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Spain 1973/74 – 2003

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Abstract

In this paper, we investigate the properties of a family of inequality measures which extends the Atkinson indices and is axiomatically characterized by a multiplicative decomposition property, where the within-group component is a generalized weighted mean with weights summing exactly to 1. This family contains canonical forms of all aggregative inequality measures; each bounded above by 1, has a useful and intuitive geometric interpretation and provides an alternative dominance criterion for ordering distributions in terms of inequality.
Taking the Spanish Household Budget Surveys (HBS) for 1973/74, 1980/81 and 1990/91 and the more recent continuous HBS for 2003, we show the advantages and possibilities of this extended family in regard to completing and detailing information in studies of inequality focussing on the tails of the distribution and on the changes in the distribution when the population is partitioned into population subgroups.

Key words

Inequality measurement, multiplicative decomposition, Atkinson indices.

Resumen

En este documento de trabajo se examinan las propiedades de una familia de medidas de desigualdad, extensión natural de la familia de Atkinson, que está caracterizada axiomáticamente por una propiedad de descomposición multiplicativa donde la componente intra-grupos es una media generalizada con pesos que suman uno. Los índices de esta familia, todos acotados superiormente por la unidad, pueden considerarse formas canónicas para generar la clase de medidas agregativas, permitiendo proponer un criterio de dominancia alternativo para ordenar distribuciones en términos de desigualdad con una útil e intuitiva interpretación geométrica.
Tomando como base las Encuestas de Presupuestos Familiares (EPF) para España de los años 1973/74, 1980/81 y 1990/91, y la más reciente encuesta continua ($ECPF_{97}$) para 2003 se muestran las ventajas y posibilidades de esta familia para completar y detallar la información en los análisis de la evolución de la desigualdad cuando la población está clasificada en grupos, prestando especial atención a las colas de la distribución.

Palabras clave

Medidas de desigualdad, descomposición multiplicativa, índices de desigualdad de Atkinson.

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The* Extended *Atkinson Family and Changes in the Expenditure Distribution: Spain 1973/74 –2003

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C O N T E N T S

1. Introduction

TWO families of relative inequality measures have been widely used in the literature: the Generalised Entropy family (Shorrocks, 1980) and the Atkinson family (Atkinson, 1970). As is well-known, the Atkinson family is ordinally equivalent to one tail of the Generalised Entropy family, hereafter called the *GE* family, and we wonder what happens with the other tail. We answer this question by providing a natural extension of the Atkinson family, so that now each member of the *GE* family can be transformed, with the same type of monotonic transformation, into one of the extended family. In addition this family contains canonical forms of all aggregative measures (Shorrocks, 1984); each bounded above by 1, and has a useful and intuitive geometric interpretation.

Obviously this extended family and the *GE* family are ordinally equivalent although they have different cardinalization functions. So what else is required to distinguish between them? A new multiplicative decomposition property sheds light on this issue. This alternative decomposition has the following features: *(i)* the between-group component is, according to the traditional approach, the equality level of a hypothetical distribution in which each person's income is replaced by the mean income of his/her group; *(ii)* the within-group component is a generalised weighted mean of the group equality levels, where the weights depend on their aggregated characteristics and their sum is 1; and *(iii)* overall equality is the product of the within- and between-group equality terms. In fact the extended Atkinson family is essentially the only class of continuous multiplicatively decomposable measures (Lasso de la Vega and Urrutia, 2006) [1].

Moreover, this family can be used as a tool for ordering distributions in terms of inequality, in a way that is similar but not equivalent to Lorenz dominance. Second order stochastic dominance and Lorenz dominance are

1. Blackorby, Donaldson and Auersperg (1981) present a multiplicative decomposition for the indices in the Atkinson-Kolm-Sen family in terms of equality indices from a welfare theory approach, using subgroup equally distributed equivalent *(EDE)* income levels to determine the between group component of overall inequality. By contrast we retain the traditional *subgroup (arithmetic) mean income* approach to between-group inequality.

considered as appropriate procedures for deciding whether one distribution is unambiguously less unequal than another as long as one subscribes to the principle of transfers. Thus this tool not only complements the information given by the Lorenz curve, but also provides a neater representation at the tails, and since smoothing is not required, as for example in nonparametric density estimation, the picture that emerges at the extremes of the distribution is not distorted by statistical procedures.

Using this extended family we take a new and original approach to ranking income distributions and measuring inequality in Spain along the period 1973/1974-2003. This approach allows us to study income inequality, paying particular attention to different parts in the distribution. Next, we study trends in inequality. Because such a family is bounded for every parameter value, with bounds that can be interpreted in terms of tails of the distribution, we can see immediately whether the evolution of inequality is driven by movements at the bottom or at the top of the distribution. Finally, an analysis in terms of subgroups of the population is also conducted, focusing on how changes in between and within equality affect overall equality.

The rest of the paper is structured as follows. The next section presents the notation and the basic definitions used in the paper. In section 3 we introduce the one parameter extended family of Atkinson inequality measures and discuss its properties. Section 4 illustrates the issues in the empirical application mentioned. Finally, section 5 offers some concluding remarks.

2. Definitions

WE consider a population consisting of $n \geq 2$ individuals. Individual i's income is denoted by $y_i \in \mathbb{R}_{++} = (0, \infty)$, $i = 1, 2, ..., n$. An income distribution is represented by a vector $\mathbf{y} = (y_1, y_2, ..., y_n) \in \mathbb{R}^n_{++}$. We let $D = \bigcup_{n=1}^{\infty} \mathbb{R}^n_{++}$ represent the set of all finite dimensional income distribution and denote the mean and population size of any $\mathbf{y} \in D$ by $\mu(\mathbf{y})$ and $n(\mathbf{y})$, respectively.

We say that distribution $\mathbf{x} \in D$ is a *permutation* of $\mathbf{y} \in D$ if $\mathbf{x} = \Pi\mathbf{y}$ for some permutation matrix Π; that $\mathbf{x}$ is an *m-replication* of $\mathbf{y}$ if $\mathbf{x} = (\mathbf{y}, \mathbf{y}, ..., \mathbf{y})$ and $n(\mathbf{x}) = m.n(\mathbf{y})$ for some positive integer m [2]; and that $\mathbf{x}$ is obtained from $\mathbf{y}$ by a *progressive transfer* if, for some i and j with $x_i \leq x_j$ we have $x_i - y_i = y_j - x_j > 0$, while for all $k \neq i, j$ we have $x_k = y_k$. We use the vector $\bar{\mathbf{y}}$ to signify the equalised version of $\mathbf{y}$, defined by $n(\bar{\mathbf{y}}) = n(\mathbf{y})$ and $\bar{y}_i = \mu(\mathbf{y})$ for all $i = 1, 2, ..., n(\mathbf{y})$.

An inequality index I is a real valued continuous function $I : D \to \mathbb{R}$, and for the purpose of this paper we take the equality index as $E(\mathbf{y}) = 1 - I(\mathbf{y})$, which is a sensible measure even if it takes negative values. Suppose that the population of n individuals is split into $J \geq 2$ mutually exclusive subgroups with income distribution $\mathbf{y}^j = (y^j_1, y^j_2, ..., \mathbf{y}^j_{n_j})$, mean incomes $\mu_j = \mu(\mathbf{y}^j)$ and population sizes $n_j = n(\mathbf{y}^j)$ for all $j = 1, 2, ..., J$. Let inequality and equality in group j be written $I_j = I(\mathbf{y}^j)$ and $E_j = E(\mathbf{y}^j)$. Let p_j and s_j be the respective population and income shares of subgroup j.

Certain properties, which can be considered to be inherent to the concept of inequality, have come to be accepted as basic properties for an inequality measure. They are listed below.

Property I. *Symmetry.* $I(\mathbf{x}) = I(\mathbf{y})$ whenever $\mathbf{x}$ is a *permutation* of $\mathbf{y}$.

Property II. *Pigou-Dalton Transfers Principle.* $I(\mathbf{x}) < I(\mathbf{y})$ whenever $\mathbf{x}$ is obtained from $\mathbf{y}$ by a *progressive transfer.*

Property III. *Normalisation.* $I(\bar{\mathbf{y}}) = 0$ for all $\mathbf{y} \in D$. Otherwise, $I(\mathbf{y}) > 0$.

Property IV. *Replication Invariance.* $I(\mathbf{x}) = I(\mathbf{y})$ whenever $\mathbf{x}$ is a *replication* of $\mathbf{y}$.

---

2. The incomes in **x** are simply the incomes in **y** repeated a finite number of times.

An inequality index corresponds to a concept of **relative inequality** if it is *scale invariant*, that is:

Property V. *Scale Invariance.* $I(\lambda \mathbf{y}) = I(\mathbf{y})$ for all $\lambda > 0$.

Finally, Shorrocks (1984) introduced the following property for any partitioning of the population into exhaustive and disjoint subgroups:

Property VI. *Aggregative Principle.* (Shorrocks, 1984) An inequality index I will be said to be aggregative if there exits an *aggregator* function Q such that

$$I(\mathbf{x}, \mathbf{y}) = Q\left(I(\mathbf{x}), I(\mathbf{y}), \mu(\mathbf{x}), \mu(\mathbf{y}), n(\mathbf{x}), n(\mathbf{y})\right)$$

for all $\mathbf{x}, \mathbf{y} \in D$, where $Q(\bullet)$ is continuous and strictly increasing in its first two arguments.

In the sequel p-order means will play a role. We use $m_p(\mathbf{y})$ to represent the mean of the order p, that is:

$$m_p(\mathbf{y}) = \begin{cases} \left(\frac{1}{n}\sum_{i=1}^{n} y_i^p\right)^{\frac{1}{p}} & p \in \mathbb{R}, p \neq 0 \\ \left(\prod_{i=1}^{n} y_i\right)^{\frac{1}{n}} & p = 0 \end{cases}$$

whence in particular $m_1(\mathbf{y})$ is the arithmetic mean, $\mu(\mathbf{y})$, and $m_0(\mathbf{y})$ is the geometric mean.

The mapping $p \to m_p$ is a non decreasing continuous function on all of $\mathbb{R}$. The limiting case at one extreme is as $p \to -\infty$, giving $m_p(\mathbf{y}) \to \min \{y_i\}_{i=1}^n$. At the other extreme, as $p \to \infty$, giving $m_p(\mathbf{y}) \to \max \{y_i\}_{i=1}^n$. Moreover, for a given p, m_p is non-decreasing in every element of $\mathbf{y}$ and also is concave for $p \leq 1$ and convex for $p \geq 1$ [3].

Two families of relative inequality measures are widely used in literature. The Generalised Entropy class (Bourguignon, 1979; Shorrocks 1980, 1984; Cowell, 1980; Cowell and Kuga, 1981a, 1981b) is given by:

3. See for example Kendall and Stuart (1977), Magnus and Neudecker (1988), Bullen (2003) or Steele (2004).

$$I_\alpha^{GE}(\mathbf{y}) = \begin{cases} \frac{1}{n} \cdot \frac{1}{\alpha^2 - \alpha} \sum_{i=1}^{n} \left[\left(\frac{y_i}{\mu} \right)^\alpha - 1 \right] & \alpha \in \mathbb{R}, \alpha \neq 0,1 \\ -\frac{1}{n} \sum_{i=1}^{n} \log \frac{y_i}{\mu} & \alpha = 0 \\ \frac{1}{n} \sum_{i=1}^{n} \frac{y_i}{\mu} \log \frac{y_i}{\mu} & \alpha = 1 \end{cases} \quad (2.1)$$ [4]

The other is the Atkinson family (Atkinson, 1970) [5] given by:

$$I_\alpha^{A}(\mathbf{y}) = \begin{cases} 1 - \left[\frac{1}{n} \sum_{i=1}^{n} \left(\frac{y_i}{\mu} \right)^\alpha \right]^{\frac{1}{\alpha}} & \alpha < 1, \alpha \neq 0 \\ 1 - \prod_{i=1}^{n} \left(\frac{y_i}{\mu} \right)^{\frac{1}{n}} & \alpha = 0 \end{cases} \quad (2.2)$$ [6]

It may be interesting to note that, as is well-known, these two families are monotonically related, I_α^A can be obtained from I_α^{GE} via the following transformation:

$$I_\alpha^{A}(\mathbf{y}) = F\left(I_\alpha^{GE}(\mathbf{y})\right) = \begin{cases} 1 - \left(1 + (\alpha^2 - \alpha)\, I_\alpha^{GE}(\mathbf{y})\right)^{\frac{1}{\alpha}} & \alpha < 1, \alpha \neq 0 \\ 1 - \exp\left(-I_0^{GE}(\mathbf{y})\right) & \alpha = 0 \end{cases}$$

---

4. Note that for $\alpha \neq 0,1$ I_α^{GE} can be equivalently written as $I_\alpha^{GE}(\mathbf{y}) = \frac{1}{\alpha^2 - \alpha} \left[\left(\frac{m_\alpha}{\mu} \right)^\alpha - 1 \right]$, and for $\alpha = 0$ $I_0^{GE}(\mathbf{y}) = \log \frac{\mu}{m_0}$.

5. In the original formulation of Atkinson (1970)

$$A_{1-\varepsilon}(\mathbf{y}) = \begin{cases} 1 - \left[\frac{1}{n} \sum_{i=1}^{n} \left(\frac{y_i}{\mu} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} & \varepsilon > 0, \varepsilon \neq 0 \\ 1 - \prod_{i=1}^{n} \left(\frac{y_i}{\mu} \right)^{\frac{1}{n}} & \varepsilon = 0 \end{cases}$$

where $\varepsilon > 0$ represents the relative inequality aversion of the society, so in terms of (2.2) $\alpha = 1 - \varepsilon$ and $\varepsilon > 0$ implies $\alpha < 1$, i.e. $A_{1-\varepsilon}(\mathbf{y}) = I_\alpha^A(\mathbf{y})$. In our formulation α is not linked to any concept of welfare.

6. Note that I_α^A can be equivalently written as $I_\alpha^A(\mathbf{y}) = 1 - \frac{m_\alpha}{\mu}$.

3. The *Extended* Atkinson Family

WE now turn to the aim of this paper. Consider the following single parameter class of measures

$$I_\alpha(\mathbf{y}) = \begin{cases} 1 - \left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{y_i}{\mu}\right)^\alpha\right]^{\frac{1}{\alpha}} & \alpha < 1, \alpha \neq 0 \\ 1 - \left(\left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{y_i}{\mu}\right)^\alpha\right]^{\frac{1}{\alpha}}\right)^{-1} & \alpha > 1 \\ 1 - \prod_{i=1}^{n}\left(\frac{y_i}{\mu}\right)^{\frac{1}{n}} & \alpha = 0 \\ 1 - \prod_{i=1}^{n}\left(\frac{\mu}{y_i}\right)^{\frac{y_i}{n\mu}} & \alpha = 1 \end{cases} \quad (3.1)$$ [7]

and note in particular that this family is defined for all $\alpha \in \mathbb{R}$.

3.1. Basic properties of this family

Firstly, it is noteworthy that the Atkinson family arises from this class when $\alpha < 1$. The following theorem shows that also the rest of the measures of the family fulfil convenient properties.

***Theorem 1**: For each $\alpha \in \mathbb{R}$ $I_\alpha(\mathbf{y})$ satisfies Symmetry, the Pigou-Dalton Transfers Principle, Normalization, Replication Invariance, the Scale Invariance Principle, the Aggregative Principle and is bounded above by 1.*

Proof. Since the Atkinson indices verify all these properties it is enough to prove that it is true for $\alpha \geq 1$. It is clear that all the members of

7. Note that for $\alpha > 1$ I_α can be equivalently written as $I_\alpha(\mathbf{y}) = 1 - \frac{\mu}{m_\alpha}$ and that for $\alpha < 1$ $I_\alpha(\mathbf{y}) = I_\alpha^A(\mathbf{y})$.

this family are bounded above by 1. Furthermore, we can mechanically transform one index $I_\alpha(\mathbf{y})$ from the family (3.1) into other one, $I_\alpha^{GE}(\mathbf{y})$, from the *GE* family, for all $\mathbf{y} \in D$, using the following formulas

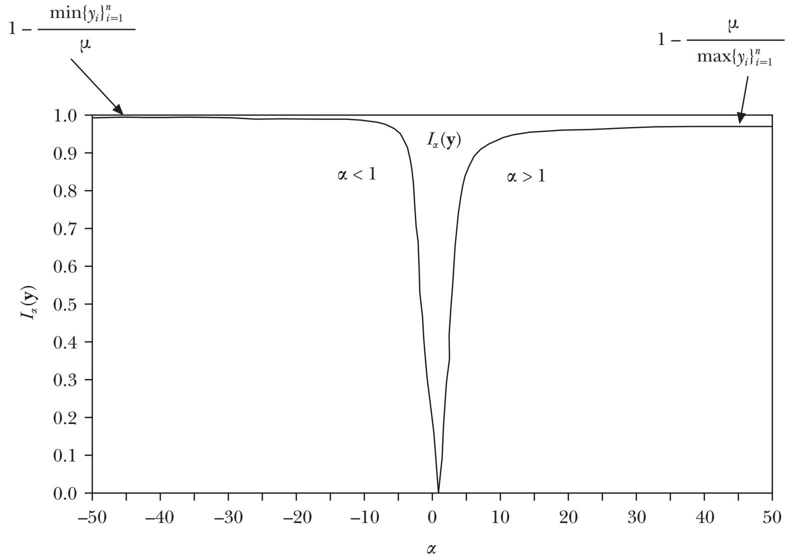
$$I_\alpha(\mathbf{y}) = F\left(I_\alpha^{GE}(\mathbf{y})\right) = \begin{cases} 1 - \left(\left(1 + (\alpha^2 - \alpha)\, I_\alpha^{GE}(\mathbf{y})\right)^{\frac{1}{\alpha}}\right)^{-1} & \alpha > 1 \\ 1 - \exp\left(-I_1^{GE}(\mathbf{y})\right) & \alpha = 1 \end{cases}$$

where it is easy to verify that, at any given α, $F\colon [0, \infty) \to \mathbb{R}$ is a continuous increasing function with $F(0) = 0$. Then from Shorrocks (1984), $I_\alpha(\mathbf{y})$ verifies the mentioned properties.

In our opinion, this result provides a natural extension of the Atkinson indices which may be referred to as *the extended Atkinson family*. Moreover, this family contains alternative canonical forms of all continuous aggregative inequality measures, each bounded above by 1, since there is a measure in this family corresponding to each continuous aggregative index which orders distributions in precisely the same way.

Another interesting characteristic of this family is that the inequality value $I_\alpha(\mathbf{y})$ varies continuously as a function of the α-parameter at each income distribution given by $\mathbf{y}$, except for $\alpha = 1$. (As $\alpha \to 1$, $I_\alpha(\mathbf{y})$ tends to a totally insensitive measure, whereas $I_1(\mathbf{y})$ is ordinally equivalent to what is commonly called the Theil inequality index.) Actually, for any given $\mathbf{y}$, the family has two tails, according to whether α is less or greater than 1. An example is given in graphic 3.1. The α-parameter is clearly a measure of the degree of relative sensitivity to transfers at different income levels. As α increases $I_\alpha(\mathbf{y})$ becomes more sensitive to transfers at the upper end than at the lower end and in the middle part of the distribution. The limiting case is as $\alpha \to \infty$, giving $I_\alpha(\mathbf{y}) \to 1 - \dfrac{\mu}{\max \{y_i\}_{i=1}^n}$, which only considers transfers among the richest income group. By contrast, as α decreases the opposite is true, in other words this family becomes more sensitive to transfers at the lower end of the distribution. The limiting case is as $\alpha \to -\infty$, giving $I_\alpha(\mathbf{y}) \to 1 - \dfrac{\min \{y_i\}_{i=1}^n}{\mu}$, which only takes account of transfers among the very lowest income group. In fact, when α is less than 1, $I_\alpha(\mathbf{y})$ satisfies the transfer sensitive principle according to Shorrocks and Foster (1987).

GRAPHIC 3.1: $I_\alpha(\mathbf{y})$ as a function of the α-parameter



In fact the interpretation of $I_\alpha(\mathbf{y})$ becomes transparent in the income space and complements the graphical representation given by Atkinson (1970). The graphic 3.2 provides such graph in two cases of interest, one for $\alpha < 1$ and other for $\alpha > 1$, in a two person society. Given the curvature properties of m_α, being strictly concave for $\alpha < 1$ and strictly convex for $\alpha > 1$, the iso-m_α curves are strictly convex for $\alpha < 1$ and strictly concave for $\alpha > 1$, and cross the 45º line at the income distribution whose entries are all equal to μ. Consequently, inequality as measure by $I_\alpha(\mathbf{y})$ is the extend to which the iso-m_α curve departs from the mean line along the perfect equality line in the income space. This distance is taken in relative terms and with respect to the maximum mean under comparison [8]. For example, for $\alpha < 1$ the distance $\mu - m_\alpha$ relative to μ is the Atkinson (1970) measure of inequality. In his welfare interpretation this is the percentage welfare loss from inequality, since m_α is the welfare of the actual distribution and μ is the maximum welfare attainable with existing resources [9]. For $\alpha > 1$ no such welfare interpretation is possible, but the distance $m_\alpha - \mu$ relative to m_α is a suitable measure of inequality. In fact given that the α-order mean, m_α, can be considered as a representative income function reflecting the average prosperity level for a giv-

8. Which results in an upper bound of 1 on $I_\alpha(\mathbf{y})$ and normalization.

9. In Atkinson's setting, m_α is the *equally distributed equivalent income*, a particular homogeneous cardinalization of the welfare function.

en distribution (Foster and Shneyerov, 2001), $I_\alpha(\mathbf{y})$ for $\alpha < 1$ is a relative measure of the spare resources available if we focus on a representative income below μ, we focus on the poor; whereas $I_\alpha(\mathbf{y})$ for $\alpha > 1$ is a relative measure of the additional resources needed if we focus on a representative income above μ, we focus on the rich this time. In all cases we keep comparisons along the perfect equality line. In both directions of μ it makes sense to measure inequality.

Graphic 3.2 also helps to understand why, even if the measure of the distance between m_α and μ is not discontinuous, the inequality family (3.1) has a discontinuity at $\alpha = 1$, since at this point the iso-m_1 curve becomes the mean line [10].

3.1.1. $I_\alpha(\mathbf{y})$-curve dominance

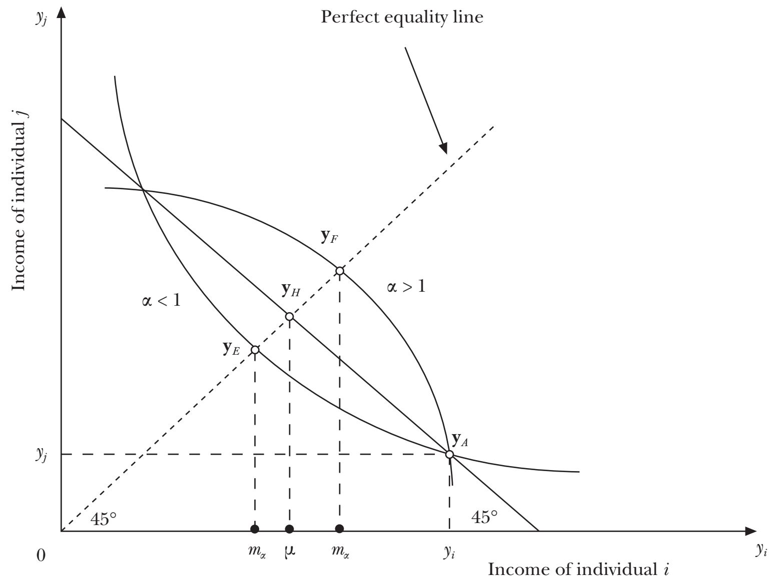
What value of α should we choose to determine the specific value of inequality? The answer is by no means simple because the resulting inequality comparisons may be sensitive to the choice of this value, so we may have to employ a variety of values of α and accept the fact that the resulting inequality comparisons may at times conflict in significant aspects.

Apart from that, a standard procedure in order to avoid any conflict is to demand unanimous agreement among classes of inequality measures. To that end we now suggest the use of $I_\alpha(\mathbf{y})$-curves as an appealing tool for ordering distributions: when the curve of one distribution lies above the curve of another one, it displays more inequality as measured by this family. However, since the members of this family, as already mentioned, can be considered as canonical forms of all continuous aggregative inequality indices, checking this $I_\alpha(\mathbf{y})$-dominance enables us to establish inequality comparisons that necessarily hold for all continuous aggregative inequality indices.

The Lorenz curve is usually used to test whether one distribution is unambiguously more unequal than another providing that one accepts the principle of transfers, since Lorenz ordering is equivalent to inverse stochastic dominance of the second order. Therefore, Lorenz dominance obviously implies $I_\alpha(\mathbf{y})$-curve dominance. However, the ranking of distributions im-

10. Note in passing that $\frac{1}{\alpha - 1}\left(1 - \frac{\mu}{m_\alpha}\right)$ is a family of measures that belong to the general class studied by Foster and Shneyerov (1999). This is defined for all $\alpha \in \mathbb{R}$, since at $\alpha = 1$ converges to the Theil (1967) inequality measure, and it is additively decomposable in the general sense of Foster and Shneyerov (1999). However it is not bounded above by 1. Presumably we can avoid the discontinuity point adopting a suitable cardinalization, but at the cost of some other properties of our family, such as the multiplicative decomposability property introduced below.

GRAPHIC 3.2: Graphical interpretation of $I_\alpha(\mathbf{y})$

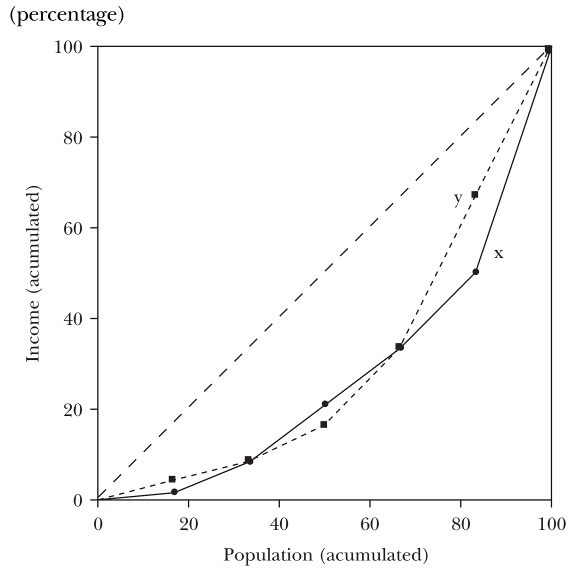


plied by Lorenz dominance and that of $I_\alpha(\mathbf{y})$-curve dominance are not equivalent. To illustrate this point, take a six-person society and consider two distributions $\mathbf{x} = (0.1, 0.4, 0.75, 0.75, 1, 3)$ and $\mathbf{y} = (0.25, 0.25, 0.5, 1, 2, 2)$. According to the Lorenz criterion distributions **x** and **y** cannot be ranked since their Lorenz curves cross twice, as shown in graphic 3.3a. By contrast it may be observed that distribution **y** dominates distribution **x** under $I_\alpha(\mathbf{y})$-curve dominance since, as shown in graphic 3.3b, $I_\alpha(\mathbf{x}) \geq I_\alpha(\mathbf{y})$ for all $\alpha \in \mathbb{R}$.

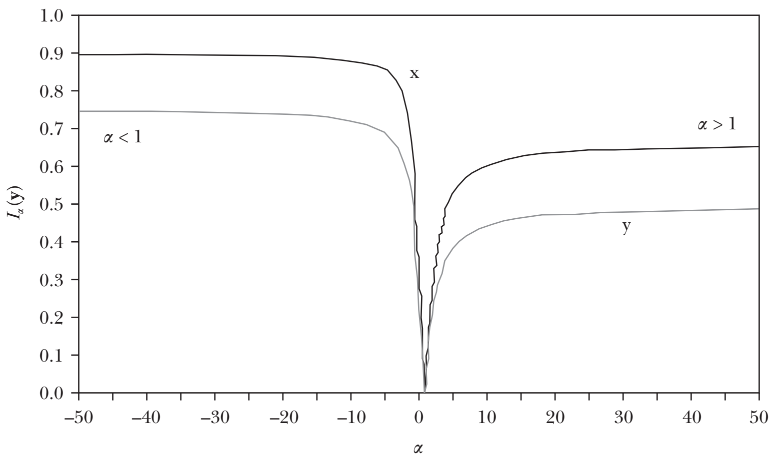
Bearing in mind that the aggregative principle has often been invoked for measuring inequality in a population split into different groups, $I_\alpha(\mathbf{y})$-curves provide a powerful tool for testing whether the unanimous agreement applied to the class of continuous aggregative indices leads to an unquestionable ruling when the Lorenz curves intersect.

However, obviously, it is not rare in practice for $I_\alpha(\mathbf{y})$-curves also to intersect. If one were interested in giving more weight to transfers at the lower end of the distribution than at the top, the measure should satisfy the transfer sensitive principle, and it is possible under special circumstances to obtain a conclusive ranking when one refers to this class of measures. Whereas Shorrocks and Foster (1987) demonstrated that third order stochastic dominance allows us to characterize unanimous agreement among measures of this class, Davies and Hoy (1995) proved that a variance condition allows distributions whose Lorenz curves intersect to be ordered unambiguously among them. As regards $I_\alpha(\mathbf{y})$-curve dominance, when the curves do not in-

GRAPHIC 3.3a: Lorenz curves: distributions *x* and *y*



GRAPHIC 3.3b: I_α(y) curves: distributions *x* and *y*



tersect for $\alpha \leq 1$, distributions may be ranked conclusively among the class of aggregative transfer sensitive inequality measures.

In any case, since the normative judgements associated with the values of α are both explicit and appealing, this I_α(**y**)-curve dominance allows us to make inequality comparisons giving a fuller description of differences in inequality. If the curve of one distribution crosses that for another one from above on the left-hand side of the graph, the first distribution is more unequal than the second according to large negative values of α, which are sensitive to the income of the people who are worst off in society. Conversely, when the curves intersect on the right-hand side with α large but positive, the rule would be particularly sensitive to the income of the richest people in society. So, using $I_\alpha(\mathbf{y})$-curves to make inequality comparisons rather than relying on summary indices alone not only becomes a useful tool for ranking distributions, but also gives us an insight into how we might explain what is observed.

3.2. Multiplicative Decomposition Property

Dominance provides an ordinal measure but not a cardinal one. The problem arises when the analysts or policy makers desire a precise measure of how big the difference between income distributions or the change between them is. If we wish to use an additively subgroup decomposable inequality index then we have to choose one from the *GE* family [11]. The extended Atkinson family does not permit additive decomposition in the standard way, although it possesses desirable properties. The following theorem shows that the measures of the extended Atkinson family permit an alternative decomposition into the between- and the within- group equality components [12].

Theorem 2*: Let consider any exhaustive collection of mutually exclusive population subgroups $j = 1, 2, \ldots, J$. For each $\alpha \in \mathbb{R}$, $I_\alpha(\mathbf{y})$ verifies the following multiplicative decomposition in terms of equality indices:*

$$E_\alpha(\mathbf{y}) = \begin{cases} \left(\sum_{j=1}^{J} \omega_j \left[E_\alpha(\mathbf{y}^j)\right]^{\alpha}\right)^{\frac{1}{\alpha}} E_\alpha(\mathbf{y}^1, \mathbf{y}^2, \ldots, \mathbf{y}^J) & \alpha < 1, \alpha \neq 0 \\ \left(\sum_{j=1}^{J} \omega_j \left[E_\alpha(\mathbf{y}^j)\right]^{-\alpha}\right)^{-\frac{1}{\alpha}} E_\alpha(\mathbf{y}^1, \mathbf{y}^2, \ldots, \mathbf{y}^J) & \alpha > 1 \\ \prod_{j=1}^{J} \left[E_\alpha(\mathbf{y}^j)\right]^{\omega_j} E_\alpha(\mathbf{y}^1, \mathbf{y}^2, \ldots, \mathbf{y}^J) & \alpha = 0,1 \end{cases} \tag{3.2}$$

11. This is true as long as we restrict ourselves to the arithmetic mean as the representative measure of the prosperity level in society. See Foster and Shneyerov (1999) for a generalization of the additive decomposition that relaxes this assumption.

12. This extends the result in Lasso de la Vega and Urrutia (2003) for the Atkinson family. The focus on equality, instead of inequality, is not new; see Blackorby and Donaldson (1978).

with weights $\omega_j = \dfrac{p_j^{1-\alpha}\, s_j^{\alpha}}{\sum_{j=1}^{J} p_j^{1-\alpha}\, s_j^{\alpha}}$ for all α.

Proof. The proof is straightforward after a few lines of standard computations and rearrangements.

The second term on the right-hand side of these equations is the equality level of a hypothetical distribution in which each person's income is replaced by the mean income of his/her subgroup and may be considered the between-group equality component according to the traditional approach.

With respect to the first term on the right-hand side of the equal sign there are three possibilities:

i) when α is less than 1 and different from 0, the term is the α-order (weighted) mean of the group equalities;
ii) when α is greater than 1 it is the $(-\alpha)$-order (weighted) mean of the group equalities;
iii) when α is equal to 0 or to 1 it is the geometric (weighted) mean of the group equalities.

In all cases, this term summarizes equality within the population subgroups and may be considered the within-group equality component. Hence, for each α-parameter value, the overall equality is the product of the between-group equality component multiplied by the within-group equality component. The decomposition coefficients for these indices are functions of the subgroup means and population sizes and their sum is equal to 1 by construction. In addition, Lasso de la Vega and Urrutia (2006) give a key characterization of the extended Atkinson family as essentially the only continuous measures with such a weighted multiplicative decomposition where weights can be general functions of the subgroup means and population sizes, summing exactly to 1.

It may be noteworthy to compare the additive decomposition for the *GE* family with the multiplicative decomposition presented here. Note that this multiplicative decomposition plays a role symmetrical to the one played by the additive decomposition. Indeed, for the *GE* indices the within-component is a weighted average of the inequality of each individual group and in the $I_\alpha(\mathbf{y})$ family the within- term is an α-order weighted mean of the group equalities. The decomposition coefficients for these indices are the same as in the additive decomposition but normalised and their sum is equal to 1.

This approach has several advantages. First, the sum of the decomposition coefficients is equal to 1. Second, with respect to the within-group component definition, note that our formulation broadens the set of possibilities to include the α-order means of the subgroup equality levels, with $\alpha \leq 1$. If the level of equality coincides in all groups, the mean and the α-order mean lead to the same result. The bigger the difference in the levels of equality of the groups, the smaller the α-order mean with $\alpha < 1$, so that the α-order mean indicates not only the mean levels of equality of groups but also the differences between those levels [13].

Moreover, the multiplicative decomposition allows us to evaluate the impact of marginal changes from a given group on overall equality. Indeed the multiplicative decomposition of these indices can be transformed through the logarithmic transformation, so that it is additive in log's. Letting, $E_{W\alpha}(\mathbf{y})$ as the within-group component in (3.2), i.e. the first term on the right hand side of these equations, and $E_{B\alpha}(\mathbf{y})$ as the between-group component in (3.2), i.e. $E_{B\alpha}(\mathbf{y}) = E_{\alpha}\ (\mathbf{y}^1, \mathbf{y}^2, ..., \mathbf{y}^J)$. Then taking log's in (3.2) we find that $\log E_{\alpha}(\mathbf{y}) = \log E_{W\alpha}(\mathbf{y}) + \log E_{B\alpha}(\mathbf{y})$, so using the fact that a percentage change in x, $\%\Delta x$, can be written as $\%\Delta x \approx 100.\Delta \log\ (x)$ for small changes in x, we find that

$$\%\Delta E_{\alpha}(\mathbf{y}) \approx \%\Delta E_{W\alpha}(\mathbf{y}) + \%\Delta E_{B\alpha}(\mathbf{y}) \qquad (3.3)$$

Equation (3.3) shows that the overall percentage of change in equality can be expressed as the sum of the percentage changes in the within- and the between- components. Note that this analysis cannot be carried out with additive decompositions, where the available decompositions for changes rely on approximations (Theil and Sorooshian, 1979).

The multiplicatively decomposition for equality (3.2) can be turned into an additive decomposition for inequality with an interaction term. Given de equality indices, $E_{W\alpha}(\mathbf{y})$ and $E_{B\alpha}(\mathbf{y})$, we can define the corresponding inequality terms as, $I_{W\alpha}(\mathbf{y}) = 1 - E_{W\alpha}(\mathbf{y})$ and $I_{B\alpha}(\mathbf{y}) = 1 - E_{B\alpha}(\mathbf{y})$, so expressing $E_{\alpha}(\mathbf{y})$ in terms of $I_{\alpha}(\mathbf{y})$ and substituting this into the general formula for our family of inequality, $I_{\alpha}(\mathbf{y}) = 1 - E_{\alpha}(\mathbf{y})$ we get,

---

13. A similar reasoning applies for $\alpha > 1$ since we can write

$$\left(\sum_{j=1}^{J} \omega_j \left[E_{\alpha}(\mathbf{y}^j)\right]^{-\alpha}\right)^{-\frac{1}{\alpha}} = \frac{1}{\left(\sum_{j=1}^{J} \omega_j \left[\dfrac{1}{E_{\alpha}(\mathbf{y}^j)}\right]^{\alpha}\right)^{\frac{1}{\alpha}}}$$

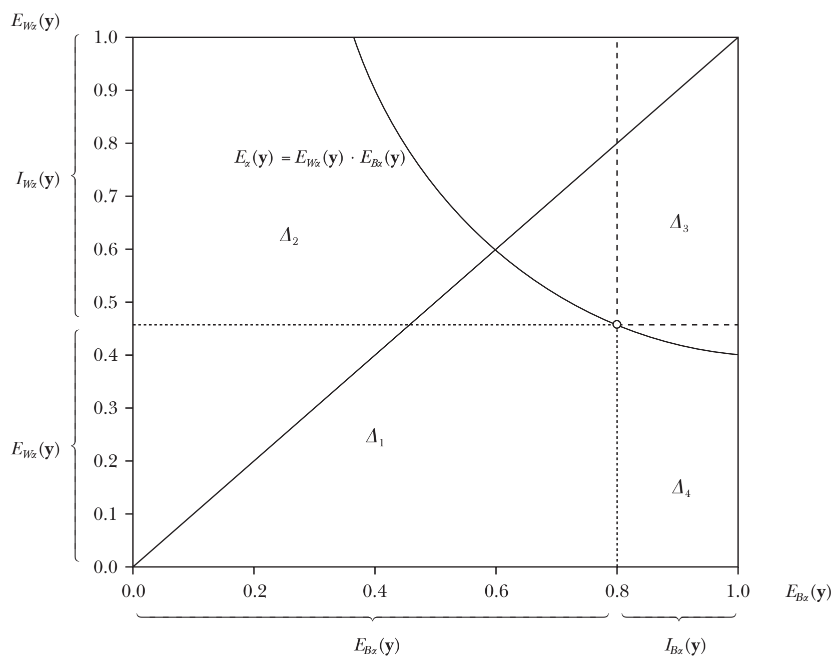
so a $(-\alpha)$-order (weighted) mean of the group equalities can be written as the inverse of the α-order (weighted) mean of the inverses of the group equalities.

$$\begin{aligned} I_\alpha(\mathbf{y}) &= 1 - E_\alpha(\mathbf{y}) \\ &= 1 - E_{W\alpha}(\mathbf{y}) \,.\, E_{B\alpha}(\mathbf{y}) \\ &= 1 - (1 - I_{W\alpha}(\mathbf{y}))\ (1 - I_{B\alpha}(\mathbf{y})) \\ &= I_{W\alpha}(\mathbf{y}) + I_{B\alpha}(\mathbf{y}) - I_{W\alpha}(\mathbf{y}) \,.\, I_{B\alpha}(\mathbf{y}) \end{aligned} \tag{3.4}$$

Each multiplicatively decomposable measure also permits a simple geometric interpretation on the unit box, as shown in graphic 3.4. This geometric interpretation helps us to understand where the interaction term in (3.4) comes from. Note that, in terms of the areas in graphic 3.4, $I_\alpha(\mathbf{y}) = 1 - \Delta_1 = \Delta_2 + \Delta_3 + \Delta_4$, $I_{W\alpha}(\mathbf{y}) = \Delta_2 + \Delta_3$, $I_{B\alpha}(\mathbf{y}) = \Delta_4 + \Delta_3$ and $I_{W\alpha}(\mathbf{y}) \,.\, I_{B\alpha}(\mathbf{y}) = \Delta_3$. So summing $I_{W\alpha}(\mathbf{y})$ and $I_{B\alpha}(\mathbf{y})$ double counts $I_{W\alpha}(\mathbf{y}) \,.\, I_{B\alpha}(\mathbf{y}) = \Delta_3$, which have to be subtracted in the additive decomposition of $I_\alpha(\mathbf{y})$ [14].

The comparison between the equality levels of two income distributions and a given partition of the population is reduced to a comparison between their respective level curves and the corresponding projections on the axes, with information being provided at all times on how much prog-

GRAPHIC 3.4: Geometric interpretation of a multiplicative decomposable measure



14. Note that this interaction term is similar to the one that appears in Blackorby, Donaldson and Auersperg (1981), and very different to the additive decomposition of the Gini coefficient (Silber, 1989; Lambert and Aronson, 1993), since it has nothing to do with the overlapping in the partition of the population and only disappears when inequality is zero in the within or in the between distribution.

ress has been made towards equality in each component, and how far there is still to go. For applied economists and policy analysts, this graphical approach can effectively convey information about inequality, although great care is needed to interpret the empirical results. On the one hand, we must certainly reference here the interesting arguments of Kanbur (2003). He warns against normative use of decomposition findings because in his opinion they cannot determine the appropriate focus for policy interventions. Policy instruments that target between-group differences and those which target within-group differences must be costed and their benefits and effectiveness compared.

On the other hand, a serious objection to the decomposition weights can be pointed out. In general changing the between-group inequality is bound to change income shares and those changes will also affect the decomposition coefficients and therefore the total within-group contribution. Thus, variations in between-group inequality result in modifications not only in the between-group component but also in the within-group one, even though there may have been no change in the inequality of the different groups. Only when these coefficients do not depend on the group means are the between- and within- components independent [15]. Theil (1967), Shorrocks (1980) and Foster and Shneyerov (2000) have highlighted this handicap as regards to the decomposition coefficients for the *GE* family. Of the family of measures in (3.1), only one satisfies this independence requirement: the index $I_0(\mathbf{y})$, for which the corresponding decomposition coefficients are the population shares. For this reason, $I_0(\mathbf{y})$ is the only multiplicatively decomposable measure which allows to break down unambiguously the overall equality change into the shares that can be attributed either to the change in the between group term or to changes in equalities within those groups [16]. In this case the additive form of the within-group component in equation (3.3) makes the contributions of the different groups to changes in overall equality particularly easy to investigate.

15. In fact, this is a well known necessary condition for the so called *path independent decomposability,* as investigated by Foster and Shneyerov (2000).

16. Lasso de la Vega and Urrutia (2005) investigate the additional properties of $I_0(\mathbf{y})$ as regard to the *path independent decomposability.*

4. Changes in Expenditure Distribution: Spain 1973/1974-2003

LET us use the foregoing approach to analyze the income distribution in Spain for the period 1973/1974 to 2003 using expenditure as a proxy variable. We use the Household Budget Surveys (HBS) from 1973/1974, 1980/1981 and 1990/1991, as well as the last available year from the Continuous Household Budget Survey, 2003, carried out by the Spanish Statistical Institute *(Instituto Nacional del Estadística*-INE) [17]. All of them are representative at the regional level (NUTS 2 regions) [18]. As the variable representative of the standard of living we use per capita total expenditure [19], defined as monetary expenditure plus non monetary expenditure arising from self-consumption, self-supply, free meals, in-kind salary and imputed rents for house ownership [20]. Total expenditure per capita is assigned to every person in the household, so even if the household is the basic statistical unit, person

17. The HBS from 1973/1974, 1980/1981 and 1990/1991 are taken from the web of the Economics Department at University Carlos III of Madrid (http://www.eco.uc3m.es/investigacion/index.html#toc4). The HBS for 2003 was retrieved directly from the INE's web (http://www.ine.es), and corresponds to merging the quarterly files of 2003 for the strong collaboration sub-sample. For methodological information see INE (1975, 1983, 1992, 1998).

18. We exclude from the analysis the autonomous cities of Ceuta and Melilla, since they were excluded in the HBS of 1973/1974.

19. See Deaton and Zaidi (2002) for arguments in favour of using consumption for distributional purposes and Atkinson and Bourguignon (2002) for arguments in favour of using income. Essentially most of the arguments rely on consumption being smoother than income, hence being a better proxy for permanent income.

20. To be more specific *Transfers to other Households and Institutions* are excluded from the definition of expenditure in the HBS of 1980/1981 and 1990/1991, since they are not included either in the actual Continuous HBS or in the HBS of 1973/1974. Moreover, the HBS for 1990/1991 includes a different valuation criterion for the non monetary expenditures that the one used by INE. See Arévalo, Cardelús and Ruiz-Castillo (1998).

weights are applied to the calculations, since we are mainly concerned with the economic well-being of individuals.

4.1. Focusing on different parts of income distribution

In table 4.1 we present the generally accepted view about the evolution of inequality in the income distribution in Spain along the period of analysis (Goerlich and Mas, 2001, 2002, 2004; Oliver-Alonso, Ramos and Raymond-Bara, 2001). We offer a plethora of inequality indices from the $I_\alpha^{GE}(\mathbf{y})$ and $I_\alpha(\mathbf{y})$ families, as well as the popular Gini (1912) index. Generally speaking all of them tell about the same story. A continuous negative trend in inequality for the period 1973/1974-1990/1991, the magnitude of the change depends, of course, on the particular index but the general tendency seems to be clear, and a more or less stable distribution in the last period, 1990/1991-2003. Contrary to what has happened in other developed countries Spain has not experienced an increase in income inequality in recent years but the income distribution seems to be stable in the last period. This is a robust fact to the examination of the data for the end of the nineties and the beginning of this de-cade, at least if we focus on expenditure (Aldás, Goerlich and Mas, 2006a, 2006b) [21].

TABLE 4.1: Inequality Indexes — Total Expenditure

	Gini	Extended Atkinson α							Generalized Entropy α						
		–2	–1	0	0.5	1	2	3	–2	–1	0	0.5	1	2	3
1973/1974	0.340	0.476	0.323	0.177	0.094	0.185	0.202	0.411	0.441	0.239	0.195	0.194	0.204	0.285	0.648
	(0.002)	(0.015)	(0.003)	(0.003)	(0.002)	(0.003)	(0.008)	(0.022)	(0.040)	(0.004)	(0.003)	(0.003)	(0.005)	(0.015)	(0.105)
1980/1981	0.333	0.462	0.317	0.171	0.090	0.174	0.185	0.370	0.409	0.232	0.187	0.184	0.192	0.252	0.501
	(0.002)	(0.011)	(0.004)	(0.002)	(0.001)	(0.003)	(0.005)	(0.020)	(0.023)	(0.005)	(0.003)	(0.002)	(0.003)	(0.009)	(0.063)
1990/1991	0.316	0.417	0.284	0.154	0.081	0.159	0.169	0.336	0.323	0.198	0.167	0.165	0.173	0.224	0.403
	(0.003)	(0.012)	(0.004)	(0.003)	(0.002)	(0.004)	(0.007)	(0.020)	(0.023)	(0.004)	(0.003)	(0.003)	(0.004)	(0.012)	(0.052)
2003	0.317	0.367	0.269	0.153	0.083	0.166	0.184	0.364	0.249	0.184	0.166	0.169	0.181	0.250	0.480
	(0.003)	(0.006)	(0.004)	(0.003)	(0.002)	(0.003)	(0.006)	(0.011)	(0.006)	(0.003)	(0.003)	(0.003)	(0.004)	(0.010)	(0.034)

Note: HBS 1973/1974, 1980/1981, 1990/1991 and 2003. Atkinson (1970) family corresponds to values $\alpha < 1$.
Simple bootstrap standard erros (Efron and Tibshirani, 1993) in brakets.

21. We are aware that the results for the last period, 1990/1991-2003, are not robust to the particular file used in the calculations. In particular using the longitudinal file from the continuous

Inequality indices balance someway the behaviour at different segments of the income distribution to be able to give you a real number summarizing the whole distribution, and these segments may be pushing inequality in different directions. It may be of interest trying to look at both sides of the distribution and see how these are trying to affect the inequality index. This can be done quite easily by varying the α parameter, since this reflects in some way the different weights attached by the index at different parts of the distribution. Even this is true for both families, $I_\alpha^{GE}(\mathbf{y})$ and $I_\alpha(\mathbf{y})$, the bounded behaviour of $I_\alpha(\mathbf{y})$ makes the $I_\alpha(\mathbf{y})$-curves very handy to study the tails of the distribution.

The tendencies shown in table 4.1 are however masking a rather different behaviour at the both sides of the distribution in the different periods covered by our study. The examination of the $I_\alpha(\mathbf{y})$-curves reveal this in a simple, graphical and intuitive form.

In graphic 4.1 we represent the $I_\alpha(\mathbf{y})$-curve for all the years covered and values of α in the range of –40 to +40. The extended Atkinson family behaves as expected. Inequality is very high for large negative values of α, but decreases steadily as α increases and the underlying inequality index becomes more and more sensitive to the circumstances of the less well-off members of the society, just the opposite is true for large positive values of α.

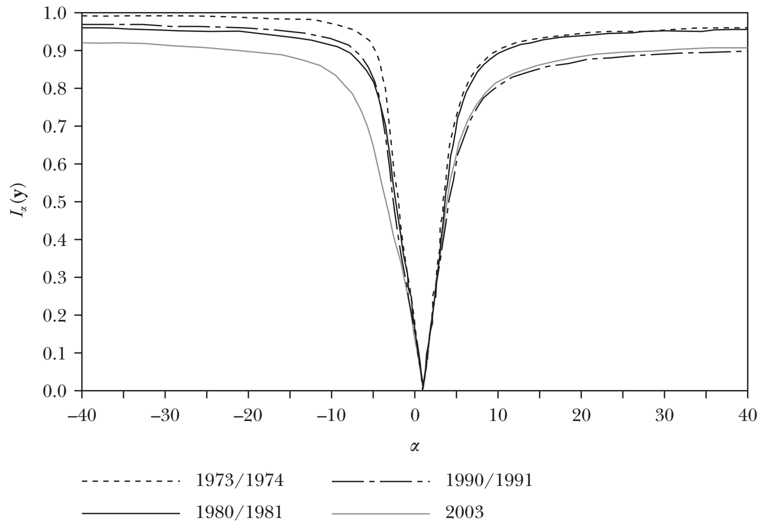
One rather conclusion emerges clearly from graphic 4.1. For every value of the sensitivity parameter, the index value $I_\alpha(\mathbf{y})$ in 2003 is lower than their equivalent value in 1973/1974, and thus the corresponding curves do not intersect. This suggests quite strongly that it has been a redistribution of income from those who are better-off to those who are worse-off throughout Spain along this period [22].

This decrease in inequality is however not monotonic along the complete period at the different tails of the distribution. So the overall trend shown in table 4.1 is picking up conflicting tendencies at different parts of the income distribution. In particular it is shown quite clearly how between 1973/1974 and 1980/1981 inequality decreases according to negative values of α, indicating that the worse off are approaching the mean income, the

HBS for 2002 (INE, 2002) results in a significant lower inequality level for this year than using the original quarterly files for the same year, either using a single quarter or merging all quarters in the calculations. We have been puzzled by some of the discrepancies we have found using both files from what is in fact the same survey, and we are in the process of investigating the origins of these discrepancies. Most of the qualitative results and the usefulness of our approach are, however, not affected by this fact.

22. Examination of the same respective figures at the regional levels reveals very different behaviours, and the absence of a single pattern at this level of aggregation.

GRAPHIC 4.1: Inequality in Spain: $I_\alpha(\mathbf{y})$ curves



middle class, whereas the size of the change in the index in this period remains fairly small as α increases.

In the next period, from 1980/1981 to 1990/1991, just the opposite behaviour is detected. Inequality significantly decreases for positive values of α, so is the richest part of the society who is approaching the average person, while for negative values of the parameter the change in the index is small and in the direction of a slightly increase in inequality. Hence the overall index, showing a reduction in inequality, is picking up mainly the distributional changes from the top part of the distribution.

Eventually, for the last period the stability observed in the Gini, Theil, $I_1^{GE}(\mathbf{y})$ and $I_0^{GE}(\mathbf{y})$, or standard Atkinson, $I_0^{A}(\mathbf{y})$, indices result from opposite tendencies at both sides of the mean. We observe an important reduction in inequality for $\alpha < 1$, reflecting the catching-up of the poor, while an slightly increase in inequality for $\alpha > 1$, which shows the tendency of the richest part of the society to move away from the *middle class.* Note that this behaviour can be observed from table (4.1) for values of α such as –2 or 3, which move in opposite directions for either $I_\alpha(\mathbf{y})$ and $I_\alpha^{GE}(\mathbf{y})$. So the apparent stability in the distribution in this last period is compensating conflicting tendencies at the different tails of the distribution [23].

23. This result is consistent with the examination of the Lorenz ordinates, and also robust to previous years (Goerlich and Mas, 2004).

Overall, this is consistent with the view that, for the whole period, the pattern of redistribution results in a substantial redistribution of income in favour of the least well off in Spain during this period, however the evolution of the changes are not monotonic and come from different parts of the distribution.

In summary, the $I_\alpha(\mathbf{y})$-curves allow us to uncover how the behaviour at both sides of the income distribution is affecting the Atkinson index for standard values of the sensitivity parameter.

4.2. Accounting for changes over time in income equality

Our next application involves the multiplicative decomposability property as indicated in Theorem 2. This, however, involves switching the mind from the standard view of inequality to the complementary view of equality, $E(\mathbf{y}) = 1 - I(\mathbf{y})$.

Contrary to the additive decomposition of the Generalised Entropy family (Shorrocks, 1980) the decomposition shown in (3.2) is not useful in a static context, but it can be very handy in a dynamic one, to account for changes in the equality index in terms of the changes inside the within and between distributions [24].

As an example consider two possible partitions of the population. In the first term, we consider a regional partition, in which we divide the population into to 17 NUTS 2 regions *(Comunidades Autónomas)* in which Spain is divided from a political perspective. So now the criterion is regional residence. In the static context, using $I_\alpha^{GE}(\mathbf{y})$, it is well known that the between term is much less important than the within term and that this importance has been diminishing over time along our period of study as a result of regional convergence (Goerlich and Mas, 2001), hence inequality is mainly in the personal distribution, not in the regional distribution of income. In a dynamic context it is not clear what has been the role of the demographic factors in altering the inequality and its contributions.

In the second term, we consider an educational partition according to the level of studies of the head of the family. We consider four groups, illiter-

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24. As a practical matter it was the case that in all the examples shown in this section the additive decomposition (3.4) gave us the same quantitative results as the additive decomposition for $I_\alpha^{GE}(\mathbf{y})$, since the interaction term is the product of two small numbers and hence it is negligible in most of the cases. Note, however, that the between and within terms in (3.4) are not the same as the corresponding terms in the additive decomposition for $I_\alpha^{GE}(\mathbf{y})$.

ates, primary school, secondary school and university studies. The importance of these groups has changed a great deal in relative terms along the period of study. The access to higher education has increased the importance of secondary school and university studies groups, while the other two have seen an important reduction. In this way we can examine how the changes in the distribution within each group have contributed to overall distributional changes [25].

Before we present the numerical examples lets consider a bit further the break of the equality changes given by (3.3) in the particular case that $\alpha = 0$. For this value the multiplicative decomposition (3.2) satisfies the path independent property for the equality index (Foster and Shneyerov, 2000; Lasso de la Vega and Urrutia, 2005) and the different contributions of the different groups to changes in overall equality are easy and intuitive to investigate.

Thus, equation (3.3) for $\alpha = 0$ can be written as $\%\Delta E_0(\mathbf{y}) \approx \%\Delta E_{W0}(\mathbf{y}) + \Delta E_{B0}(\mathbf{y})$, or alternatively as $\Delta \log E_0(\mathbf{y}) = \Delta \log E_{W0}(\mathbf{y}) + \Delta \log E_{B0}(\mathbf{y})$, but given that from (3.2) $\log E_{w0}(\mathbf{y}) = \Sigma_{j=1}^{J} p_j \log E_0(\mathbf{y}^j)$ we can eventually write [26],

$$\begin{aligned} \Delta \log E_0(\mathbf{y}) &= \Delta \left(\sum_{j=1}^{J} p_j \log E_0(\mathbf{y}^j) \right) + \Delta \log E_{B0}(\mathbf{y}) \\ &= \sum_{j=1}^{J} \left(\Delta \left(p_j \log E_0(\mathbf{y}^j) \right) \right) + \Delta \log E_{B0}(\mathbf{y}) \end{aligned} \tag{4.1}$$

which shows what is the particular contribution of a given group to changes in the overall index.

However these contributions depend on changes in both, the within-group equalities, $E_0(\mathbf{y}^j)$, and their population shares, p_j. We suggest further decomposing the change in the within-group component, $\Delta \log E_{W0}(\mathbf{y})$, into both effects by means of a *shift-share* analysis. Using this, we can write

$$\begin{aligned} \Delta \left(\sum_{j=1}^{J} p_j \log E_0(\mathbf{y}^j) \right) &= \sum_{j=1}^{J} \Delta p_j \left[\frac{\log E_0(\mathbf{y}^j)_t + \log E_0(\mathbf{y}^j)_{t-1}}{2} \right] \\ &+ \sum_{j=1}^{J} \left[\frac{p_{j,t} + p_{j,t-1}}{2} \right] \Delta \log E_0(\mathbf{y}^j) \end{aligned} \tag{4.2}$$

25. Two other population partitions according to the sex or to the age of the head of the family are available from the authors upon request. In both of these cases external inequality plays almost no role and most of the observed inequality is attributed to the internal component.

26. A similar analysis could be done for $\alpha = 1$ using income shares, s_j, instead of population shares, p_j. For other values of $\alpha \neq 0,1$ the contribution of each individual group to changes in the overall equality cannot be determined, as can be seen from (3.2).

where the subscript t means a point in time and the first term on the right hand side of the equal sign in (4.2) represents the contribution to changes in the within-group due to changes in the population shares *(demographic factors)* and the second term represents the contribution due to changes in the equality *(distributional factor)*. This could be further disaggregated for each individual group.

The decomposition (3.3) and the within contributions, either by groups and by factors, in the regional partition for $\alpha = 0$ is offered in table 4.2 for the period 1973/1974-2003. The corresponding equality indexes are obtained directly from table 4.1 and the breakdown (3.3) shows that 60% of the change in the equality during the period can be attributed to within contribution. This is quantitatively similar to the changes in the period 1973/1974-1990/1991. Hence, even if the static decomposition of $I_0^{GE}(\mathbf{y})$ in the between and within terms shows that the latter is more important, and has gained importance along the period (Goerlich and Mas, 2004), the improvement in distributional terms comes mostly from the within distribution. Moreover the contribution of the between term, which is non-negligible, reflects regional convergence.

If we further disaggregate the within term by looking at the contribution of the different groups we discover again that there is no single regional experience. Some regions appear to have a negative contribution to improvements in the overall distribution, *Madrid*[27], *Canarias* and *Comunidad Valenciana*. Others, like *Illes Balears, Cantabria, Cataluña, Región de Murcia, Navarra* or *La Rioja*, shows no contribution, either negative or positive. For the rest of the cases it does not seem to be a general pattern, with important contributions, either from rich, *País Vasco,* or poorer regions, *Castilla y León* or *Andalucía*.

Looking at the contribution of the different factors, equation (4.2), we see that the improvements in the within distribution are driven solely by distributional changes and not by demographic factors, so we have a truly distributional improvement.

The decomposition (3.3) and the within contributions, either by groups and by factors, in the educational partition for $\alpha = 0$ is offered in table 4.3 for the period 1973/1974-2003. In this case we have a reversal in the contributions to the improvements in distribution. A slightly higher contribution comes from the between distribution, which indicates the important

27. The negative contribution from *Madrid* comes from the important increase in the population share over the period.

TABLE 4.2: Regional Partition

	Global	External	Internal
Changes 1973/1974-2003	**2.93%**	**1.19%**	**1.75%**
		40.42%	**59.58%**
Internal decomposition			
by group			
Andalucía			0.54%
Aragón			0.24%
Asturias (Principado de)			0.10%
Balears (Illes)			–0.05%
Canarias			–0.10%
Cantabria			0.01%
Castilla y León			0.48%
Castilla-La Mancha			0.19%
Cataluña			0.03%
Comunidad Valenciana			–0.10%
Extremadura			0.25%
Galicia			0.21%
Madrid (Comunidad de)			–0.23%
Murcia (Región de)			–0.08%
Navarra (Comunidad Foral de)			0.02%
País Vasco			0.23%
Rioja (La)			0.01%
Total			**1.75%**
by factor			
Demographic			–0.03%
Distributional			1.78%
Total			**1.75%**

Source: HBS 1973/1974 and 2003.

convergence of the different groups. The within contribution accounts however for almost 48% of the change in equality, so both of them are in fact quite important.

Further disaggregating the within term in (4.1) shows that lower educated groups have contributed positively to the improvement in distribution while the higher educated groups have contributed negatively. Essentially this is picking up a compositional effect. Moreover, these are again pure distributional changes, with little aggregate effects from the changes in the composition of the two groups.

TABLE 4.3: Educational Partition

	Global	External	Internal
Changes 1973/1974-2003	**2.93%**	**1.56%**	**1.37%**
		53.24%	**46.76%**
Internal decomposition			
by group			
Illiterate			2.75%
Primary School			5.11%
Secondary School			–4.58%
University Studies			–1.91%
Total			**1.37%**
by factor			
Demographic			–0.03%
Distributional			1.40%
Total			**1.37%**

Source: HBS 1973/1974 and 2003.

5. Conclusions

THIS paper highlights the properties and the underlying possibilities of the extended Atkinson family, a class of inequality measures which is axiomatically characterized by an alternative multiplicative decomposition property. We have presented the main theoretical points and have illustrated the use of this family with an application from the HBS for 1973/1974, 1980/1981, 1990/1991 and 2003.

Essentially we have used the class of measures to show how the negative trend in inequality along the 1973/1974-2003 period can be identified as coming from different parts of the distribution in the different periods analyzed, and how the apparently stable distribution in the recent years is the result of two opposite forces, an improvement in distribution from the bottom and a worsening from the top.

Second we have conducted a decomposition analysis that breaks down the change in the equality index in a between and a within component. Using two alternative partitions of the population we show how the distribution is improving mostly from the within component in a regional partition, but mostly from the between component in an educational partition.

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