* $(x, y, z) \in X^3$, $\backslash W(\rho_{\{x, y, z\}})\ | \leq 2$.

Definition 20.5 (α-Restriction): *A preference profile ρ is α-restricted if there do not exist two agents, i, j ∈ N, and four alternatives w, x, y, and z such that*

1. *The preferences over w, x, and z are opposite:* wP_ixP_iz *and* zP_jxP_jw.
2. *The players agree about the ranking of y and x:* yP_ix *abd* yP_jx.

Definition 20.6 (Single-peakedness): *A preference profile is single-peaked if and only if it satisfies worst-restriction and α-restriction (Ballester and Haeringer 2007).*

It is important to note at this point that the domain S^n is the set of *all* single peaked preference profiles. In other words, in *a priori* terms, any ordering of the alternatives is possible.[4]

[4] This point is a technical one, but important for broader considerations of the results in this paper. In particular, for any given linear ordering of the alternatives, $Q \in$ P, one can identify the set of prefer-

Free triples. Several authors have examined domain restrictions related to the heterogeneity of "triples'" of preferences in any realized preference profile.[5] To be precise, the *k*-free-triple restriction (where $k \in \{1, 2, 3, 4, 5, 6\}$) is formally defined as follows.

Definition 20.7 (*k*-free triple domains): *For any $k \leq n$, define the k-free triple domain, $\mathrm{T}_k^n \subseteq \mathrm{R}^n$, as*

$$\mathrm{T}_k^n \equiv \{\rho \in \mathrm{R}^n \colon |R_i|_{abc} \in \rho\,|_{abc}| \leq k\} \text{ for all } (a, b, c) \in X^3$$

In words, T_k^n is the set of preference profiles such that for each triple, $(a, b, c) \in X^3$, at most k different orderings on those triples are allowable. A domain D satisfies the *k*-free triple domain restriction if $\mathrm{T}_k^n \subseteq \mathrm{D}$. The principal interest of much of the literature examining free triple restrictions is the minimal amount of preference homogeneity that one must presume to ensure that majority preference in acyclic.

Ubeda (2003) has recently used two-free triple domain restriction, demonstrating that on any domain satisfying the two-free triple restriction, weakly Paretian and IIA imply neutrality, a conclusion that mirrors our own (Theorem 20.3, below). The key distinction between Ubeda's result and Theorem 20.3 is that the two-free triple domain and the single-peaked domain are not nested. Specifically, for all $n \geq 2$, $\mathrm{T}_2^n \not\subseteq \mathrm{S}^n$ and $\mathrm{S}^n \not\subseteq \mathrm{T}_2^n$. In other words, satisfaction of either the *k*-free triple restriction nor single-peakedness does not imply satisfaction of the other. With the preliminaries in hand, we are now in a position to state and prove our main result.

Theorem 20.3. *Let F be a preference aggregation rule defined on S^n. Then F is weakly Paretian and IIA only if F is neutral.*

Proof: The proof is adapted from Ubeda (2003) to the case of domain S^n. Consider two strict profiles ρ_1 and $\rho_2 \in \mathrm{S}^n$ with $\rho_2 = \sigma(\rho_1)$. We can get from one ρ to any $\sigma(\rho)$ by switching alternatives one at a time, and so we may limit ourselves to considering permutations that only switch one pair of alternatives. Thus, ρ_1 and ρ_2 are such that for one pair x, y with $x \neq y$, $\rho_1\,|_{ab} = \rho_2\,|_{ab}$ for all $a, b \neq x, y$,

ences that are single-peaked with respect to Q, this set is denoted by S_Q, and the set of all profiles of such preferences is denoted by S_Q^n. This space is widely discussed in the political economy literature. For a succinct and lucid overview of the power of the assumption that Q is known a priori, see Chapter 2.4 of Austen-Smith and Banks (2004), in particular Theorem 2.4.

[5] See, among others and in addition to those cited elsewhere in this paper, Blau (1957), Murakami (1961), Kalai et al. (1979), Bordes and Le Breton (1990), Campbell and Kelly (1993), Redekop (1993), Kelly (1994), and Bordes et al. (1995).

and $\rho_1|_{xz} = \rho_2|_{yz}$, for all z. In other words, ρ_1 is identical to ρ_2 up to a permutation of x, y. By IIA we know that if $xR_F(\rho)\, y$ then $xR_F(\rho|_{xy})\, y$. It suffices to show that for all distinct triples $a, x, y \in X$, $xR_F(\rho_1)\, a \Rightarrow yR_F(\rho_2)\, a$ and $aR_F(\rho_1)\, x \Rightarrow aR_F(\rho_2)\, y$. Construct a new profile $\acute{\rho} \in S^n$ with $P(x, a; \acute{\rho}) = P(y, a; \acute{\rho}) = P(x, a; \rho_1) = P(y, a; \rho_2)$. Suppose $aR_F(\rho_1)\, x$. Let $P(x, y; \acute{\rho}) = N$. F IIA implies $aR_F(\acute{\rho})\, x$, because $\acute{\rho}|_{ax} = \rho_1|_{ax}$. F weakly Paretian and transitive implies that $aR_F(\acute{\rho})\ xR_F(\acute{\rho})\, y$. F IIA again implies $aR_F(\rho_2)\, y$, because $\acute{\rho}|_{ay} = \rho_2|_{ay}$. Thus, $aR_F(\rho_1)\, x \Rightarrow aR_F(\rho_2)\, y$. The case where $xR_F(\rho_1)\, a$ follows similarly, with $P(x, y; \acute{\rho}) = \varnothing$.

We last need to check that the constructed $\acute{\rho}$ is indeed an element of S^n. We required that $P(x, a; \acute{\rho}) = P(y, a; \acute{\rho}) = P(x, a; \rho_1)$, and that $P(x, y; \acute{\rho}) = N$ or $\varnothing$. $\acute{\rho}$ clearly satisfies α - restriction because it only specifies preferences over three elements of X. And $\acute{\rho}$ satisfies worst-restriction because $P(x, y; \acute{\rho}) = N$ or $\varnothing$ implies that either x or y is never ranked last by any individual.

20.3. Conclusion

A theory of political institutions necessarily must deal with the possibility that policy choices in the future may have no natural structure that is known *a priori*—even if some such ordering is presumed to structure all political choices. For example, while the ordering of preferences over the marginal rate of a flat income tax may be presumed (with at least some implicit heroism) to be single-peaked according to the usual ordering of the real line, the general presumption that preferences are single-peaked with respect to a set of political alternatives does not provide enough information to declare what the "median most-preferred alternative" is, even after individuals have informed you of their most-preferred alternatives. So long as this ordering is not known when the institution is designed, the mere fact that some such ordering will exist does not obviate the need to be careful in one's choice of institutional details. In particular, if *ex post* interpretation/rationalizability of collective preference is a desideratum, then the institution in question *must* be neutral with respect to the alternatives of political choice. This is an important point particularly once one acknowledges that many features of policymaking institutions, such as bicameral requirements, supermajoritarian quotas, separation-of-powers systems, and gatekeeping institutions such as legislative committees necessarily lead to violations of neutrality.[6]

[6] This is particularly true of the equilibrium policy outcomes predicted to occur within such systems by non-cooperative game theoretic analysis. This linkage is a deep one that is beyond the scope of the

Theorem 20.3 implies that one must be careful in interpreting collective will (even insofar as being "well-behaved") in any institution that is non-neutral. This point is highly relevant for those scholars who insist that majority rule cycles are infrequent and/or untroubling (*e.g.*, Mackie 2003) and either explicitly or implicitly then rely upon appeals to aggregate outcomes such as vote totals and the passage or failure of proposed legislation as being indicative of collective will. The point of this paper, at some level, is that even in the realm of well-defined *majority will*, many institutions that are simultaneously and unambiguously democratic and relevant (*i.e.*, extant) must lead one to question whether the behaviors and/or outcomes produced within are necessarily "representative". To be even more forceful—the normative, prescriptive, descriptive, and inferential issues raised by Arrow's theorem (among others) are more than simple mathematical curiosities dreamed up for the purpose of scholarly debate. In very precise and simple terms, a democratic institution may be defended as truly "representative"—even with the presumption of single-peaked individual preferences only if its rules are themselves invariant to the alternatives under consideration.

References

ARROW, K. J. *Social Choice and Individual Values.* New York, NY: John Wiley and Sons, 1951.

AUSTEN-SMITH, D. and J.S. BANKS. *Positive Political Theory II: Strategy & Structure.* Ann Arbor, MI: University of Michigan Press, 2004.

BALLESTER, M. A. and G. HAERINGER. *A Characterization of the Single-Peaked Domain.* Working paper, Universitat Autònoma de Barcelona, 2007.

BLACK, D. "On the Rationale of Group Decision-making." *Journal of Political Economy* 56 (1948): 23-34.

BLAU, J. H. "The Existence of Social Welfare Functions." *Econometrica* 250, no. 2 (1957): 302-313.

BORDES, G. and M. LE BRETON. "Arrovian Theorems for Economic Domains. The Case Where There Are Simultaneously Private and Public Goods." *Social Choice and Welfare* 70, no. 1 (1990): 1-17.

BORDES, G., D. E. CAMPBELL and M. LE BRETON. "Arrow's Theorem for Economic Domains and Edgeworth Hyperboxes." *International Economic Review* 360, no. 2 (1995): 441-454.

CAMERON, C. M. *Veto Bargaining.* New York, NY: Cambridge University Press, 2000.

CAMPBELL, D. E. and J. S. KELLY. "t or 1-t. That is the Trade-Off." *Econometrica* 610, no. 6 (1993): 1355-1365.

CROMBEZ, C., T. GROSECLOSE and K. KREHBIEL. "Gatekeeping." *Journal of Politics* 68, no. 2 (2006): 322-334.

DAVIS, O., M. HINICH and P. ORDESHOOK. "An Expository Development of a Mathematical Model of the Electoral Process." *American Political Science Review* 54 (1970): 426-448.

---

current project. An excellent discussion of this relationship is offered in Chapter 8 of Schofield (2006). Related recent models of political institutions with one or more of these features include Tsebelis and Money (1997), Krehbiel (1998), Epstein and O'Halloran (1999), McCarty (2000), Cameron (2000), and Crombez et al. (2006).

DOWNS, A. *An Economic Theory of Democracy*. New York: Harperand Row, 1957.

EPSTEIN, D. and S. O'HALLORAN. *Delegating Powers: A Transaction Cost Politics Approach to Policy Making Under Separate Powers*. New York, NY: Cambridge University Press, 1999.

KALAI, E., E. MULLER and M. A. SATTERTHWAITE. "Social Welfare Functions when Preferences are Convex, Strictly Monotonic, and Continuous." *Public Choice* 340, no. 1 (1979): 87-97.

KELLY, J. S. "The Free Triple Assumption." *Social Choice and Welfare* 110, no. 2 (1994): 97-101.

KREHBIEL, K. *Pivotal Politics: A Theory of U.S. Lawmaking*. Chicago, IL: University of Chicago Press, 1998.

MACKIE, G. *Democracy Defended*. New York, NY: Cambridge University Press, 2003.

MAY, K. O. "A Set of Independent Necessary and Sufficient Conditions for Simple Majority Decision." *Econometrica* 200, no. 4 (1952): 680-684.

MCCARTY, N. M. "Presidential Pork: Executive Veto Power and Distributive Politics." *American Political Science Review* 94, no. 1 (2000): 117-129.

MCCUBBINS, M., R. NOLL and B. WEINGAST. "Legislative Intent: The Use of Positive Political Theory in Statutory Interpretation." *Journal of Law and Contemporary Problems* 570, no. 1 (1994): 3-37.

MOULIN, H. "On Strategy-Proofness and Single Peakedness." *Public Choice* 350, no. 4 (1980): 437-455.

MURAKAMI, Y. "A Note on the General Possibility Theorem of the Social Welfare Function." *Econometrica* 290, no. 2 (1961): 244-246.

POOLE, K. and H. ROSENTHAL. *Congress: A Political-Economic History of Roll-Call Voting*. New York, NY: Oxford University Press, 1997.

REDEKOP, J. "Arrow-Inconsistent Economic Domains." *Social Choice and Welfare* 100, no. 2 (1993): 107-126.

SCHOFIELD, N. *Architects of Political Change: Constitutional Quandaries and Social Choice Theory*. New York: Cambridge University Press, 2006.

SEN, A. K. "A Possibility Theorem on Majority Decisions." *Econometrica* 340, no. 2 (1966): 491-499.

SEN, A. K. and P. K. PATTANAIK. "Necessary and Sufficient Conditions for Rational Choice Under Majority Decision." *Journal of Economic Theory* 10, no. 2 (1969): 178-202.

TSEBELIS, G. and J. MONEY. *Bicameralism*. New York, NY: Cambridge University Press, 1997.

UBEDA, L. "Neutrality in Arrow and Other Impossibility Theorems." *Economic Theory* 230, no. 1 (2003): 0-195.

List of Figures and Graphs

List of Tables

About the Authors

Enriqueta Aragonès is research professor at the Institute of Economic Analysis, CSIC, and affiliated faculty of the Barcelona Graduate School of Economics. She was previously assistant professor in the Department of Economics and Business at Pompeu Fabra University, researcher of CBRSS at Harvard University, visiting fellow of the Center in Political Economy at Washington University, and visiting professor at the Autonomous University of Barcelona and Pompeu Fabra University. She obtained her Ph.D. in managerial economics and decision sciences at the Kellogg Graduate School of Management, Northwestern University in 1995. Her research interests are political economy, social choice, decision theory, and game theory. Her published works include articles in journals like *American Economic Review, American Political Science Review*, and the *Journal of Economic Theory*. She is an associate editor of the *Spanish Economic Review.*

Dan Bernhardt is IBE Distinguished Professor of the Departments of Economics and Finance at the University of Illinois. He has been a professor in the Department of Economics at Queen's University. He obtained his Ph.D. in economics at Carnegie Mellon University in 1986. His research interests are applied microeconomic theory, political economy, finance, labor economics and industrial organization. He has published articles in *American Economic Review, Journal of Economic Theory* and *Journal of Finance.*

Carmen Beviá is associate professor of the Department of Economics and Economic History of the Autonomous University of Barcelona and affiliated faculty of the Barcelona Graduate School of Economics. She was previously visiting professor at the University of Alicante, and visiting fellow at the University of Rochester, and University of California, Davis. She obtained her Ph.D. in economics at the University of Alicante in 1994. Her research interests are social choice, game theory, public economics, and political economy. She has published articles in journals like *Journal of Economic theory, Games and Economic Behavior* and *Social Choice and Welfare.* She is a member of the editorial board of *Mathematical Social Sciences.*

Carles Boix is professor of politics and public affairs in the Department of Politics and the Woodrow Wilson School of Public and International Affairs at Princeton University. He was previously professor of the Department of Political Science at the University of Chicago, and visiting professor of the Departments of Political Science and Economics at Pompeu Fabra University. He obtained his Ph.D. in political science at Harvard University in 1995. His main research interests are political economy and comparative politics, and he has published several books as well as articles in reviews like *American Political Science Review, World Politics,* and *Journal of Law, Economics and Organization.* He has served on the editorial board of *Series on European Politics.*

Laurent Bouton is a Ph.D. student of the European Centre for Advanced Research in Economics and Statistics at the Free University of Brussels. His research interests are focused on voting theory, public economics and political economy. He has published in the *European Journal of Political Economy* and a chapter in *L'impôt et la politique fiscale en Belgique.*

Daniel Cardona is professor of the Department of Applied Economics at the University of the Balearic Islands. He has been visiting professor of the Department of Foundations of Economic Analysis at the University of Alicante. He obtained his Ph.D. in economics at the Autonomous University of Barcelona (IDEA) in 1997. His research interests are political economy, social choice and voting theory. He has published articles in journals like *Journal of Economic Theory, Economic Theory,* and *Theory and Decision.*

Micael Castanheira is researcher and professor of the European Centre for Advanced Research in Economics and Statistics at the Free University of Brussels. He was also previously researcher at the University of Bocconi. He obtained his Ph.D. in economics at the Free University of Brussels in 1998. His research fields are political economy, voting theory, economic reforms, and information management. His publications include papers in the *Journal of the European Economic Association, Economic Journal,* and *European Journal of Political Economy.*

Josep Colomer is research professor of the Institute of Economic Analysis of CSIC, visiting professor of the Department of Economics and Business at Pompeu Fabra University, and affiliated faculty of the Barcelona Graduate School of Economics. He obtained his Ph.D. in economics at the University of Barcelona in 1984. His main research interests are institutions, comparative politics, and democratization. He has published several books as well as articles in journals like *Political Studies, Public Choice* and the *European Journal of Political Research.* He is a member of the editorial board of *Journal of Elections, Public Opinion and Parties, Journal of Globalization, Competitiveness and Governability,* and the *Journal of Theoretical Politics.* He is also a member of the international advisory committee for the journals *Political Studies* and *Political Studies Review.*

Luis C. Corchón is professor at the Department of Economics at Carlos III University of Madrid. He was previously professor of economics at the University of Alicante and Pompeu Fabra University and visiting scholar at the universities of Rochester, Harvard, Stanford and New York. He holds a Ph.D. in economics from the London School of Economics. His research interests are game theory, industrial organization, mechanism design, political economy, and economics of conflict. His published works include two books and articles in journals like *Quarterly Journal of Economics*, *Journal of Economic Theory*, and *Games and Economic Behavior*. He is a member of the editorial board of *Journal of Public Economic Theory* and *Review of Economic Design*.

Pilar Dellunde is associate professor of the Department of Philosophy at the Autonomous University of Barcelona. She obtained her Ph.D. in mathematical logic at the Autonomous University of Barcelona in 1996. Her research interests are fuzzy and graded logics, normative multi-agent systems and multi-agent automated negotiation. Her articles have been published in *Lecture Notes in Computer Science*, *Frontiers in Artificial Intelligence*, and *Journal of Symbolic Logic*.

John Duggan is professor of the Departments of Political Science and Economics, and director of the W. Allen Wallis Institute of Political Economy at the University of Rochester. He was also previously visiting associate professor of the Division of Humanities and Social Sciences at California Institute of Technology and assistant professor in the Department of Economics at Queen's University. He received his Ph.D. in social sciences from the California Institute of Technology in 1995. His research interests are economic theory, political economy, and social choice theory. He has published articles in *Games and Economic Behavior*, *American Economic Review*, and *Social Choice and Welfare*. He is part of the editorial board of *Social Choice and Welfare*, *Mathematical Social Sciences*, and *Journal of Economic Theory*.

Jon X. Eguia is assistant professor in the Departments of Politics and Economics at New York University. He was previously visiting professor of the Department of Economics and Economic History at the Autonomous University of Barcelona. He obtained his Ph.D. in social sciences at California Institute of Technology in 2007. His research interests are formal political theory, political economy, social choice, public economics, collective decision-making, coalition formation, and voting and elections. He has published articles in journals like *Quarterly Journal of Political Science*, *Social Choice and Welfare*, and *Journal of Public Economic Theory*.

Alexander Elbittar is researcher of the Economics Division at the Center for Economic Research and Study (CIDE). He obtained his Ph.D. in economics at the University of Pittsburgh in 2000. His research work is focused on applied game theory, auction theory, experimental economics, econometrics and numeric methods. His publications include papers in *Advances in Economic Design* and books like *Teoría Conductual de la Elección: Decisiones que se revierten*.

David Epstein is professor in the Political Science Department at Columbia University. He was also previously visiting professor of the Graduate School of Business at Stanford University. He obtained his Ph.D. in political economics from the Graduate School of Business at Stanford University in 1992. His research interests are political science, political economy and game theory. He has written books on political economy and has published articles in journals like *Quarterly Journal of Law, Economics, and Organization*, *American Journal of Political Science*, and *American Political Science Review.*

Sean Gailmard is Charles and Louise Travers Assistant Professor of the Department of Political Science at the University of California, Berkeley. He was previously assistant professor in the Department of Political Science at Northwestern University and in the Harris School of Public Policy Studies at Chicago University. He obtained his Ph.D. in social sciences at California Institute of Technology in 2002. His research interests are American political institutions, particularly the bureaucracy and congress, game theory, statistical modeling, and collective decision making. He has published articles in journals like *American Journal of Political Science*, *Journal of Politics*, and *Quarterly Journal of Political Science.* He is member of the editorial board of *e-Journal on Bureaucracies and Public Administration.*

Andrei Gomberg is professor and researcher of the Center for Economic Research at the Instituto Tecnológico Autónomo de México (ITAM). He was also previously visiting professor at University Carlos III. He obtained his Ph.D. in economics at New York University in 2000. His research interests are microeconomic theory, political economy, public finance and experimental economics. His publications include papers in the *Journal of Economic Theory*, *Journal of Public Economic Theory*, and *Social Choice and Welfare.*

Jens Großer is assistant professor of the Department of Political Science and Economics at Florida State University. He obtained his Ph.D. in economics at the University of Amsterdam, CREED and the Tinbergen Institute in 2006. His major research interests are experimental economics, game theory, political economy, collective action and voting, and decision-making. His publications include papers in *Economic Theory* and *American Political Science Review.*

Matias Iaryczower is assistant professor of the Division of Humanities and Social Sciences at California Institute of Technology. He obtained his Ph.D. in economics at the University of California, Los Angeles in 2005. His research interests are political economy, institutions and organizations and applied game theory. His publications include papers in *Quarterly Journal of Political Science, American Political Science Review* and *American Journal of Political Science.*

Gilat Levy is reader of the Department of Economics at the London School of Economics and Political Science. She was previously a visiting fellow at Princeton University. She obtained her Ph.D. in economics at Princeton University in 1999. Her main research fields are microeconomic theory, political economy, and law and economics. Her publications include papers in *Econometrica*, *American Economic Review*, and *Quarterly Journal of Economics*. She has been part of the board of directors of the *Review of Economics Studies*, and associate editor of the *Journal of Public Economics* and the *Journal of European Economics Association*.

Alessandro Lizzeri is professor of the Department of Economics at New York University. He was previously assistant professor of the Department of Economics at Princeton University and research fellow of the Center for Economic Policy Research. He obtained his Ph.D. in managerial economics and decision sciences at Northwestern University in 1995. His research interests are economic policy, political institutions, contract theory and game theory. He has published articles in *American Economic Review*, *Quarterly Journal of Economics*, and *Review of Economic Studies*. He is co-editor of the *American Economic Review*, and he has been editor of *Journal of Economic Theory* and associate editor of *Journal of the European Economic Association* and *International Economic Review*.

Humberto Llavador is associate professor and researcher of the Department of Economics and Business at Pompeu Fabra University and affiliated faculty of the Barcelona Graduate School of Economics. He was previously visiting professor of the Departments of Political Science and Economics at Yale University. He obtained his Ph.D. in economics at the University of California, Davis in 2000. His research interests are political economy, social choice and welfare, and public economics. He has published in journals like *The Quarterly Journal of Economics*, *Social Choice and Welfare*, and *Mathematical Social Sciences*.

Andrea Matozzi is assistant professor of the Division of Humanities and Social Sciences at California Institute of Technology. He obtained his Ph.D. in economics at the University of Pennsylvania in 2004. His research fields of interest are political economy, public economics and applied microeconomics. His publications include papers in *American Economic Review, Journal of Public Economics, and Games and Economic Behavior.*

Massimo Morelli is professor of the Department of Political Science at Columbia University. He was previously associate professor of economics and political science at the Ohio State University, Deutsche Bank member of the Institute for Advanced Study, School of Social Sciences at Princeton University and visiting professor at the University of Minnesota. He obtained his Ph.D. in economics at Harvard University in 1996. His research interests are game theory, mechanism design, political economy, governance institutions, behavioral and public economics, comparative politics and international relations. He has published articles in journals like *Econometrica*, *American Economic Review*, and *Journal of Economic Theory*. He is editor of *Review of Economic Design* and associate editor of *Berkeley Electronic Journals for Theoretical Economics* and *Economics*.

Sharyn O'Halloran is George Blumenthal Professor and director of the Advanced Policy and Economic Analysis Concentration in the Department of Political Science and the School of International and Public Affairs at Columbia University. She was previously visiting professor of the Graduate School of Business at Stanford University. She obtained her Ph.D. in political science at the University of California, San Diego in 1991. Her research interests are political institutions and public policy, electoral politics, U.S. trade policy, and globalization and international law. She has published some books and articles in journals like *American Journal of Political Science*, *Journal of Law, Economics & Organization*, and *American Political Science Review*.

Thomas R. Palfrey has been Flintridge Foundation Professor of Economics and Political Science of the Division of Humanities and Social Sciences at California Institute of Technology since 2006. He was professor of politics and economics at Princeton University and professor of economics and political economy at GSIA, Carnegie-Mellon University. He obtained his Ph.D. in social sciences at Caltech in 1981. His major research interests are economic theory, mechanism design, public economics, formal political theory, experimental economics and political science, game theory, voting and elections. His publications include papers in *Econometrica*, *American Economic Review*, and *Journal of Political Economy*. Among various editorial positions, he has been associate editor of *Econometrica*, *Games and Economic Behavior* and *Journal of Public Economics*.

John W. Patty is assistant professor of the Department of Government and the Institute for Quantitative Social Science at Harvard University. He was previously assistant professor of political economy and decision sciences at Carnegie Mellon University and visiting fellow of the Center in Political Economy at Washington University. He obtained his Ph.D. in economics and political science at California Institute of Technology in 2001. His research interests are formal political theory, game theory, congress, and bureaucracy. He has published articles in journals like *American Journal of Political Science*, *Social Choice and Welfare*, and *Games and Economic Behavior*. He is member of the editorial board of *Journal of Politics* and *Political Analysis*.

Elizabeth Maggie Penn is assistant professor of the Department of Government at Harvard University. She was previously assistant professor of the Department of Social and Decision Sciences at Carnegie Mellon University, and visiting fellow of the Center in Political Economy at Washington University. She obtained her Ph.D. in social sciences at California Institute of Technology in 2003. Her research interests are social choice theory, institutional organization and political economy. She has published articles in journals like *American Journal of Political Science*, *Social Choice and Welfare*, and *Public Choice*. She is member of the editorial board of *Journal of Politics*.

Nicola Persico is professor of the Department of Economics at New York University. He was previously associate professor of the Department of Economics at the University of Pennsylvania, assistant professor of the Department of Economics at UCLA, and NBER research associate. He obtained his Ph.D. in economics at Northwestern University in 1996. His research interests are in microeconomic theory, political economy, public economics, mathematical economics and game theory. He has published articles in journals like *American Economic Review*, *Econometrica*, and *Quarterly Journal of Economics*. He has been member of the editorial board of *International Economic Review*.

Clara Ponsati is director and research professor at the Institute of Economic Analysis, CSIC and affiliated faculty of Barcelona Graduate School of Economics. She was previously associate professor of the Department of Economics and Economic History at the Autonomous University of Barcelona and visiting professor at Pompeu Fabra University, University of Toronto, and University of California San Diego. She obtained her Ph.D. in economics at the University of Minnesota. Her research interests are political economy, social choice and game theory. She has published articles in journals like *Games and Economic Behavior*, *International Journal of Game Theory*, and *Journal of Economic Theory*. She is associate editor of *Review of Economic Design* and *International Journal of Game Theory*.

Pablo Querubin is Ph.D. candidate of the Department of Economics at Massachusetts Institute of Technology. His research interests are in political economy and development economics. He has published an article in *Brookings Papers on Economic Activity*.

Ronny Razin is reader of the Department of Economics at the London School of Economics and Political Science. He obtained his Ph.D. in economics at Princeton University in 2001. His research interests are applied microeconomics, political economy and industrial organization. His publications include papers in *Econometrica*, *American Economic Review*, and *Journal of Economic Theory*. He is an associate editor of the review *Economic Journal*.

John E. Roemer is Elizabeth S. and A. Varick Stout Professor of the Departments of Political Science and Economics at Yale University. He was previously visiting professor at Université de Paris I, Ecole Polytechnique de Paris, New York University, Harvard University, and the University of California, Los Angeles; associate professor and director of the Program on Economy, Justice and Society at the University of California, Davis. He is fellow of the American Academy of Arts and Sciences, the British Academy and the Econometric Society. He obtained his Ph.D. in economics from the University of California at Berkeley. His research interests are in political economy, distributive justice and theory of socialism. He has published many books and papers in journals like *Econometrica, American Economic Review* and *Quarterly Journal of Economics*. He was previously associate editor of *Journal of Economic Perspectives* and *Review of Economic Design*, and member of the editorial board of *Social Choice and Welfare*, and *Ethics*.

Guillermo Rosas is assistant professor of the Department of Political Science at Washington University. He obtained his Ph.D. in political science at Duke University. His research interests include comparative political economy, determinants of budget deficits, banking crises, and legislative politics in Latin America. He has published articles in journals like *International Organization*, *Economic Inquiry*, and *Economics & Politics*.

Norman Schofield is director of the Centre in Political Economy William R. Taussig and professor of political economy in the Department of Economics and Political Science at Washington University. He was previously Fulbright Distinguished Professor at Humboldt University, fellow of the Centre for Advanced Study in the Behavioral Sciences at Stanford University, and fellow of the American Academy of Arts and Sciences. He obtained his Ph.D. at Essex University and Doctorat d'Etat in economic science at the University of Caen. His main interests are constitutional political theory, social choice, mathematical economics, international politics and political economy. He has written several books, including *Multiparty Government* and *Social Choice and Democracy*. His publications include papers in *Econometrica, The Review of Economic Studies* and *The American Journal of Political Science*. He is an associate editor of *Social Choice and Welfare* and *Politics, Philosophy and Economics*, and former associate editor of *European Journal of Political Economy*.

Yael Shomer is a Ph.D. student of the Department of Political Science at Washington University. His research interests are comparative politics: institutions, legislative studies, parties and party system, electoral systems and political methodology. He has written and presented papers at the annual meeting of the American Political Science Association and the European Consortium for Political Research.

James M. Snyder, Jr. is Arthur and Ruth Sloan Professor of the Departments of Political Science and Economics at Massachusetts Institute of Technology. He was previously STIC-

ERD distinguished visitor at the London School of Economics and assistant professor of the Department of Economics at University of Chicago. He obtained his Ph.D. in economics at California Institute of Technology in 1985. His research interests are political science (American politics), elections, legislative behavior and institutions, and democracy. His articles have appeared in journals like *American Journal of Political Science*, *Econometrica* and *American Political Science Review.* He is a member of the editorial board of *American Political Science Review*, *American Journal of Political Science*, *Economics and Politics*, and *Business and Politics*.

Francesco Squintani is professor and researcher of the Department of Economics at Essex University and the Department of Economics at the University of Brescia. He was also previously assistant professor of ELSE at University College London and of the Department of Economics at the University of Rochester. He obtained his Ph.D. in economics at Northwestern University in 2000. His main research interests are political economy, voting models, applied game theory, dynamic games, and contract theory. He has published in journals like *Games and Economic Behavior*, *Journal of Economic Theory*, and *American Economic Review.*

Fundación BBVA

Plaza de San Nicolás, 4
48005 Bilbao
España
Tel.: +34 94 487 52 52
Fax: +34 94 424 46 21

Paseo de Recoletos, 10
28001 Madrid
España
Tel.: +34 91 374 54 00
Fax: +34 91 374 85 22

publicaciones@fbbva.es
www.fbbva.es

ISBN 978-84-96515-91-8