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Abstract

I study the relative role of technological and physical obsolescence in the determination of the timing of adoption and the monopolist's incentives to innovate. I show that depreciation of durable goods (physical obsolescence) makes decisions about the timing of adoption non-trivial. If the monopolist cannot perfectly restrict the timing of purchases (using purchase deadlines, trade-in allowances, etc.) then its optimal pricing policy will inefficiently delay adoption, which in turn will reduce the incentives to innovate.

Key words

Durable goods, obsolescence, adoption, innovation, monopoly.

Resumen

En este trabajo estudio el papel de la obsolescencia física y la obsolescencia tecnológica en la determinación de la estructutura temporal de la adopción y los incentivos del monopolista a innovar. Muestro como la depreciación de los bienes duraderos (la obsolescencia física) convierte las decisiones sobre el momento ideal para adoptar en un asunto nada trivial. Si el monopolista no es capaz de restringir el espacio temporal de las compras (utilizando fechas límite, descuentos por entrega de modelos antiguos, etc.) entonces su política de precios óptima retrasaría las adopciones de forma ineficiente, lo cual a su vez reduciría los incentivos a innovar.

Palabras clave

Bienes duraderos, obsolescencia, adopción, innovación monopolio.

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Technological and Physical Obsolescence and the Timing of Adoption

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1. Introduction

TECHNOLOGICAL innovations are very often embedded into new generations of durable goods. Thus, adoption of new technologies is associated with lumpy investment decisions that involve replacement of old units by new units. The timing of investment will be affected by various factors; in particular, by the relative technological disadvantage of the old unit (technological obsolescence), and the depreciation of the old unit (physical obsolescence), since typically maintenance costs and/or the frequency of break downs increase over time and, as a result, the net flow of services provided by the durable good decreases with the age of the unit. In some cases (software, for instance) the advantage of the new units is mainly associated to the technological developments incorporated, whereas physical deterioration of the old units has little importance. In some other cases (like cars or manufacturing machines) technological development is present but physical deterioration plays a much more significant role. In this paper I examine the relative role of technological and physical obsolescence in affecting the timing of adoption and the monopolist's incentives to innovate.

The optimal timing must balance the benefits from moving forward the lumpy investment (the increase in production services) with the costs (the price of the new unit). In the absence of depreciation (if obsolescence is purely technological), the benefits will not tend to increase over time (they may even decrease if further technological developments are expected to take place at some point in the future). Thus, a new technology should in principle be adopted as soon as it becomes available [1]. As a result, the timing of innovations will exclusively depend on *supply* factors, i.e., the incremental costs associated with accelerating innovations.

In contrast, if the flow of services provided by durable goods shrink over time then the optimal timing of adoption is no longer trivial. Depreciation affects the timing of adoption by reducing the value of the old unit and

1. The timing of adoption may also be affected by strategic considerations (Fudenberg and Tirole, 1985) and by consumer learning (Vettas, 1998). Our model abstracts from these issues by focusing on a monopoly enviroment with independent consumers and complete information.

hence increasing, over time, consumers' willingness to pay for the new unit. In fact, if the monopolist is unable to restrict the timing of purchases then it faces a trade-off, with a higher price inducing consumers to postpone their purchases. As a result, the timing of adoption is inefficiently delayed, which at the same time delays innovation.

I explore these issues in two different set-ups. In Section 2 I analyze a continuous-time infinite-horizon model, where at the beginning consumers are using a low quality durable good that depreciates over time. Consumers are heterogeneous only because they bought the low quality good at different dates. A monopolist can develop and produce a high quality good. Once the high quality good is available the monopolist sets a price and consumers choose whether and when to adopt the new good. In the absence of depreciation the monopoly solution is efficient. The reason is that the monopolist has full control over the timing of adoption and, as a result, he can appropriate the entire surplus, which provides the proper incentives to innovate. In contrast, with depreciation and heterogeneous consumers, monopoly pricing and the timing of purchases interact in such a way that in equilibrium both adoption and innovation are delayed. If the monopolist sets a constant price then monopoly profits are relatively low. In fact, the monopolist has incentives to restrict the timing of purchases. It is shown that commitment to a purchase deadline increases monopoly profits and it might increase efficiency (especially if consumer heterogeneity is limited). More complicated pricing policies can also have positive effects on profits and efficiency. In particular, the monopolist could offer a discount if consumers trade in their old units. If discounts decrease with the level of physical deterioration of the old unit (age) then the monopolist can induce consumers to bring their purchases forward and raise profits and efficiency.

In Section 3 I study a similar model but now the monopolist can sequentially create cumulative innovations and hence consumers make multiple purchases. Unfortunately, the model generates stationary equilibria only for particular parameter values. In the absence of depreciation the model (a pure model of technological obsolescence) is a simplified version of Fishman and Rob (2000) (FR from now onwards) that I discuss below. Under positive depreciation but no technological innovation (a pure model of physical obsolescence) I need to restrict myself to the case of homogeneous consumers (synchronized purchases) for tractability reasons. The latter model is closely related to the model of (non-durable) *cyclical* goods analyzed in Caminal (2004). I consider two extreme pricing games. In the first, the monopolist can restrict the timing of purchases (at no cost since consumers are perfectly synchronized). As in FR, the equilibrium frequency of purchases is inefficiently low because the monopolist cannot observe the individual history of purchases. In the second game the monopolist sets a constant price and cannot restrict the timing of purchases. In this case, the inefficiency is much larger. These two games can be thought of as representing the bounds of more realistic set-ups. It is also shown that the seller's commitment capacity pushes the two outcomes in opposite directions. Under restricted timing if the monopolist can commit to future actions then the outcome is more inefficient than in the time consistent equilibrium. In contrast, under unrestricted timing, commitment power reduces the inefficiency of the monopoly outcome. Discounts based on the age of the old unit are also shown to raise profits and efficiency, at the cost of reducing the consumer surplus. Finally, it is shown that leasing, as opposed to selling, may be a profitable strategy for the monopolist, although the design of the lease is not a trivial matter.

The literature has emphasized two reasons that induce the monopolist to under-invest in innovation. The first is the classic (static) partial appropriability problem of a non-discriminating monopoly: if consumers are heterogeneous and the monopolist cannot price discriminate then profits associated to the innovation are lower than total surplus, and as a result the incentives to innovate are below the efficient level (See, for instance, chapter 10 in Tirole, 1988). The second one is more sophisticated and has to do with the dynamic nature of innovation. FR present an infinite horizon model with homogeneous consumers, where a monopolist produces a sequence of cumulative innovations. Consumers are willing to pay only for the incremental value of the innovation. Also, since consumers expect that the current innovation will be available for free at the purchase of the next innovation, the incremental value of the current innovation is valued only in the time interval between two consecutive innovations. Therefore, the monopolist cannot appropriate the entire surplus and again will under-invest in innovation.

In this paper I emphasize a third mechanism, which has to do with the interaction between prices and the timing of adoption, and the role played by physical obsolescence. The monopolist has incentives to use any direct or indirect device available in order to influence consumers' timing of purchases. Such ability will depend on the degree of synchronization of consumers (one type of heterogeneity) but also on its commitment capacity and the administrative costs associated with various marketing strategies [2].

2. In a sense this paper generalizes FR's result, by pointing out that inefficency of the monopoly outcome arises in any model with repeated purchases of a durable good, and it is not restricted to the existence of cumilative innovations.

This paper is also related to the literature on new product introduction in durable-goods markets: Waldman (1993, 1996), Choi (1994), Fudenberg and Tirole (1998), Lee and Lee (1998) [3]. Unlike those papers, I am particularly concerned about the timing of adoption.

There is also a literature that aims at explaining the aggregate time series of investment on durable goods, based on the cross-sectional distribution of the ages of durable goods and the stochastic properties of the environment (See, for instance, Adda and Cooper, 2000, and its references). In contrast to this literature, I am mainly concerned with the pricing problem of the supplier of durable goods and the associated incentives to innovate.

3. See also the survey by Waldman (2003).

2. A model with a single innovation

TIME is a continuous variable and horizon is infinite. There is a continuum of infinitely-lived agents (mass one), who at time 0 have been using a low quality durable good (a machine that incorporates an old technology) for z units of time. Agents are heterogenous with respect to z, i.e., z is distributed according to the density function $h(z)$ in the interval $[0, \bar{z}]$. The monopolist cannot discriminate among consumers with different z's. The profit flow obtained from using the old technology at time t is $q_0 e^{-\delta(z+t)}$, i.e., it depends on the *age* of the low quality durable good, $z + t$. Thus, $\delta > 0$ is the rate of depreciation.

A monopolist is able to produce a new, higher quality, durable good (which incorporates some technological development), that generates a profit flow of $q_n e^{-\delta s}$ after s units of time of adopting the new technology. The monopolist faces two types of costs. Firstly, the costs of developing the new technology (innovation costs), which vary with the gestation period (the length of time required to develop the new technology and create the prototype). Thus, if we let $\eta(x)$ be the accumulated costs, as of time x, of making the new technology available at time x, we assume that $\eta'(x) < 0$ and $\eta''(x) > 0$, $\lim_{x\to\infty} \eta(x) = 0$. In other words, costs decrease with the gestation period at a decreasing rate [4]. It will be convenient to work with its present value, which we denote by $\Psi(x)$, $\Psi(x) \equiv e^{-rx}\eta(x)$. Note for future reference that $\Psi(x)$ is also negative and $\dfrac{\Psi'(x)}{e^{-rx}}$ increases with x. Secondly, the monopolist also faces a unit production cost, c, $c > 0$. Production is instantaneous and capacity is sufficiently large. Thus, for all $t \geq x$ the monopolist can sell as many units of the high quality good as demanded.

For simplicity I rule out repeated purchases of the new durable good. Thus, consumers choose when to adopt the new technology but, indepen-

4. The same results hold if the limit of $\eta(x)$ as x goes to infinity is a positive number, but not too large.

dently of how much the high quality good deteriorates, they cannot purchase a new unit. We postpone the analysis of repeat purchases to the next section [5].

I restrict the possible values of c as follows (Assumption 1):

$$\frac{q_n}{\delta + r} > c \geq \frac{q_n}{\delta + r} - \frac{q_0}{r} e^{-\delta z}$$

The first inequality eliminates the trivial case that the first best involves no innovation. The second inequality is convenient but not essential, since it avoids considering corner solutions.

2.1. The first best

The optimal plan can be characterized in two steps. First, once the new technology is available, we must determine when consumers should adopt it. The social planner maximizes the present value of consumers' profit flow minus production costs. The net utility of a consumer of age z that adopts the new technology at time t, and faces an adoption cost c, is given by:

$$U_0\ (z,\ t,\ c) = \frac{q_0 e^{-\delta z}}{\delta + r}\left[1 - e^{-(\delta+r)t}\right] + e^{-rt}\left[\frac{q_n}{\delta + r} - c\right]$$

The first order condition characterizes the optimal timing (provided $t > 0$):

$$\frac{\partial U_0}{\partial t}\ (z,\ t,\ c) = e^{-rt}\left[q_0 e^{-\delta(z+t)} - r\left(\frac{q_n}{\delta + r} - c\right)\right] = 0 \qquad (2.1)$$

Thus, the optimal timing balances two effects. By delaying adoption the consumer still enjoys the profit flow derived from using the old technology, $q_0 e^{-\delta(z+t)}$, but misses the returns of the new asset, i.e., the interest on $\frac{q_n}{\delta + r} - c$. It is important to note that the optimal timing depends only on the age of the low quality durable good, $z + t$. Let us denote the

5. Alternatively, I could have considered a finite horizon framework. Unfortunately, in this case the presentation would be less transparent.

value of $(z + t)$ that satisfies equation (2.1) by w. Note that by Assumption 1, $w > \bar{z}$ and hence the solution is always interior. Also, w increases with q_0 and c, and decreases with q_n.

The second step is to determine when the new technology should be made available. Clearly, it is not optimal to innovate before any of the consumers are ready to switch to the new technology, i.e., in the optimal plan $x \geq w - \bar{z}$, otherwise nobody would adopt the new technology for a certain time interval and innovation costs would be unnecessarily high. This implies that when the high quality good is available there may be a mass of consumers of different ages that should be adopting the new technology immediately. More specifically, let $\hat{z}$ be the type of agents whose optimal adoption time coincides with the marketing of the high quality good, i.e., $\hat{z} = w - x$. Then, for all $z \in [\hat{z}, \bar{z}]$ adoption should take place immediately, at time x, while for $z \in [0, \hat{z}]$ the optimal timing is $t = w - z$.

The optimal timing of innovation is the solution to the following optimization problem: choose x in order to maximize:

$$W_0\ (x) = \int_0^{\hat{z}} U_0\ (z,\ w - z,\ c)\ dH\ (z) + \int_{\hat{z}}^{\bar{z}} U_0\ (z,\ w - z,\ c)\ dH\ (z) - \Psi\ (x)$$

The solution is given by the first order condition:

$$e^{rx} \int_{\hat{z}}^{\bar{z}} \frac{\partial U_0}{\partial t}\ (z,\ x,\ c)\ dH\ (z) = \frac{\Psi'\ (x)}{e^{-rx}} \tag{2.2}$$

Note that from equation (2.1) $\frac{\partial U_0}{\partial t}(\hat{z}, x, c) = 0$, and for all $z > \hat{z}$, $\frac{\partial U_0}{\partial t}(z, x, c) < 0$. Also, it is always optimal to make the new technology available only when there exist a sufficient mass of users, $1 - H\ (\hat{z}) > 0$, who are ready to adopt it.

2.2. The monopoly outcome: commitment to a constant price

It will be useful to start the investigation of the monopoly outcome by considering the simple case in which the monopolist must charge a constant price for the high quality good. Below, I consider more complicated pricing policies and also the game with complete price flexibility (no commitment power).

Let us first analyze consumer behavior for a given price, p. Consumer z's optimization problem consists of choosing the timing of the purchase, t, in order to maximize:

$$U_0\ (z,\ t,\ p) = \frac{q_0 e^{-\delta z}}{\delta - r} \left[1 - e^{-(\delta+r)t}\right] + e^{-rt} \left[\frac{q_n}{\delta - r} - p\right]$$

The first order condition is:

$$\frac{\partial U_0}{\partial t}\ (z,\ t,\ p) = e^{-rt} \left[q_0 \mathrm{e}^{-\delta(z+t)} - \mathrm{r} \left(\frac{q_n}{\delta + r} - p\right)\right] = 0 \qquad (2.3)$$

Note that, for a given price, the above equation determines the consumer's optimal value of $(z + t)$, which we denote by $v\ (p)$. In other words, all agents want to adopt the new technology after having used the old technology for v units of time. Moreover, the optimal v increases with the price. In fact, if $p = c$ the timing is efficient, $v\ (c) = w$, but as the price increases above marginal cost consumers are induced to delay the adoption of the new technology beyond the efficient timing. Thus, monopoly pricing affects the timing of adoption, and the monopolist faces a trade-off: a higher mark up associated with longer adoption delays.

The monopolist's optimization problem consists of choosing x and p in order to maximize:

$$\prod\nolimits_0\ (x,\ p) = \int_0^{\hat{z}} (p - c)\ e^{-r(v-z)}\ dH\ (x) + [1 - H\ (\hat{z})]\ (p - c)\ e^{-rz} - \Psi\ (x)$$

where $\hat{z} = v - x$.

The first order condition with respect to x can be written as:

$$e^{rx}\ [1 - H\ (v - x)]\ \frac{\partial U_0}{\partial \mathrm{t}}\ (v - x,\ x,\ c) = \frac{\Psi'\ (x)}{e^{-rx}} \qquad (2.4)$$

The next result follows from comparing equations (2.2) and (2.4) (See the Appendix for details):

Proposition 1. *If the monopolist sets a constant price for the high quality good then innovation is inefficiently delayed.*

The intuition is the following. Equation (2.3) characterizes consumers' willingness to pay at a given date (under the expectation of a constant price). Note that willingness to pay increases over time: $\frac{dp}{dt} > 0$ (Analogously, a higher price induces consumers to postpone adoption). As a result the seller is willing to postpone sales in order to get a higher mark-up, and hence innovation is also delayed.

Figure 1 plots the timing of adoption of consumers with different z, both in the first best (thick line) and in the constant price monopoly outcome (discontinuous line). Note that in spite of consumer heterogeneity, no consumer is excluded: those consumers with a lower willingness to pay at time x adopt later.

No depreciation

In order to understand the role of depreciation on the timing of adoption and its interactions with monopoly pricing, let us consider the extreme case of no depreciation, $\delta = 0$. First note that if $\delta = 0$ a positive surplus exist if, an only if (Assumption 1'):

$$c < \frac{q_n - q_0}{r}$$

It is immediate to adapt the equations that characterize the optimal plan to this case. First, note that (because of Assumption 1'):

$$\frac{\partial U_0}{\partial t}\,(z,\ t,\ c) = e^{-rt}\,(q_0 - q_n + rc) < 0$$

Thus, in the absence of depreciation it is optimal to adopt the new technology as soon as it becomes available, independently of z. Therefore, in this case, the optimal speed of innovation depends only on *supply side* factors, i.e., innovation costs. More specifically, equation (2.2) becomes:

$$q_0 - q_n + rc = \frac{\Psi'\ (x)}{e^{-rx}}$$

Let us now turn to the monopolist's optimization problem. At time 0 the monopolist must choose the timing of innovation, x, and the price of the high quality good, p. Consumers' willingness to pay at time x is given by:

$$R\ (z,\ x) = \frac{q_n - q_0}{r}$$

Note that willingness to pay is independent of both x and z. In other words, in the absence of depreciation consumers are homogeneous. In this case the monopolist sets $p = R\ (z, x)$ and chooses x in order to maximize:

$$\Pi_0 = e^{-rz}\left[\frac{q_n - q_0}{r} - c\right] - \Psi\ (x)$$

Since the monopolist is able to capture the entire surplus associated to innovation the next result follows.

Remark 1. *If $\delta = 0$ then the timing of innovation of a monopolist is efficient.*

In the absence of depreciation, monopoly pricing does not affect the timing of adoption since consumer willingness to pay is constant over time (also in this case consumers are homogeneous). As a result, only supply side factors (innovation costs) determine the timing of adoption, and hence the monopolist has the proper incentives to innovate.

Lack of commitment power

Let us consider the opposite scenario. Suppose that prices can be changed after an arbitrarily small time interval. In this case it can be shown (see Appendix) that there may exist an equilibrium with a constant price close to marginal cost. This result is reminiscent of the Coase conjecture, but actually the mechanism is different. In our case, all consumers are alike but they started using the old technology at different dates (they are asynchronized). If consumers expect a constant price then they behave according to equation (2,3). However, their behavior out of the equilibrium path is different. If consumers observe a slightly lower price then they expect that such a price will last for a very short period of time and therefore will have incentives to purchase the good immediately. In other words, consumers are very sensitive to price cuts, and as a result the only *credible* price is one sufficiently close to marginal cost, such that the monopolist does not have incentives to undercut consumers' expected price.

In this scenario, if the monopolist has access to a commitment technology that allows him to set, for instance, a constant price, then he will use it.

2.3. Commitment to a constant price and a purchase deadline

Suppose now that the monopolist can commit to a constant price plus a purchase deadline. In other words, the monopolist must set a price, p, and a deadline, y, so that no purchase is allowed for all $t > y$. In this case, the specific characterization of the equilibrium varies depending on the distribution of consumers, $h(z)$ and other parameters of the model. Because of this the formal analysis is postponed to the Appendix, and here I informally discuss the main results. Firstly, the purchase deadline is binding for a positive mass of consumers, in the sense that in equilibrium those consumers buy at the deadline but they would prefer to delay their purchases at the posted price. Secondly, the equilibrium price is higher than in the absence of deadlines. The reason is, that in the absence of deadlines, the equilibrium price is relatively moderate in order to avoid delaying sales excessively. The purchase deadline brings many of these purchases forward and hence weakens the motives behind moderate prices. Both the higher price and earlier sales increases the monopolist's profits, which improves incentives to innovate. The timing of innovation is still inefficient but less than in the absence of deadlines. Figure 2 plots the equilibrium timing of adoption with (hard line) and without (discontinuous line) a purchase deadline. The hard line is plotted for a particular region of the parameter space (no consumers are excluded). Note that, in the equilibrium with deadlines, some consumers purchase the good later than in the absence of deadlines, because of the higher price associated with the deadline. As a result the effect of purchase deadlines on total welfare is ambiguous [6].

Proposition 2. *If the monopolist can commit to a purchase deadline (on top of a constant price) this results in a higher price, higher profits and better incentives to innovate.*

Restricting the timing of purchases may be particularly useful if consumers are not very dispersed. Let us next examine an extreme example.

6. Note that those consumers with $z < \hat{z}$ are worse off when the monopolist can commit to a purchase deadline: they face a higher price and the timing of purchases may be restricted. However, the effect on the welfare of consumers with a high z is ambiguous: since they pay a higher price but may enjoy the new good earlier.

Synchronized consumers

Suppose that the distribution of consumers is degenerate (all consumers have the same z). For simplicity suppose that $z = 0$. Consumers are homogeneous and their willingness to pay at the deadline, y, is given by:

$$R\ (y) = \frac{q_n - q_0 e^{-\delta y}}{\delta + r}$$

Thus, the monopolist can leave consumers with zero rents by setting $x = y$ and $p = R\ (x)$. As a result, the innovation decision is efficient and the monopolist captures the entire surplus.

Remark 2. *If all consumers are perfectly synchronized (same z) and the monopolist can commit to a purchase deadline, then the timing of innovation is efficient.*

Thus, depreciation per se is not sufficient to produce inefficient delays. If the monopolist is able to discriminate among different consumer types and restrict the timing of purchases then the fact that consumer willingness to pay increases over time does not prevent the monopolist from extracting the entire surplus.

It is not obvious how the monopolist can acquire the ability to commit to purchase deadlines. In the case of perfectly synchronized consumers the same outcome can be obtained in a different (extreme) pricing game. Suppose the monopolist can change his price instantaneously. Then (see the Appendix for a discrete time version of this proposition) the unique subgame perfect equilibrium consists of a price schedule $p = R\ (t)$ \$ and consumers purchasing at the efficient timing. Thus, once again the monopolist captures the entire surplus from innovation.

Neither of these two approaches (a purchase deadline and flexible pricing) works sufficiently well if consumers are not perfectly synchronized [7]. The reason is that consumers are heterogeneous at any point in time. Hence, it cannot set a price function that tracks consumer willingness to pay. Also, if the monopolist commits not to trade for all $t > x$, then he faces a downward sloping demand function and hence monopoly profits are below total welfare.

7. Also, note that the monopolist can appropriate the entire surplus by setting $p = R\ (t)$ for all t, $t \geq x$, only if there is complete information and no costs of changing prices.

2.4. Contingent pricing policies

2.4.1. Commitment to a time varying price

The previous analysis considered simple pricing policies (a constant price up to a deadline, and no trade from there onwards) that presume a great deal of commitment power. We could go all the way and allow the monopolist to commit to any arbitrary price schedule $p\,(t)$. This is formally analogous to the problem of a multiproduct monopolist facing heterogeneous consumers. The analysis of this case would not provide any new insights. The monopolist would set an increasing price function in order bring forward the purchases of some consumers at the cost of delaying (perhaps excluding) the purchases of others.

Thus, the general lesson is that if the monopolist can somewhat restrict the timing of purchases through deadlines or more flexible intertemporal pricing schemes, the timing of innovations will be more efficient, although some consumers are likely to be worse off than in the equilibrium of Section 2.2 .

2.4.2. Trade-in allowance

In some durable good industries (e.g., automobile), sellers offer a discount to those buyers that trade in their old units. Typically, the size of the discount depends on the age of the old unit [8]. In our model we assume a deterministic depreciation rate and hence age and physical deterioration are perfectly correlated. As a result, if we allow the seller to set prices as a function of the age of the old unit then it can appropriate the entire surplus and induce the efficient timing of purchases. In particular, the optimal pricing policy consists of setting:

$$p\,(z+t) = \frac{q_n - q_0 e^{-\delta(z+t)}}{\delta + r}$$

8. For instance, Goldberg (1996) reports that 50 percent of the sample of the Consumer Expenditure Survey (which is representative of the US population) trade in an old car when buying a new one. She suspects that the trade-in allowance is related to the price in the wholesale second hand market.

Given such pricing policy consumers are indifferent about the timing of their purchase and hence they are willing to buy the good at the efficient timing. Consequently, the monopolist has the right incentives to innovate.

If we relax the assumption of perfect correlation between depreciation and age then things are a bit more complicated. Suppose, for instance, that right after purchasing the durable good the rate of depreciation is zero for an interval of time of length $\tilde{z}$, and the rate of depreciation becomes δ at the end of this interval. Suppose also that $\tilde{z}$ is a random variable. If only the age of the old unit is observable then the optimal pricing is more complicated. In general, the seller may still use discounts as a function of the age of the old unit, although if the dispersion of $\tilde{z}$ is sufficiently large then he may prefer very flat pricing schemes.

In contrast, if the industry is very competitive and equilibrium prices are close to marginal costs, then there is no room for trade-ins and price discrimination (although in this case incentives to innovate are very weak).

2.5. Discussion

What have we learned from this simple model? In the absence of depreciation the timing of adoption is not an issue, since consumers wish to adopt the new model as soon as it becomes available. As a result, the incentives to innovate depend exclusively on supply side factors. However, under depreciation, the timing of adoption crucially interacts with the seller pricing policies. The monopolist will extract surplus from consumers at the cost of delaying their purchases, which in turn reduces the incentives to innovate. Finally, whenever possible, the monopolist will try to restrict the timing of purchases, either by setting purchase deadlines or by structuring its intertemporal pricing policies. Those devices will tend to increase monopoly profits and improve incentives to innovate, although most consumers will be hurt.

Unfortunately, the model was very restrictive (a single innovation and a single purchase of the high quality good). In the case of multiple innovations new issues arise. For instance, we know from previous work (FR) that, even in the absence of depreciation, the timing of innovations is inefficient, and that the optimal strategy of the monopolist is time inconsistent. Also, the timing of adoptions and the timing of innovations may interact in more complicated ways. In particular, a credible purchase deadline may endogenously arise every time the monopolist markets a new model. Finally, the monopolist may develop loyalty-inducing pricing policies and choose between leasing and selling. In the next section we extend the previous analysis to the case of sequential and cumulative innovations. Unfortunately, for tractability reasons I must restrict myself to the case of synchronized consumers.

3. A repeat purchase framework

TIME is a continuous variable and horizon is infinite. A monopolist is able to develop successive generations of a durable good. If the previous model had an initial quality q then the monopolist can develop a model of quality $q + \Delta$, $\Delta > 0$, by paying an innovation cost equal to $\eta\ (t)$ where t is the gestation period (the time interval between two innovations), and measured in time t units. Similarly to the previous section, we assume that $\eta'(t) < 0$, $\eta''\ (t) > 0$, $\lim_{t\to 0} \eta\ (t) = -\infty$, $\lim_{t\to\infty} \eta\ (t) = 0$. Production is instantaneous and unit costs, c, are constant.

There is a continuum of buyers (mass one), who are perfectly synchronized, each one obtains a profit flow of $qe^{-\delta s}$ from a machine of quality q adopted s units of time ago. Thus, δ again denotes the rate of depreciation.

For simplicity we assume that at time 0 buyers have just bought a good of quality q_0. The seller must choose the gestation period of the new model, t_1. At time t_1, the seller sets a pricing scheme (see below) and the gestation period of the next model, t_2, and buyers decide whether and when to adopt the new good. The same pattern is repeated indefinitely.

Thus, the model encompasses both technological obsolescence and physical obsolescence in a repeated purchase framework. Unfortunately, stationary equilibria do not exist in general, but below I consider two extreme cases where stationary equilibria do exist.

3.1. Pure technological obsolescence

Suppose that $\delta = 0$ and $\Delta > 0$. This is a simplified version of the model in FR [9]. Let us briefly review their main insights. For simplicity, in this subsection we set $c = 0$. Note that it is always optimal to innovate since innovation

9. In their model the monopolist can choose both the timing of the next innovation as well as the incremental value, Δ.

costs go to zero as the gestation period goes to infinite and there are no production costs.

The optimization problem of the social planner can be set up as follows: choose t in order to maximize:

$$W(q_0) = \frac{q_0}{r}(1 - e^{-rt}) + e^{-rt}\left[W^*(q_0 + \Delta) - \eta(t)\right]$$

where $W^*(q_0 + \Delta)$ is the optimal value of total welfare at the moment of the next innovation. Notice that the formulation implicitly assumes that there is perfect synchronization between innovation and adoption, which must be the case in the optimal plan since there is no reason to delay adoption.

The solution is stationary (see FR). The optimal length of the gestation period is characterized by the first order condition:

$$-\frac{\Delta}{r} - \frac{1 - e^{-rt}}{r}\eta'(t) + \eta(t) = 0 \tag{3.1}$$

The first term represents the social value of a single innovation. Since innovations accumulate the social value of a single innovation is the present value (infinite horizon) of the incremental profit flow, Δ. The second and third terms reflect the costs of innovation.

Let us turn to the monopoly pricing problem. If the last innovation was made available s units of time ago, and consumers expect that the length of the period between two consecutive innovations is t^e, then consumers' willingness to pay is given by:

$$\mathrm{R}(\mathrm{s}) = \frac{\Delta}{r}\left(1 - e^{-r(t^e - s)}\right)$$

The reason is that the consumer is willing to pay only for the additional value provided by the current model which will be enjoyed only for a limited time. Therefore, consumers do not have any reason to delay the adoption of the new technology, since $R'(s) < 0$. The current model is analogous to the model of Subsection 2.2. for $\delta = 0$, although there are some differences. First, in Subsection 2.2. $t^e = \infty$, which implied that $R(s)$ is constant. Second, $R(s)$ was equal to the social value of innovation.

The seller's optimization problem at time 0 is to choose t in order to maximize:

$$\Pi_0 (q_0) = e^{-rt} \left[\frac{\Delta}{r} (1 - e^{-rt^e}) - \eta (t) + \Pi^* (q_0 + \Delta) \right]$$

where $\Pi^* (q_0 + \Delta)$ is the seller's continuation value after the next innovation.

Note that the solution is stationary and the gestation period is characterized by the first order condition:

$$-\frac{\Delta}{r} (1 - e^{-rt}) - \frac{1 - e^{-rt}}{r} \eta' (t) + \eta (t) = 0 \tag{3.2}$$

If we compare equations (3.1) and (3.2) we realize that the main difference comes from the fact that consumers are not willing to pay the entire value of innovation, $\frac{\Delta}{r}$ since they will be able to acquire such innovation for free when they buy the next model of the durable good. In other words, if the consumer skips the current model then at the next purchase she can get the current plus the next innovation at the same price. Hence, consumer willingness to pay is below the total surplus which reduces the seller's incentives to innovate. Hence, the following proposition holds:

Proposition 3. *(Fishman and Rob, 2000) In the absence of depreciation, the speed of innovation of a monopolist is inefficiently low.*

In the simple model of Section 2, if $\delta = 0$ then the monopoly solution was efficient. In contrast, with multiple innovations, the monopolist cannot appropriate the entire surplus because innovations are cumulative.

Note that the seller's optimization problem is time inconsistent. If the seller can commit to a constant time interval between any two future innovations then the speed of innovation is further reduced (t is higher). The reason is that without commitment the seller does not take into account that moving the next innovation forward reduces the consumer's return on the previous innovation. If this is anticipated by consumers, their willingness to pay is reduced. Under commitment, the seller internalizes such effect.

3.2. Pure physical obsolescence

Let us now set $\Delta = 0$ and $\delta > 0$. This is a stationary environment where new purchases take place whenever the previous model has sufficiently deteriorated. For simplicity, and compatible with the nature of this extreme case, we set $\eta\ (t) = 0$, but $c > 0$. The following assumption creates a positive demand for the durable good (Assumption 2):

$$c < \frac{q}{\delta + r}$$

This model displays close analogies to the model of non-durable *cyclical* goods presented in Caminal (2004), in the sense that consumers willingness to pay increases with the length of the time period since the last purchase and follow the same pattern exactly after every purchase. However, there are also important differences. In particular, in the current model (durable goods) it makes a difference whether the good is sold or rented, since if it is sold consumer willingness to pay for a new unit is restricted by the value of the old unit. Also, the price of a durable good can be a function of the age of the previous unit.

3.2.1. The first best

The social planner's optimization problem can be written as choosing t in order to maximize:

$$W_0 = \frac{q}{\delta + r}\left[1 - e^{-(\delta+r)t}\right] + e^{-rt}\,(W^* - c)$$

where W^* is the continuation value after the next purchase. Note that the optimization problem is exactly the same after each adoption. Hence the solution is stationary.

The first order condition characterizes the efficient timing of purchases, t^*:

$$\frac{q}{r}\,e^{-\delta t} - \frac{q}{\delta + r}\,\frac{1 - e^{-(\delta+r)t}}{1 - e^{-rt}} + \frac{c}{1 - e^{-rt}} = 0 \tag{3.3}$$

The optimal timing balances costs and benefits. The benefit from postponing a purchase is the profit flow from the previous unit, $qe^{-\delta t}$, and the costs are the interest on the continuation value, $\frac{1}{1 - e^{-rt}}\left\{\frac{q}{\delta + r}\left[1 - e^{-(\delta+r)t}\right] - c\right\}$.

3.2.2. Equilibrium prices under restricted timing

The analysis of this subsection is analogous to the case of synchronized consumers and purchase deadlines studied in Section 2. Suppose that the seller can restrict trading to discrete points in time, t_1, t_2,.. One possible interpretation is that at each point in time the seller supplies a new variety of a durable good and sets a very short purchase deadline. The game evolves as follows. At time 0, right after the buyer has acquired a new unit, the seller sets the timing of the next purchase, t_1, and the price, p_1. At time t_1 buyers decide whether to purchase the good or not. Immediately after, the seller sets the next trading time and price (t_2, p_2), and at time t_2 buyers decide whether to purchase the good or not. And the same pattern is repeated indefinitely.

I focus on (pure) strategies that depend only on state variables that are directly payoff relevant. In particular, consumers' purchasing decisions depend only on the posted price and the length of the time period since the previous purchase. If the seller offers the good at a price p after t units of time since the last purchase, and the next purchase is expected to take place t^e units of time later, then the net gain from purchasing the good right away is:

$$\frac{q}{\delta + r}\left(1 - e^{-\delta t}\right)\left(1 - e^{-(\delta+r)t^e}\right) - p$$

In other words, consumers will buy the good if, and only if $p \leq \overline{p}(t, t^e)$, where the latter is the consumer willingness to pay and is given by:

$$\overline{p}\,(t, t^e) = \frac{q}{\delta + r}\,(1 - e^{-\delta t})\,(1 - e^{-(\delta+r)t^e}) \tag{3.4}$$

Note that $\overline{p}$ increases with t, i.e., as the current model deteriorates consumer willingness to pay increases. The monopolist's strategy is simply a pair (p, t), which is only conditional on equation (3.4). Hence, the monopolist's optimization problem consists of choosing t in order to maximize:

$$\prod\nolimits_0 = e^{-rt}\,[\overline{p}\,(t, t^e) - \mathrm{c} + \prod\nolimits^*]$$

where $\prod^*$ is the monopolist continuation value. The first order condition evaluated at $t = t^e$, characterizes the equilibrium length of the interpurchase time periods, $\overline{t}$:

$$\frac{q}{r}\, e^{-\delta t} \frac{\delta}{\delta + r} \left[1 - e^{-(\delta+r)t}\right] - \frac{q}{\delta + r} \frac{1 - e^{-(\delta+r)t}}{1 - e^{-rt}} \left(1 - e^{-\delta t}\right) + \frac{c}{1 - e^{-rt}} = 0 \qquad (3.5)$$

The next result comes from comparing equations (3.3) and (3.5) (See Appendix):

Proposition 4. *The length of the interpurchase time periods are inefficiently long; i.e.,* $\overline{t} > t^*$.

This result can be compared to Remark 2 and Proposition 3. With a single innovation (and synchronized consumers), if the monopolist was able to restrict the timing of purchases then the first best allocation could be implemented (Remark 2). In contrast, Proposition 4 indicates that with cumulative innovations this is no longer the case. The intuition is analogous to the one given in relation to Proposition 3 (FR's result). If a consumer skips the opportunity to purchase a new unit at time t_1, she will have another opportunity at time t_2. At that point she will pay the same price for the new model as those other consumers that did purchase the good at time t_1 and hence have a lower willingness to pay for the good at time t_2. Hence, depreciation (plus restricted timing) and cumulative innovations play a similar role in restricting the monopolist's ability to appropriate the entire surplus. Therefore, it was not cumulative innovations that were behind Proposition 3, but a more general phenomenon. What matters in both cases is the fact that consumers make multiple purchases, and their willingness to pay is higher for those who skipped the previous trading point. If the monopolist cannot discriminate among consumers with different histories of purchases then consumers can always guarantee a positive surplus for themselves.

Once again, it can be argued that restricting the timing of purchases is problematic. Firstly, consumers may be non-synchronized. In this case, the role of purchase deadlines is much weaker. Secondly, restricting the timing of purchases involves a certain commitment power. In the next section I analyze the opposite extreme case, which is analogous to the model analyzed in Subsection 2.2. The seller sets a uniform price and stands ready to sell the good at that price with not restriction on timing.

3.2.3. Equilibrium prices with unrestricted timing

In the model of Section 2.2 the monopolist was assumed to set a constant price for the high quality good and stood ready to sell it at any time. This was a somewhat reasonable assumption given that the monopolist faced consumer heterogeneity. In order to make the current model tractable we got rid of consumer heterogeneity. However, we still wish to consider situations where the monopolist cannot directly or indirectly control the timing of purchases. With this idea in mind, let us consider the following interpretation of the above framework. Suppose that right after consumers purchase a new unit, the seller sets the price for the next purchase. Consumers choose the timing of the next purchase and immediately after the seller sets a new price. The same pattern is repeated indefinitely [10]. Once again, I focus on strategies that depend exclusively on payoff-relevant state variables. Thus, the seller always sets the same price and buyers choose the timing of the next purchase as a function of the current price. The optimization problem of a representative buyer consists of choosing t in order to maximize:

$$U_0 = \frac{q}{\delta + r}\left[1 - e^{-(\delta+r)t}\right] + e^{-rt}\left(-p + U^*\right)$$

where U^* is the consumers' continuation value, which is independent of t. Hence, the first order condition provides the consumer's willingness to pay for every t:

$$\underline{p}\,(t) = U^* - \frac{q}{r}\,e^{-\delta t} \tag{3.6}$$

Note that the consumer is willing to pay a higher price as t increases and the durable good deteriorates. The seller's optimization problem can be written as choosing t in order to maximize:

$$\Pi_0 = e^{-rt}\left[\underline{p}\,(t) - c + \Pi^*\right]$$

where again, the seller's continuation value, Π^* is independent of t. The first order condition characterizes the length of the interpurchase time period when the monopolist is unable to restrict the timing of purchases, $\underline{t}$.

10. Alternatively, I could have assumed that the seller must keep the price fixed for a time interval of exogenous length. Unfortunately, stationary equilibria of such game do not generically exist, although I conjecture that the qualitative properties of the equilibrium would be the same.

$$\frac{q}{r}\frac{\delta+r}{r}e^{-\delta t}-\frac{q}{\delta+r}\frac{1-e^{-(\delta+r)t}}{1-e^{-rt}}+\frac{c}{1-e^{-rt}}=0 \qquad (3.7)$$

Comparing equations (3.5) and (3.7) (See Appendix) we obtain:

Proposition 5. *The length of the interpurchase time periods are inefficiently long. In fact, such inefficiency is larger than under restricted timing, i.e.,* $\underline{t} > \overline{t}$.

The intuition goes as follows. If the seller is able to restrict the timing of purchases then she faces a more favorable trade-off between mark-ups and the frequency of purchases. As a result, the seller can afford to induce consumers to buy more frequently (which generates a higher total surplus).

Thus, if the seller cannot restrict the timing of purchases, perhaps because she faces a large degree of consumer heterogeneity, then purchases are inefficiently delayed, which adds to the inefficiency associated to the repeat purchase of durable goods, described in Proposition 4.

3.3. Discussion

3.3.1. The relative role of physical and technological obsolescence

The previous section has considered two extreme versions of the general repeat purchase model, where either δ or Δ has been set to zero. In the former case, consumers' adoption decision was trivial, and the seller did not need to restrict the timing of purchases. However, the speed of innovation was inefficiently low because the seller could not keep track of the history of individual purchases. Similarly, in the latter case if the seller is not able to discriminate across consumers depending on their history of purchases then he can only appropriate a fraction of the total surplus and the frequency of purchases is inefficiently low. On the top of that, there is an additional source of inefficiency. If the seller cannot restrict the timing of purchases then he faces a less favorable trade-off between prices and frequency of purchases and, as a result, the frequency of purchases is further reduced.

We can only speculate about the patterns of equilibria in intermediate cases. However, it looks plausible that if the ratio of Δ to δ is sufficiently high (technological obsolescence dominates) then consumers' adoption decisions are trivial and the seller does not need to restrict the timing of purchases. However, if the ratio of Δ to δ is sufficiently low (physical obsolescence dominates) then monopoly pricing interacts with consumers' adoption decisions and the seller has incentives to restrict the timing of purchases. If he fails to do so then incentives to innovate are further reduced and the average quality of durable goods is far below efficient levels.

3.3.2. Size of inefficiency

What is the relative weight of each of the two sources of inefficiency? (The seller's inability of price discriminating among consumers with different history of purchases, on the one hand, and the seller's inability to restrict the timing of purchases, on the other). Let us consider the model of Section 3.2 with the following parameter restriction: $\delta = r$. In this case, if we let $\alpha \equiv e^{-rt}$ and $\Omega \equiv 2rcq^{-1}$, then equations (3.3), (3.5), and (3.7) become respectively:

$$(1 - \alpha^{*})^{2} = \Omega$$

$$(1 - \bar{\alpha})^{2}\,(1 - \bar{\alpha}^{2}) = \Omega$$

$$(1 - \underline{\alpha})\,(1 - 3\underline{\alpha}) = \Omega$$

These functions are plotted in Figure 3. Note that for most parameter values the inefficiency associated with a monopolist that is able to restrict the timing of purchases is relatively small. However, if the seller sets a constant price then the frequency of purchases is substantially below the efficient level.

3.3.3. Time consistency

Let us first examine the time consistency of the monopolist optimization problem with restricted timing. From equation (3.4) it is clear that future decisions on trading time affects the current willingness to pay. In contrast to Subsection 3.2.2, suppose now that the monopolist can commit to a constant interval between purchasing periods. Hence, the objective function can be written as:

$$\Pi_0 = \frac{e^{-rt}}{1 - e^{-rt}}\,(\bar{p} - c)$$

where now $\bar{p}$ is given by:

$$\bar{p} = \frac{q}{\delta + r}\left(1 - e^{-\delta t}\right)\left(1 - e^{-(\delta+r)t}\right)$$

Therefore, consumers' reservation price is now more sensitive to the distance between two consecutive trading points. As a result, the monopolist has incentives to induce consumers to purchase less frequently. If we let $\bar{t}^c$ be the length of the period between two consecutive purchases under commitment, then (See Appendix):

Proposition 6. *If the monopolist can restrict the timing of purchases, the frequency of purchases is lower under commitment: $\bar{t}^c > \bar{t} > t^*$. As a result, total welfare decreases with commitment power.*

The same result holds in FR's model. The reason in both cases is that when the monopolist chooses the next sales point he does not take into account its effect on the value of the current unit for consumers. As a result, in the absence of commitment, the monopolist chooses trading points which are too close to each other in terms of profit maximization. A similar effect can also be found in models of planned obsolescence (See, for instance, Waldman, 1996). In the absence of commitment, the monopolist's decisions about the quality of a new product do not take into account that a higher quality reduces the value of existing (lower quality) units. These incentives are anticipated, which reduces consumer willingness to pay and total profits.

Let us now turn to the case of unrestricted timing. Equation (3.6) indicates that the effect of the price on the timing of the next purchase is relatively small if consumers expect that the current price does not affect the continuation value, U^*. Alternatively, suppose that the monopolist can commit to a constant price. Then consumers' optimization problem consists of choosing t in order to maximize:

$$U_0 = \frac{1}{1 - e^{-rt}}\left\{\frac{q}{\delta + r}\left(1 - e^{-(\delta+r)t^e}\right) - p\right\}$$

The solution is characterized by the first order condition:

$$\frac{q}{r}\, e^{-\delta t}\left(1 - e^{-rt}\right) - \frac{q}{\delta + r}\left[1 - e^{-(\delta+r)t}\right] + p = 0$$

Note that the timing of purchases, t, is more sensitive to the price, p, than in the case of no commitment (consumer willingness to pay increases with t but at a lower rate) [11]. Consequently, if we let $\bar{t}^c$ be the solution of the monopolist optimization problem under commitment, then we obtain the following result (See Appendix):

Proposition 7. *If the monopolist cannot restrict the timing of purchases, the frequency of purchases is higher under commitment but still inefficiently low,* $t^* < \underline{t}^c < \underline{t}$. *As a result, total welfare increases with commitment power.*

Note that the effect of commitment capacity on total welfare has different signs depending on whether the timing of purchases can or cannot be restricted. However, commitment does not destroy the positive effect of restricting the timing of purchases on total welfare. In the case $\delta = r$, it is immediate to check that $\underline{t}^c > \bar{t}^c$.

3.3.4. Alternative marketing strategies

a) *Trade-ins*

Consider the model of Section 3.2 and suppose that the seller can offer discounts if the old unit is traded in, and such a discount varies with the age of the old. Once again, the monopolist can appropriate the entire surplus and implement the first best. In particular, the monopolist can set the following pricing policy:

$$p(t) = \frac{q}{\delta + r}\left(e^{rt} - e^{-\delta t}\right)$$

where t now refers to the age of the old unit traded in. Under such pricing policy, consumers are indifferent about the timing of their purchase (they make a zero surplus everywhere) and hence are willing to buy at the efficient timing. Once again, if physical deterioration and age are imperfectly correlated then trade-ins are less useful devices.

11. This is analogous to the result obtained in the case of *cyclical goods* (Caminal, 2002).

b) *Leasing*

It has long been recognized that monopolists are likely to prefer leasing over selling in order to avoid Coasian dynamics. In the context of cumulative innovations FR show that the monopolist can appropriate the entire surplus if he leases his products instead of selling them. Leasing is also a profitable strategy for the monopolist in the context of the model of Section 3.2. However, the pricing scheme needed to implement the first best requires at least three parameters: a rental rate, a fee associated to the replacement of an old unit, and an exit fee. That is, if the consumer wishes to cancel the leasing contract then he must pay a certain fee to the seller. If an exit fee cannot be used (for instance, because of a moral hazard problem) then a contract with only two parameters (a rental price and a replacement fee) will induce excessively frequent replacements. The reason is that in the absence of exit fees consumers will turn the unit in whenever $qe^{-\delta t}$ is lower or equal to the rental price. Thus, the monopolist can charge a higher price only if it allows for more frequent replacements. An increase in the frequency starting at the efficient level causes a second order loss in total surplus, but it allows the monopolist to charge a higher price, which implies a first order gain in profits.

c) *Loyalty-inducing mechanisms*

By analogy to the case of cyclical goods (Caminal, 2004), most of the devices that lock consumers in will also be useful to increase profits and efficiency in the context of the model of Section 3.2. An obvious example are subscriptions, i.e., contracts that specify a price and the frequency of purchases. However, it is quite unlikely that this type of contract can actually be used in markets where the rate of innovation is uncertain.

Similarly, a commitment to a lower price in successive purchases can be shown to raise monopoly profits and efficiency. Once again, it is difficult to commit to the price of a good with uncertain characteristics and that will be available at some uncertain future time.

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5. Appendix

5.1. Proof of Proposition 1

The first order conditions of the monopolist's optimization problem are:

$$(p - c)\ [1 - H(\hat{z})] = \frac{-\Psi'(x)}{re^{-rx}} \tag{5.1}$$

$$[1 - H(\hat{z})]\ e^{-rx} + \int_0^{\hat{z}} e^{-r(v-z)} dH(z) - \int_0^{\hat{z}} (p - c)\ re^{-r(v-z)} \frac{dt}{dp}(v)\ dH(z) = 0 \tag{5.2}$$

where (from equation (2.3) in the text):

$$\frac{dt}{dp}(v) = \frac{r}{\delta q_0 e^{-\delta v}} \tag{5.3}$$

From equations (5.2) and (5.3) we derive that $p > c$, and hence $v > w$, and that $\hat{z} > 0$. Plugging equation (2.3) in the text into equation (5.1) we obtain equation (2.4) in the text. This implies that $\hat{z} < \bar{z}$. Finally, remember that $\frac{\partial U_0}{\partial t}$ (w – x, x, c) = 0, and $\frac{\partial^2 U_0}{\partial t^2} < 0$. Therefore,

$$\int_{w-x}^{\bar{z}} \frac{\partial U_0}{\partial t}(z, x, c)\ dH(z) < \int_{v-x}^{\bar{z}} \frac{\partial U_0}{\partial t}(z, x, c)\ dH(z) < [1 - H(v - x)]\ \frac{\partial U_0}{\partial t}(v - x, x, c)$$

As a result the left hand side of equation (2.4) is higher than the left hand side of equation (2.2), and since the right hand side increases with x, this implies that the timing of innovation under monopoly is inefficiently delayed. QED

5.2. Price flexibility and asynchronized consumers

Consider the discrete time version of the model of Section 2.2. Trade can only take place at discrete and equally distant points: t_1, t_2,... That is, $t_n = t_{n-1} + \tau$. For symmetry, assume that z can only take discrete values, with the same difference between two consecutive values. That is, $\bar{z}$, $\bar{z} - \tau$, $\bar{z} - 2\tau$, ..., 0. Suppose that consumers expect a constant price p^*. Then the optimal timing of purchases will depend only on the age of the old units. Let us denote by v the optimal age, i.e., consumers purchase the new unit at time $v - z$. Similarly, we can define the reservation prices only as a function of the age of the old unit, $z + t$:

If $t < v - z$

$$U_0\ (z,\ t,\ \bar{p}\ (z + t)) = U_0\ (z,\ v - z,\ p^*)$$

If $t > v - z$

$$U_0\ (z,\ t,\ \bar{p}\ (z + t)) = U_0\ (z,\ t + \tau,\ p^*)$$

and $\bar{p}\ (v) = p^*$.

Suppose that the seller can price discriminate among consumers with different z's. Then the seller finds it optimal to charge the same price to all types and trade with them when the old units are the same age.

Next, I characterize the value of p^*. The seller can bring all purchases one period forward by setting $\bar{p}\ (v - \tau)$ and leaving consumers indifferent. Such a deviation will be profitable if, and only if the total surplus of trading when old units are age $v - \tau$ is higher than at v. Thus in order to be an equilibrium v must be the efficient age. Hence, as τ goes to zero the limit of (p^* – c) is also zero.

Thus if an equilibrium with a constant price exist and τ is small then the price must be close to marginal cost. However, such an equilibrium does not always exist. Define $\underline{v}$ as follows:

$$U_0\ (z,\ \underline{v} - z,\ c) = U_0\ (z,\ \underline{v} - z + \tau,\ c)$$

Then the equilibrium candidate must satisfy $v \in [\underline{v},\ \underline{v} + \tau]$ otherwise the seller has incentives to deviate and bring purchases forward undercutting p^*. Depending on how close v is to each of these bounds equilibrium does or does not exist.

Take the following limit case $v = \underline{v} + \tau$, i.e.,:

$$U_0\ (z,\ v - z,\ c) = U_0\ (z,\ v - z - \tau,\ c) \tag{5.4}$$

Thus, by construction there are no incentives to bring purchases forward undercutting p^*. Let us examine incentives to delay purchases.

By definition of equilibrium:

$$U_0\ (z,\ v - z,\ p^*) = U_0\ (z,\ v - z + \tau,\ p^*) \tag{5.5}$$

Let us check the incentives to postpone purchases until $v - z + \tau$ (the same procedure can be used to check that no further delays are profitable).

Define:

$$U_0\ (z,\ v - z + \tau,\ \overline{p}) = U_0\ (z,\ v - z + 2\tau,\ p^*) \tag{5.6}$$

Equations (5.4), (5.5) and (5.6) fully characterize the three endogenous variables: v, p^* and $\overline{p}$. Manipulating these three equations it can be checked that the seller's incentives to deviate are negative: $e^{-r\tau}\,\overline{p} - p^* < 0$. Thus in this case the equilibrium with a constant price does exist.

Consider the other extreme example, $v = \underline{v}$, i.e.,

$$U_0\ (z,\ v - z,\ c) = U_0\ (z,\ v - z + \tau,\ c) \tag{5.7}$$

Again, there are no incentives to bring purchases forward. However, in this case the seller has incentives to delay purchases. From equations (5.5), (5.6) and (5.7) it can be checked that incentives to deviate are now positive. Hence a constant price equilibrium cannot exist.

5.3. Purchase deadlines (Proposition 2)

Consider the model of Subsection 2.3 and suppose that the monopolist sets the innovation date, x, the price of the high quality good, p, and the purchase deadline, y. That is, purchases can only occur in the time interval $[x, y]$. In other words, the price schedule $p\ (t)$ is equal to p if $t \in [x, y]$, and goes to infinite otherwise. We can define the following points: $0 \le z_0 < z_1 \le z_2 < \bar{z}$, where z_0 denotes the consumer with the lowest value of z that is willing to purchase the high quality good at time y:

$$\frac{q_n - q_0 e^{-\delta(z_0+y)}}{\delta + r} = p \tag{5.8}$$

The variable z_1 denotes the highest value of z that purchases the good at time y:

$$\frac{\partial U_0}{\partial t} (z_1, y, p) = 0 \tag{5.9}$$

From equations (5.3) and (5.8) we obtain that:

$$z_1 - z_0 = -\frac{1}{\delta} \ln \frac{r}{\delta + r} \equiv m \tag{5.10}$$

Finally, z_2 denotes the lowest value of z that purchases the good at time x:

$$\frac{\partial U_0}{\partial t} (z_2, x, p) = 0 \tag{5.11}$$

The various possibilities that arise in equilibrium are easily illustrated in the case of a uniform distribution of consumers, i.e., let us assume in this subsection that $h(z) = \frac{1}{\bar{z}}$. The monopolist chooses (x, p, y) in order to maximize:

$$\Pi_0 = \Lambda (p - c) - \Psi (x)$$

where

$$\Lambda \equiv \left\{ [1 - H(z_2)]\, e^{-rx} + \int_{z_1}^{z_2} e^{-r(v-z)}\, dH(z) + [H(z_1) - H(z_0)]\, e^{-ry} \right\}$$

The first order conditions are:

$$[1 - H(z_2)]\, (p - c) = -\frac{\Psi'(x)}{re^{-rx}} \tag{5.12}$$

$$\begin{aligned} &[1 - H(z_2)]\, e^{-rx} + \int_{z_1}^{z_2} e^{-r(v-z)}\, dH(z) + [H(z_1) - H(z_0)]\, e^{-ry} - \\ &- h(z_0)\, e^{-ry} (p - c) \frac{dz_0}{dp} - (p - c) \frac{dv}{dp}\, r \int_{z_1}^{z_2} e^{-r(v-z)}\, dH(z) = 0 \end{aligned} \tag{5.13}$$

$$r\left[H\left(z_1\right)-H\left(z_0\right)\right]=h\left(z_0\right) \tag{5.14}$$

In the case of a uniform distribution, generically we do not have an interior solution. In particular, equation (5.14) cannot hold. There are two cases. If $rm < 1$ then $z_0 = 0$, and $z_1 = m$. Let us evaluate the LHS of equation (5.13) at the (x, p) that satisfy equation (5.2), i.e., the solution to the monopoly problem without a deadline. The result is:

$$[1-H(\hat{z})]\ e^{-rx}+\int_m^{\hat{z}} e^{-r(v-z)}\ dH(z)+H(m)\ e^{-ry}-(p-c)\ \frac{dv}{dp}\ r\int_m^{\hat{z}} e^{-r(v-z)}\ dH(z)$$

which is positive. Because of the second order conditions, the optimal price with a deadline is higher than without it. Finally, from equation (5.12), and also because of second order conditions, x is lower than without a deadline. Analogously to the proof of Proposition 3, from equations (2.2) and (5.12) we conclude that x is still inefficiently high, but it is closer to the efficient value of x than in the absence of commitment to a deadline. Those consumers whom in the case of no deadline purchase the good at $t > x$ will be worse off with a deadline since they face a higher price and perhaps are forced to buy earlier than they wish. However, the effect on the welfare of those consumers who in the case of no deadline purchase the good at x is ambiguous. On the one hand, they face a higher price, but on the other hand, the new good is available earlier.

If $rm > 1$ then $y = x$, and hence $z_1 = z_2$. Thus, all purchases take place at the moment the new good is made available. The objective function of the monopolist can be written as:

$$\Pi_0=[1-H(z_0)]\ e^{-rx}\ (p-c)-\Psi\ (x)$$

In other words, if we keep (p, x) constant, by shifting from $z_0 = 0$ to $z_0 = z_2 - m$ the present value of demand is maximized. On the top of that, the price is chosen to maximize $[1 - H(z_0)\ (p - c)$. As a result, the LHS of equation (5.12) is higher than the LHS of equation (5.1) and hence the equilibrium value of x with a deadline is lower than in the absence of a deadline.

5.4. Price flexibility and synchronized consumers

Let us consider the discrete time version of the model of Subsection 2.3 with synchronized consumers ($z = 0$). Suppose that trade can only take place at discrete and equally distant points: t_1, t_2, ..., $t_n = t_{n-1} + \Delta$. Let β be the discount factor, i.e., $\beta = e^{-r\Delta}$. A strategy for each buyer is a sequence of reservation prices $\bar{p}\,(t)$ and, similarly, a strategy for the seller is a sequence of posted prices $p\,(t)$.

Proposition 8. *In the unique subgame perfect equilibrium the timing of purchases is efficient and the seller appropriates the entire surplus.*

Let us consider a candidate to a subgame perfect equilibrium. Given the seller's strategy, let us denote by x_1 the lowest value of t such that $p\,(t) \le \bar{p}\,(t)$, i.e., x_1 is the equilibrium timing of transactions. Similarly, let us denote by x_n the lowest $t > x_{n-1}$ such that $p\,(t) \le \bar{p}\,(t)$. By definition consumers' reservation prices must satisfy:

$$R\,(x_n) - \bar{p}\,(x_n) = \beta^{x_{n+1}-x_n}\,[R\,(x_{n+1}) - \bar{p}\,(x_{n+1})] \tag{5.15}$$

Since the seller's strategy must be the best response to buyers' strategy we must have that $p\,(t) = \bar{p}\,(t)$. Plugging this into 26 and by iteration we get that for all n:

$$R\,(x_1) - p\,(x_1) = \beta^{x_{n+1}-x_1}\,[R\,(x_{n+1}) - p\,(x_{n+1})]$$

As n goes to infinity the right hand side goes to zero. Hence, in any subgame perfect equilibrium consumers cannot get a strictly positive payoff. In fact, the same reasoning can be applied to all x_n, i.e., $p\,(x_n) = R\,(x_n)$. Consequently, $\bar{p}\,(t) = R\,(t)$ for all t, and hence the seller's best response is $p\,(t) = R\,(t)$ for all $t > x^e$, where x^e is the time that maximizes total surplus. Since consumers are indifferent to the timing of purchases it is optimal for them to buy at x^e.

5.5. Proof of Proposition 4

If we evaluate the LHS of equation (3.5) at $t = t^*$, then we get:

$$\left\{\frac{q}{\delta+r}\frac{\delta}{r}\left[1-e^{-(\delta+r)t}\right]e^{-\delta t} - (1-e^{-\delta t})\frac{q}{\delta+r}\frac{1-e^{-(\delta+r)t}}{1-e^{-rt}}\right\} + \frac{q}{\delta+r}\frac{1-e^{-(\delta+r)t}}{1-e^{-rt}} -$$
$$-\frac{q}{r}e^{-\delta t} = \frac{q}{\delta+r}\frac{e^{-(\delta+r)t}}{r\,(1-e^{-rt})}\left[r\left(1-e^{-\delta t}\right) - \delta e^{-\delta t}\left(1-e^{-rt}\right)\right]$$

The expression between square brackets is zero if $r = 0$ and it increases with r. As a result the LHS of x at $t = t^*$ is positive, and because of second order condition, this implies that $\bar{t} > t^*$. QED

5.6. Proof of Proposition 5

Evaluate the LHS of equation (3.5) at $t = \underline{t}$:

$$\frac{q}{\delta + r} e^{-\delta t} \left\{ \frac{\delta}{r} \left[1 - e^{-(\delta+r)t}\right] - \left(1 + \frac{\delta}{r}\right)^2 + \frac{1 - e^{-(\delta+r)t}}{1 - e^{-rt}} \right\}$$

which is negative. The reason is the following. The expression between brackets can be written as:

$$\Omega\ (a) \equiv ab + cb - (a+1)^2$$

where

$$a \equiv \frac{\delta}{r}$$

$$b \equiv 1 - e^{-(\delta+r)t}$$

$$c \equiv \frac{1}{1 - e^{-rt}}$$

Note that: (i) $b < 1$, (ii) $c > 1$, (iii) $a + 1 > cb > 1$.

These inequalities are immediate, except that $a + 1 > cb$. Define:

$$\Gamma\ (\delta) = \frac{\delta + r}{r} - \frac{1 - e^{-(\delta+r)t}}{1 - e^{-rt}}$$

Then, $\Gamma\ (0) = 0$, and $\Gamma'(\delta) > 0$. Therefore, $\Gamma\ (\delta) > 0$.

Finally, note that $\Omega\ (cb - 1) = (cb - 1)\ b\ (1 - c) < 0$, and $\Omega'\ (a) = b - 2\ (a + 1) < 0$. Therefore, $\Omega\ (a) < 0$. Because of the second order conditions, $\underline{t} > \bar{t}$. QED

5.7. Proof of Proposition 6

The solution to the monopolist optimization, denoted by $\bar{t}^c$, is given by the first order condition:

$$\frac{q}{r}\, e^{-\delta t} \left\{ e^{-rt}\,(1 - e^{-\delta t}) + \frac{\delta}{\delta + r}\,[1 - e^{-(\delta + r)\,t}] \right\} - \frac{q}{\delta + r}\,\frac{1 - e^{-(\delta + r)\,t}}{1 - e^{-rt}}\,(1 - e^{-\delta t}) +$$

$$+ \frac{c}{1 - e^{-rt}} = 0$$

If we evaluate the left hand side at $t = \bar{t}$ then this is positive, and because of the second order condition we conclude that $\bar{t}^c > \bar{t}$. QED

5.8. Proof of Proposition 7

The first order condition of the monopolist optimization problem characterizes the equilibrium frequency of purchases with unrestricted timing and commitment to a constant price, $\bar{t}^c$:

$$\frac{q}{r}\, e^{-\delta t}\,[1 + \frac{\delta}{r}\,(1 - e^{-rt})] - \frac{q}{\delta + r}\,\frac{1 - e^{-(\delta + r)\,t}}{1 - e^{-rt}} + \frac{c}{1 - e^{-rt}} = 0$$

Comparing this expression with equations (3.3) and (3.7) we check that $t^* < \underline{t}^c < \underline{t}$. QED

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