Fundación BBVA

Documentos de Trabajo

8

2009

Francesc Obiols-Homs

Parameterizing Expectations for Incomplete Markets Economies

Fundación BBVA

Parameterizing Expectations for Incomplete Markets Economies

Francesc Obiols-Homs

AUTONOMOUS UNIVERSITY OF BARCELONA

Abstract

This working paper combines the Parameterizing Expectations Approach developed by Marcet (1988) and Den Haan and Marcet (1990), and the approach proposed by Krusell and Smith (1998), to approximate the equilibrium of incomplete markets economies with aggregate uncertainty. The algorithm is tested by solving a version of the economy in Krusell and Smith where shocks take values on a continuum. The results show that the combined algorithm is fast and accurate.

Key words

Idiosyncratic shocks, incomplete insurance, computable general equilibrium.

Resumen

Este documento de trabajo combina el método de las expectativas parametrizadas desarrollado por Marcet (1988) y Den Haan y Marcet (1990), y el enfoque propuesto por Krusell y Smith (1998), para aproximar el equilibrio de las economías con mercados incompletos e incertidumbre agregada. Se comprueba el algoritmo solucionando una versión de la economía en Krusell y Smith donde los *shocks* toman valores continuos. Los resultados demuestran que el algoritmo compuesto es rápido y exacto.

Palabras clave

Choques idiosincrásicos, mercados incompletos, equilibrio general computable.

La serie Documentos de Trabajo tiene como objetivo la rápida difusión de los resultados del trabajo de investigación entre los especialistas de esa área, para promover así el intercambio de ideas y el debate académico. Cualquier comentario sobre sus contenidos será bien recibido y debe hacerse llegar directamente a los autores, cuyos datos de contacto aparecen en la *Nota sobre los autores*.

The Working Papers series is intended to disseminate research findings rapidly among specialists in the field concerned, in order to encourage the exchange of ideas and academic debate. Comments on this paper would be welcome and should be sent direct to the authors at the addresses provided in the About the authors *section.*

Todos los documentos de trabajo están disponibles, de forma gratuita y en formato PDF, en la web de la Fundación BBVA. Si desea una copia impresa, puede solicitarla a través de publicaciones@fbbva.es.

All working papers can be downloaded free of charge in pdf format from the BBVA Foundation website. Print copies can be ordered from publicaciones@fbbva.es.

La serie Documentos de Trabajo, así como información sobre otras publicaciones de la Fundación BBVA, pueden consultarse en: **http://www.fbbva.es**

The Working Papers series, as well as information on other BBVA Foundation publications, can be found at: **http://www.fbbva.es**

Parameterizing Expectations for Incomplete Markets Economies

EDITA / PUBLISHED BY
Fundación BBVA, 2009
Plaza de San Nicolás, 4. 48005 Bilbao

C O N T E N T S

1. Introduction

THIS working paper is motivated by the interest in understanding and quantifying time-series properties of equilibrium dynamics of economies in which the representative-agent abstraction does not hold. A typical example would be an economy in which production is subject to aggregate shocks to technology, and where a continuum of utility-maximizing consumer/workers, which are subject to idiosyncratic shocks of unemployment, do not have access to perfect insurance markets [1]. In these economies agents become heterogeneous as a result of the market incompleteness, and therefore, both the distribution of wealth and its law of motion are determined endogenously. Since generally equilibrium-prices depend on the distribution of wealth, the implication is that agents need to know the law of motion of that distribution to form expectations about the future [2]. Therefore approximating the equilibrium dynamics of an incomplete markets economy using numerical techniques is difficult because the distribution of wealth is an infinite dimensional object that changes over time, and it requires a manageable way to describe these changes.

The methods used to solve the class of incomplete markets economies with aggregate uncertainty, such as versions of the Parameterizing Expectations Approach (PEA) developed by Marcet (1988) and Den Haan and Marcet (1990), and the approach followed by Krusell and Smith (KS) (1998), are either costly in computing time, or assume a very specific structure for the stochastic shocks (for instance, limiting to a small number the possible states of nature), or both. Nevertheless, we show in this paper that an appropriate combination of these numerical procedures is able to get rid of the above limitations all at once.

In brief, our algorithm consists of dividing the computations into *inner* and *outer* iterations as follows: the inner iterations follow KS and take as

1. A complete list of papers studying the effects of incomplete markets would be too long to be included here. Some well known examples are Aiyagari (1994), Huggett (1993), Huggett (1997) and Krusell and Smith (1998), among others.

2. This is the key difference with respect to economies where a representative-agent holds (and thus the distribution of wealth is trivial, and its law of motion is simply the optimal policy of the representative-agent).

given the laws of motion of several moments of the distribution of wealth, but find an accurate approximation to the conditional expectations characterizing the choices of an agent's problem in several regions of the state space using PEA. The outer iterations, then, perform simulations of a large number of agents over time and update the laws of motion of the selected moments. The combination of *PEA à la KS* results in a fast and accurate algorithm which is not subject to the typical limitations of discretizing methods (such as the course of dimensionality), and it can be easily implemented to approximate a rich class of stochastic, nonlinear, incomplete markets economies.

The outline of the working paper is as follows. Section 2 presents a benchmark economy with incomplete markets similar to that in KS, but in which both the aggregate shock to technology and the idiosyncratic shocks take values on a continuum. Section 3 briefly reviews PEA and the approach in KS, and describes in detail the combination of the two procedures (PEA-KS). Section 4 reports results both about accuracy and equilibrium dynamics. Section 5 extends the previous results and introduces an endogenous labor supply decision. Finally, section 6 concludes. The appendices of the working paper contain a detailed description of practical issues regarding the implementation of PEA-KS, parameter values, and several additional results about accuracy.

2. Economy

THE model economy is a version of the model in Krusell and Smith (KS) [3]. Markets are competitive and there is a representative firm which carries out the production of the consumption/investment good in the economy. Output per capita in each period is given by $y_t = z_t f(k_{t-1}, h_t)$, where f is a constant returns to scale technology satisfying the usual assumptions about concavity and differentiability, and where k_{t-1} and h_t represent respectively the amounts of capital and labor in operation in period t (both in per capita terms). $z_t = \exp(\hat{z}_t)$ represents the realization of a stochastic shock to technology which follows a stationary first order Markov process:

$$\hat{z}_t = \rho\, \hat{z}_{t-1} + \varepsilon_t,\ 0 \leq \rho < 1, \tag{2.1}$$

where the innovation ε_t is a continuous random variable with cdf λ. Using the consumption good as the numeraire, competitive prices for capital and labor satisfy $r_t = z_t f_1(k_{t-1}, h_t)$ and $w_t = z_t f_2(k_{t-1}, h_t)$, respectively.

There is a continuum of agents with names i in the unit interval. These agents choose consumption and saving in every period to maximize the expected present value over an infinite horizon of their utility from consumption: $E[\Sigma_{t=0}^{\infty} \beta^t u(c_t^i) \mid \Omega_0^i]$, where Ω_0^i for $t = 0, 1,...$ represents the set of available information to agent i in every time t. Preferences $u(\cdot)$ are differentiable, strictly increasing and concave, and satisfy the usual Inada conditions. These preferences together with the discount factor $\beta \in (0,1)$, are the same for all i. Agents are endowed with some units of capital k_{i-1} at $t = 0$, and in each period they receive an idiosyncratic endowment of labor productivity $s^i \in \{0,1\}$ which is inelastically supplied as labor. Labor productivity endowments follow a first order Markov chain described by a transition matrix Π_t, as it is explained below. Capital is the only asset in the economy; thus the budget constraint of agent i in period t reads $c_t^i + k_t^i \leq w_t s_t^i + (r_t + 1 - \delta) k_{t-1}^i$, where $\delta \in (0,1)$, is the depreciation rate. Consumption is required to be

3. To fix notation, a subindex t indicates when a variable is known. A superscript i on a variable indicates individual values, and distinguishes them from aggregate values (in per capita terms).

nonnegative, and we also assume that there is a borrowing limit $B > -\infty$ such that $k_t^i \geq B$ for all i and t.

We interpret a realization $z_t > 1$ in period t as the economy being in *good* times, and $z_t \leq 1$ as the economy being in *bad* times. We assume that $\Pr\{s' = s_1 | s = s_0, z' > 1\}$ $\forall$ $s_1, s_0 \in \{0,1\}$ is given by a matrix Π^g, and if the economy remains in the good times state forever, ϕ^g would be the average endowment of labor implied by Π^g. Similarly, in bad times $\Pr\{s' = s_1 | s = s_0, z' \leq 1\}$ $\forall$ $s_1, s_0 \in \{0,1\}$ is given by a matrix Π^b, and in this case, $\phi^b < \phi^g$ would be the long-run average endowment of labor. Since the economy fluctuates between good and bad times, the actual probabilities of transition in the next period are given by:

$$\Pi_t = \Pi^b \cdot G(z_{t+1} \leq 1 | z_t) + \Pi^g \cdot (1 - G(z_{t+1} \leq 1 | z_t)), \tag{2.2}$$

where $G(\cdot)$ is the conditional cdf for z implied by the process in (2.1). Notice that as Π_t changes over time, also the aggregate endowment of labor smoothly fluctuates over time. In particular, the aggregate endowment of labor in each period is denoted ϕ, and it satisfies:

$$\phi_t = \pi_{1|1,\, t-1}\, \phi_{t-1} + \pi_{1|0,\, t-1}\, (1 - \phi_{t-1}). \tag{2.3}$$

In the previous economy, markets are incomplete because trading contracts contingent on the realization of shocks is exogenously precluded. This is a natural assumption given the presence of both idiosyncratic risks and a large number of agents. The implication is that at any point in time agents are heterogeneous in their idiosyncratic endowments and stocks of capital, and, therefore, in their consumption/saving decisions. This heterogeneity is described by $\Lambda_t(A)$, a probability measure defined on an appropriate family of subsets of $[B, +\infty) \times \{0,1\}$ indicating the mass of agents in period t with $x = (k_{t-1}^i, s_t^i) \in A$. Associated to the distribution Λ_t, there is a transition function $Pt(x, A; z_t, \phi_t)$ such that:

$$\Lambda_{t+1}(A) = \int P_t(x, A; z_t, \phi_t)\, d\Lambda_t. \tag{2.4}$$

The transition function $P_t(x, A; z_t, \phi_t)$ gives the probability that an agent in idiosyncratic state $x = (k_{t-1}^i, s_t^i)$ in period t lies in a set A in period $t+1$ given that the current aggregate state is (z_t, ϕ_t, Λ_t).

The information available to a given agent t in a period t, Ω_t^i, consists of $k_{\tau-1}^i, s_\tau^i, \pi_\tau^i, z_\tau, \phi_\tau, w_\tau, r_\tau, \Lambda_t$ and the transition functions for Λ, z, and s for $\tau = 0, 1, 2\ldots, t$. That is, agents know their own state and the aggregate state of the economy, but they do not know others' agents capital stock nor their realization of idiosyncratic shocks.

Assuming that a stationary competitive equilibrium (SCE) exists, it can be defined as follows:

An SCE for the economy consists of stochastic processes $\{c_t^i, k_t^i\}$, $\{w_t, r_t\}$, and a sequence of functions Λ_t and P_t, such that for all t:
1) budget constraints, non-negativity constraints for consumption, and the following first order conditions of the utility maximization problem are satisfied for all i:

$$u'(c_t^i) = \beta E[(r_{t+1} + 1 - \delta)\ u'(c_{t+1}^i) \mid \Omega_t^i] \text{ if } k_t^i > B, \\ u'(c_t^i) \geq \beta E[(r_{t+1} + 1 - \delta)\ u'(c_{t+1}^i) \mid \Omega_t^i] \text{ otherwise;} \tag{2.5}$$

2) markets for capital and labor clear; 3) prices are competitive; 4) the transition P_t is consistent with optimal decisions of agents; and 5) $P_t = P$.

In the previous definition, 2) requires that $h_t = \phi_t$ and that k_{t-1} equals the integration of k_{t-1}^i with respect to Λ_t; 3) requires that r_t and w_t are given by the corresponding marginal productivities of k_{t-1} and h_t; and 4) requires equation (2.4) be satisfied for all sets A. The assumptions on preferences imply that budget constraints are satisfied with equality. Thus, the integration of the agent's budgets constraints with respect to Λ_t together with (2.2) and (2.3) imply that feasibility is satisfied in each period.

3. Approximating the Stationary Competitive Equilibrium (SCE)

GIVEN a set of parameter values characterizing stochastic shocks, technologies, preferences and constraints, the objective is to provide a numerical method able to find an accurate approximation to the SCE of the economy just described. It is well known that finding such an approximation is difficult for two reasons: first, the equations in (2.5) represent a set of nonlinear, stochastic difference equations which, in general, cannot be solved for using analytical methods; second, to be able to evaluate the conditional expectations in (2.5), we need to know the transition function P. However, this transition function depends on the optimal decisions of the agents, which is what we are looking for.

The procedure we describe below combines ideas from Parameterizing Expectations Approach (PEA) and from the approach in Krusell and Smith (KS). For convenience, we briefly outline the main features of these algorithms.

3.1. The approach in KS

The procedure in KS is based on dynamic programming and on searching on a grid of state variables. The difficulty from the perspective of computation is that the distribution Λ_t, an infinite-dimensional object, should be included among the state variables. The main insight in KS can be summarized as *only the mean matters:* rational agents knowing only a few moments of the distribution Λ_t would make very small mistakes when trying to predict the mean of Λ_{t+1}, which is what they need to forecast prices in the future and to take optimal consumption/saving decisions today. That is, the approach in KS consists of reducing the dimension of the state space by considering only a subset of the moments that characterize the distribution of wealth. KS, then, postulate log-linear rules to approximate the laws of motion for the selected moments of the distribution Λ_t (they use only the mean) and find optimal policies for an agent's decision problem by performing iterations on the corresponding value function (i.e., searching on a manageable grid of state variables). The last step of this procedure consists of simulating the optimal policies for a large number of agents over time, compute the actual mean of the distribution in each period, and update the coefficients of the prediction rules by running OLS regressions. This process continues until decision rules and prediction rules are mutually consistent. KS report that this strategy produces very accurate prediction rules [4]. The interpretation of these results is that the differences between the decisions of fully rational agents (that would use all moments of the distribution Λ_t) and boundedly rational agents (that use only the first moment) are negligible.

In principle, grid methods, such as iterations on the value function or on its derivative, can produce an arbitrarily accurate solution by refining the grid. However, these methods become very expensive in computing time, and they easily exhaust the computer's capacity when the grid is very fine and/or when the dimension of the state space is large, like in this case. For this reason, KS restrict z and ϕ to take on two values only and relay on various interpolation schemes to reduce the number points in the grid.

3.2. PEA

This procedure follows a time series approach and exploits the fact that the conditional expectation is that function of the data in Ω_t^i that minimizes the prediction error. Thus the procedure consists of finding the best approximation to the conditional expectation in the right hand side of the equations in (2.5), as follows. Postulate a function of a subset of variables in Ω_t^i characterized with a vector of initial coefficients $\bar{\theta}_1$, and use it to replace the conditional expectations of the equations in (2.5). A typical choice for this function would be the exponential of a polynomial, which is always positive as is $u'(c)$. With this information and values for k_{-1}^i, s_0^i, k_{-1}, ϕ_0, and z_0, compute current equilibrium prices and obtain the consumption of a large number of agents in the first period, which together with prices and budget constraints, deliver (after aggregation) next period stock of capital. This stock of capital and next period realization of shocks determine equilibrium prices in that period. Repeat this process for a long time series. Use the sim-

4. For instance, the R^2 of the OLS regressions is about 0.999998, and the standard deviation of the regressions errors are about 0.003%. KS also report high accuracy by including the second moment of Λ_t in an extended version of the model with endogenous labor supply decisions.

ulated series to update the coefficients $\bar{\theta}$ by running NLLS regressions of the observed marginal utilities in $t+1$ on the postulated function using as regressors the variables selected from Ω_t^i. The process continues until the rate of change of the coefficients $\bar{\theta}$ is sufficiently small [5].

Notice that with PEA there is no need to relay on approximations of the law of motion for the mean (or other moments) of Λ_t: the stock of capital in the following period can be computed just by adding up the stocks of capital at the end of the current period of the agents in the economy. Another advantage of this procedure is that the course of dimensionality does not represent a limitation [6]. Finally, it is also known that approximations found with PEA for representative-agent economies are very accurate (see, for instance, the examples in Den Haan and Marcet, 1994). However, the procedure suffers from two serious problems when applied to incomplete markets economies. First, to obtain high accuracy and mimic the behavior of a continuum of agents, one needs to simulate a large number of agents and over a long time series. This is extremely costly in terms of computing time. Second, in the data generated in the simulations, there are lots of observations in the region of k^i where the agents' stocks of capital are close to the average capital in the economy (i.e., where most agents spent most time periods). The implication of this fact is that since for most agents their capital stock is highly correlated with the average capital in the economy, then the $X'X$ matrix that one needs to invert in the regression step is close to singular. This makes the algorithm unstable.

3.3. PEA-KS

An algorithm, which combines the advantages of the previous approaches, consists of implementing the KS strategy of reducing the dimension of the state space and iterating at an inner level (given prediction rules for some moments of Λ_t, find the implied optimal decision rules) and at an outer level (given decision rules, find the implied prediction rules), while using PEA. Since the marginal utility of consumption is dramatically different for low and high levels of capital, it is difficult to approximate its conditional ex-

5. For examples of such an application, see Den Haan (1996) and Obiols-Homs (2003).

6. The reason is that an additional state variable just represents an additional dimension in the $X'X$ matrix that is inverted in the regression step. Therefore, in principle, we can directly compute many moments (or percentiles) of Λt in each time period and use them as regressors to approximate the right hand side of the equations in (2.5).

pectation using a single function over the entire domain of k^i. Instead, it is more accurate to use several functions to approximate the conditional expectations in several regions of the (k^i, s^i) directions of the state space. This is done by performing many short-run simulations taking as given the aggregate states of the economy [7]. A precise outline of the algorithm is as follows:

- Step 1: Create a long time series realization for stochastic shocks, and postulate laws of motion for a few moments of the distribution Λ_t (say, polynomials with a vector of initial coefficients $\bar{\gamma}_1$) as functions of the aggregate state. Given this information and an initial condition for k_{-1}, compute a sequence of equilibrium prices for capital and labor over time. Divide the direction corresponding to k^i of the state space into m intervals, and postulate m functions of the elements in Ω_t^i (for instance polynomials, as before) characterized with a vector of initial coefficients $\bar{\theta}_1^m$. These functions will be used to replace the conditional expectations of the equations in (2.5) in step 2.
- Step 2: Obtain N short-run simulations for a single agent, using many initial conditions for its k^i in each of the m intervals. The prices for capital and labor used in these simulations are the ones generated in step 1.
- Step 3: Update the coefficients $\bar{\theta}^m$ by running NLLS regressions of the observed marginal utilities on the postulated functions while using as regressors the variables selected from Ω_t^i.
- Step 4: Repeat j-times steps 2 and 3 until the rate of change between $\bar{\theta}_j^m$ and $\bar{\theta}_{j-1}^m$ meets the accuracy requirement for each m. (At this stage, we obtain accurate approximations to the conditional expectations in equation [2.5] for the given prediction rules.)
- Step 5: Create a long-run realization for stochastic shocks, fix initial conditions for a large number of agent's capitals, and obtain a long-run realization of the equilibrium paths for the selected moments of the distribution Λ_t. In this step, agents approximate expected marginal utility according to the coefficients obtained in step 4.
- Step 6: Run an OLS regression of the moments of $\Lambda_t + 1$ on the observed aggregate states in period t, and update $\bar{\gamma}$. (At this stage,

7. A similar type of short-run simulations was used to compute transitional dynamics in Marcet and Marimon (1992).

we obtain accurate prediction rules for the given parameterization of conditional expectations.)

— Step 7: Repeat j'-times steps 2 to 6 until the rate of change between $\bar{\gamma}_{j'}$ and $\bar{\gamma}_{j'-1}$ meets the accuracy requirement.

Notice that in step 2, the realization of shocks is always the same one, which, for the given laws of motion for the moments of Λ_t, always deliver the same equilibrium prices. Likewise, step 5 always uses the same realization of shocks to determine actual equilibrium dynamics. Finally, also notice that the algorithm approximates different conditional expectations for $s_t^i = 1$ and for $s_t^i = 0$. This is convenient because it allows identical agents, differing only in their idiosyncratic shock of labor productivity, to choose different consumption levels [8].

The potential advantages of the algorithm outlined above are that 1) the inner iterations are cheap because we simulate the decisions of a single agent as in KS, but using PEA; 2) the outer iterations are cheap because the long-run simulations do not need to be extremely long; and 3) the course of dimensionality does not represent a binding constraint, and there is no need to restrict the set of states of nature.

8. It may seem that assuming, for instance, $s_t^i \in [0,1]$ would create a problem. This is not so since, in that case, the realization of s_t^i could be included among the regressors.

4. Results

WE first describe the calibration and the accuracy of the numerical solution. Then we discuss two applications and relate the results to the literature.

4.1. Calibration

We choose $u(c) = \log(c)$, $f(k, h) = k^{\theta} h^{1-\theta}$ with $\theta = 0.36$, $\delta = 0.025$, $\beta = 0.99$, $B = 1$, $\lambda \sim N\,(0, 0.00712)$, $\rho = 0.95$, and $\phi^{g} = 94\%$, $\phi^{b} = 90\%$, with the transition matrices as in table 4.1. These transition matrices imply an average duration of unemployment of 13 weeks in good times and 17 weeks in bad times. The previous choices are in line with those in the related literature simulating quarterly data for the U.S. economy (see, for instance, Hansen [1985], Imrohoroglu [1989] and Krusell and Smith [KS] [1998]).

TABLE 4.1: Transition matrices

$$\Pi^{g} = \begin{bmatrix} 0.9411 & 0.0589 \\ 0.9231 & 0.0769 \end{bmatrix} \qquad \Pi^{b} = \begin{bmatrix} 0.9216 & 0.0784 \\ 0.7059 & 0.2941 \end{bmatrix}$$

4.2. Accuracy

To solve for the stationary competitive equilibrium (SCE), in the inner iterations we fix $m = 5$, and we approximate the right hand side of the equations in (2.5) with functions of the form:

$$RHS^{s}\,(\Omega_t^i) = \exp\left\{\sum_{j=1}^{J} \theta_j^s \log\,(\omega_{t,j}^i)\right\},\ s = 0,1,$$

where $\omega_{t,j}^i$ corresponds to the j-component of the vector $\Omega_t^i = [1, k_{t-1}^i, z_t, \phi_t, k_{t-1}, \pi_t^i, k_{t-1}^i z_t, z_t\phi_t, k_{t-1}z_t]$, and where θ_j^s corresponds to the coefficient associated to $\omega_{t,j}^i$. That is, in each of the m regions of the k^i direction, we use a different function depending on whether the agent is in the productive or unproductive idiosyncratic state. Following KS, in the outer iterations we approximate only the law of motion for the mean of the stock of capital, as follows [9]:

$$\log(k_t) = \gamma_1 + \gamma_2 \log(k_{t-1}) + \gamma_3 \log(z_t) + \gamma_4 \log(\phi_t). \qquad (4.1)$$

The R^2 corresponding to the OLS regressions performed to update the coefficients in equation (4.1) is 0.999991. The standard deviation of the regression error is 0.013%. That is, the goodness of fit of the regression is very close to the one reported by KS, even though the realizations of both z_t and ϕ_t take values on a continuum.

To check for the overall accuracy of the solution, we use the test developed by Den Haan and Marcet (1994). This test checks whether the residual from the Euler equation (2.5) that holds with equality is statistically different from zero along certain directions. More precisely, we define the following objects:

$$\xi^i_{t+1} = u'(c^i_t) - \beta\,(r_{t+1} + 1 - \delta)\,u'(c^i_{t+1}),\ h(\omega^i_t) = [1, k_{t-1}, k_t, k_{t-1}, z_t, \phi_t, k^i_t, \pi^i_t],$$

$$B_T = T^{-1} \sum^T \xi^i_{t+1}\, h_t(\omega^i_t), \text{ and } A_T = T^{-1} \sum^T (\xi^i_{t+1})^2\, h_t(\omega^i_t)\, h'_t(\omega^i_t),$$

and, we test whether the statistic $TB'_T A^{-1}_T B_T$ is approximately distributed as an X^2 with j (the dimension of $h(\omega^i_t)$) degrees of freedom, as it should under the null hypothesis that the solution is exact and $T \to \infty$ (see proposition 1 in Den Haan and Marcet, 1994) [10]. In practice, performing the test amounts to obtain a large number of observations of the statistic $TB'_T A^{-1}_T B_T$ and to check if the values belong too often to the lower and upper critical regions of the corresponding X^2_j distribution. If so, this is evidence against the accuracy of the solution.

In the tests we report next, we use $h^j(\omega^i_t)$, with $j \leq 8$ indicating the first j-directions of the vector $h(\omega^i_t)$. For instance, by considering $j = 2$ and $j = 3$, we test whether the solution is accurate along the directions of aggregate capital. Table 4.2 reports the percentage of draws in the lower and upper 5% tails (denoted respectively l/u-5%) corresponding to 500 realiza-

9. See the appendices for further details about the implementation of Parameterizing Expectations Aproach (PEA)-KS and parameter values for $\bar{\theta}^s$ and $\bar{\gamma}$.

10. That is, the test checks whether the prediction error is orthogonal to the elements in the information set in period t.

tions of the statistic, with 3,000 observations in each one (the realizations of the stochastic shocks are different from the ones used in the short/long-run simulations) [11].

TABLE 4.2: Accuracy with PEA-KS (T = 3,000)

J	6		7		8		9	
h^j	1-5%	u-5%	1-5%	u-5%	1-5%	u-5%	1-5%	u-5%
2	7	3	6.6	3	7.2	3	7	2.8
3	5.4	5	6.2	5	6.4	5.4	5.6	4.6
4	5.6	5	5.6	4.4	5.8	3.8	6.2	4.6
5	6.4	4.2	8.2	4	7.4	3.8	7.2	3.8
6	4.2	7.2	4.4	7	4.4	8	4	8
7	5.6	7	4.4	8	4.2	8.2	4.2	8.8
8	5.6	6	4	8.2	4.4	9	4.4	8

Note: Columns J indicate the number of regressors used in the inner iterations according to the vector Ω_t^i. Rows indicate the directions of the joint test according to the elements in h^j (w_t^i).

The rough message from table 4.2 is that the relative frequencies are close to the theoretical 5%. In particular, the accuracy of the solution using a first order polynomial (J = 6) is remarkable along the joint directions of (k_{t-1}, k_t). This happens in spite of the *bounded rationality* of agents when predicting aggregate capital (remember that they use only the first moment of the distribution Λ_t, and that k_t is in Ω_t^i but that it is *not* used as a regressor in period t). For J = 6, the overall accuracy of the joint test using h^8 (ω_t^i) appears to be also very high. If anything, it seems that adding state variables (regressors) tends to produce slightly less accurate approximations [12].

In addition to the examples reported above, we have tried to solve the SCE using many other combinations of the state variables and higher order polynomials by including square terms. The problem with this strategy is that the matrix $X'X$ becomes singular, because several regressors—like those

11. We performed simulations of 5,000 periods and discarded the first 1,500 observations to reduce the effect of initial conditions. The test was, therefore, time-consuming due to the large number of realizations of the statistic (but it would not be appropriate to conduct the test along the directions of k_t, z_t, ϕ_t, or any other aggregate variable, using the residuals of many agents but only a few realizations for the previous aggregate states). Also, in the simulations the borrowing constraint binds some times for agents with s = 0 and low capital levels; therefore, in the tests we use only the residuals corresponding to an agent with s = 1.

12. See the appendices for additional results about accuracy.

involving cross-products between z, ϕ and π—are highly correlated [13]. These results suggest that using several first order polynomials to approximate the expected marginal utility on different regions along the k^i direction of the state space provides an accurate approximation to the SCE.

4.3. Applications

4.3.1. Equilibrium dynamics

We compute the averages and standard deviations over time of equilibrium variables in per capita terms, and we briefly look at the differences between the incomplete markets economy and its complete markets counterpart (the representative-agent economy). The results reported in tables 4.3a and 4.3b correspond to the statistics from a realization of equilibrium dynamics over 10,000 periods for each economy (we simulated 11,500 periods and discarded the first 1,500 observations), using in both cases the same sequences of stochastic shocks [14].

TABLE 4.3a: Aggregate time-series
(averages)

	Incomplete markets	Complete markets
y	3.4421	3.4413
c	2.5571	2.5572
inv.	0.8846	0.8841
k	35.3857	35.3657
R	1.01003	1.01004
sr	0.2566	0.2565

Note: Averages over time of aggregate variables in per capita terms (output, consumption, investment, capital, return factor defined as $r_t + 1 - \delta$, and saving rate).

It is clear from table 4.3a that output, investment, capital, and the saving rate are slightly larger under incomplete markets because of the precautionary saving role of capital. Perhaps the most interesting result in the

13. In those experiments, we remove π_t^i from the regressors, but then in the accuracy test, we find that the statistic is not too often in the lower 5% tail, and too many times in the upper 5% tail. For instance, the joint test with $h^8\ (\omega_t^i)$ delivers about 1% in the lower 5% tail, and more than 28% in the upper 5% tail.

14. The complete markets version of the economy is solved using PEA with a second order polynomial.

table is that the average stock of capital and the saving rate under incomplete markets are only 0.056 and 0.04% (respectively) larger than under complete markets. It is also interesting to see that the return factor is on average essentially the same across market arrangements (in spite of the larger amount of capital under incomplete markets). Table 4.3b shows that fluctuations under incomplete markets are slightly smaller than under complete markets (the only exception is consumption, but it fluctuates more precisely because of the lack of insurance markets) [15].

TABLE 4.3b: Aggregate time-series
(standard deviations)

	Incomplete markets	Complete markets
y	0.1553	0.1575
c	0.0932	0.0931
inv.	0.0749	0.0777
k	1.6016	1.6836
R	0.0009	0.0009

Note: Standard deviations of aggregate variables in per capita terms (same variables as in table 4.2).

The results in tables 4.3a and 4.3b are in line with previous findings in the related literature. They suggest that the effects of heterogeneity created by the lack of insurance markets washes-out in the aggregate.

4.3.2. Adding divisible labor

In the previous model agents do not value leisure, and, therefore, they supply their endowment of time whenever $s_t^i = 1$. In this section we extend the previous model by assuming that agents do value leisure: $u(c, l) = \log c + A \log l$. In the first example, we assume that labor is perfectly divisible, hence $l \in [0,1]$, and agents in state $s_t^i = 1$ are allowed to choose the amount of time supplied to the market in exchange of the wage rate. Following KS, we solve the equilibrium by approximating the law of motion of average capital and hours worked with log-linear functions. The accuracy of these approximations is $R^2(k) = 0.999997$ and $\sigma(k) = 0.007\%$ for the case of capital, and $R^2(h) = 0.99991$ and $\sigma(h) = 0.03\%$ for hours worked. The following table presents summary statistics of aggregate dynamics under complete and incomplete markets.

15. Díaz-Giménez (1997) and Obiols-Homs (2003) explore related economies and show that endogenous labor supply decisions give rise to larger fluctuations under incomplete markets.

Again, the results in tables 4.4a and 4.3b are similar to those in Obiols-Homs (2003).

4.3.3. Indivisible labor

We now study an incomplete markets economy with indivisible labor economy in the vein of Hansen (1985) and Rogerson (1988). In this version of the model, we assume that workers and firms exchange a lottery such that a worker i commits to work h_0 with probability θ_t^i in period t when the market wage rate is w_t. Since it is assumed that these contracts are signed once the labor productivity endowment has been realized, an agent i gets paid $w_t\, s_t^i\, \theta_t^i\, h_0$, where $h_0 \in (0,1)$ is a prespecified amount of hours of work (and the realization of a lottery finally determines whether the agent must actually go to work or not). Notice that because the contract between the firm and a worker involves a lottery, it is *as if* the firm completely insures the worker against labor income uncertainty *but only* once its labor productivity endowment is known. In particular, it is not possible to write contracts contingent on the realization of s_t^i, hence workers are still subject to labor productivity uncertainty. Given our assumed market structure, the only way workers can smooth out consumption fluctuations is by actively saving and dissaving in the market for capital (as before, the borrowing limit precludes

TABLE 4.4a: Aggregate time-series
(averages)

	Incomplete markets	Complete markets
y	1.0976	1.0341
k	11.3382	10.6071
h	0.2936	0.2777
c	0.8145	0.7689

Note: Averages over time of aggregate variables in per capita terms (output, capital, hours worked and consumption).

TABLE 4.4b: Aggregate time-series
(standard deviations)

	Incomplete markets	Complete markets
y	0.0511	0.0592
k	0.511	0.6207
h	0.0056	0.0091
c	0.0805	0.033

Note: Standard deviations of aggregate variables in per capita terms (same variables as in table 4.3a).

having access to complete insurance by issuing an arbitrarily large amount of debt).

We use $\delta = 0.025$ and $\beta = 0.99$ as before. The technology in the production side of the economy is a Cobb-Douglas with a capital output share $\theta = 0.3224$ (see Prescott, 2004). The utility function is $u(c) + v(l) = \log(c) + A \log(l)$ as in Hansen (1985). The parameters A and h_0 are selected to match important observations of the labor market. In particular, we fix $h_0 = 0.4$ reflecting that there are 40 hours of work in a normal workweek, and that there are 13 weeks in a quarter. The value of A is selected to obtain that in equilibrium $h_t = 0.25$, which is approximately the hours of work observed in the U.S. economy in terms of per-working-age person (Prescott, 2002). The endowment of labor productivity is given by $E = \{0,1\}$, $\pi_{1|1} = 0.9305$, and $\pi_{1|0} = 0.8$. This specification for the idiosyncratic endowments is similar to the one in Krusell and Smith (1998) and many other papers in the incomplete markets literature. Finally, the persistence of the aggregate shock to technology is $\rho = 0.99$, and ε is distributed according to a $N(0,0.00712)$ (Hansen, 1985).

We investigate first the effect of a reduction in the number of hours of work over a week, similar to the labor market reforms undertaken in France during the nineties. In table 4.5 we report the results of such an experiment for a version of the economy in which there is no aggregate uncertainty (there are no aggregate shocks to technology). The table shows that a reduction in h_0 leads to an increase in the stock of capital, hours of work, output (all measured in per capita terms), and W (average welfare forms the equilibrium distribution of agents over states at the steady state), irrespectively of the borrowing limit. The explanation for these changes is that with a smaller number of hours of work, agents choose to work more often (i.e., there is more people working), and also increase their savings for precautionary reasons. Hence, average stock of capital and labor increase, and so does average output. Notice also that the saving rate essentially does not change with a smaller h_0, and yet, the capital/labor ratio also increases. This means that the wage rate increases, and it explains the increase in welfare.

We finally restore the aggregate shocks to technology and investigate the cyclical properties of equilibrium time series. This exercise is interesting for at least two reasons. First, the indivisible labor model with complete markets is well known for predicting large fluctuations in hours of work, and it would be interesting to know to what extent this property of the model is sensitive to the market arrangement. Second, and related to the previous point, there is an ongoing debate in the profession about the elasticity of la-

TABLE 4.5: Equilibrium allocations

		B = 0.5	B= 1
	K	2.3779	2.3714
	H	0.2499	0.2500
$h_0 = 0.4$	*Y*	0.2583	0.2581
	sr	0.2301	0.2296
	W	–2.4942	–2.4956
		B = 0.5	**B= 1**
	K	2.4721	2.4701
	H	0.2592	0.2592
$h_0 = 0.35$	*Y*	0.2681	0.2681
	sr	0.2301	0.303
	W	–2.4571	–2.4572

bor supply and aggregation theory (see, for instance, Browning, Hansen and Heckman [1999], Prescott [2004] and Ljungqvist and Sargent [2007]). Under the assumption of complete markets, it is possible to perfectly aggregate the behavior of individual households and obtain the so called *representative consumer.* This artificial agent can be shown to have an arbitrarily large labor supply elasticity even though, consistently with micro observations, individual households have a small labor supply elasticity (Hansen, 1984).

The fact that markets are incomplete in our model precludes perfect aggregation, so it is not possible to obtain the *representative consumer.* Hence, our model can be used to asses whether the type of aggregation that complete markets facilitate is in fact quantitatively relevant for the predictions of the model. Table 4.6 reports standard deviations and correlations in the usual way in the RBC tradition [16]. The first column in the table reports standard deviation of consumption, investment, capital stock, hours of work, and labor productivity relative to the standard deviation of output. The second column reports the correlations with output.

A salient feature from table 4.6 is that even with incomplete markets, the indivisible labor model still predicts large fluctuations of hours worked, together with large fluctuations in output. These fluctuations are substantially larger than in the usual divisible labor model with complete markets,

16. That is, averages of 100-time series with 120 periods in each of them, logged and detrended using the HP-filter. We discarded the first 1,000-time periods in each series to reduce the effects of initial conditions. In the examples reported in the table, we assume $h_0 = 0.4$ and $B = 1$.

TABLE 4.6: Deviations and correlations

x	*C*	Inv.	*K*	*H*	*Y/H*
% sdl $(Y) = 2.09$					
std (x)/std (Y)	0.33	4.21	0.27	0.90	0.33
corr (x, Y)	0.39	0.96	0.48	0.94	0.43

and also larger than in the Hansen economy. Notice also that there is a high correlation between hours and output, and yet, the correlation between consumption and output is rather small. This later fact suggests that there is substantial heterogeneity among households, and that at the aggregate level aggregation is far from perfect (in the complete markets version of the model this correlation is close to 0.9). These findings suggest that whether markets are complete or not, i.e., whether aggregation is perfect or not, does not seem to alter significatively some basic properties of the indivisible labor model.

5. Conclusion

THIS working paper shows that the combination of Parameterizing Expectations Approach (PEA) with the approach in Krusell and Smith (KS) provides an accurate procedure to approximate equilibrium dynamics of incomplete markets, stochastic economies. It is encouraging that the quantitative results in sections 4.3.1 and 4.3.2, using the combined approach, replicate previous findings about the effects of incomplete markets in a production economy without leisure. Also, we used the numerical procedure to solve a model economy in which insurance markets are incomplete, and in which labor is indivisible. The results from this model, in section 4.3.3, suggest that the magnitude of the fluctuations of hours of work and of output are unaffected by the market arrangement.

Appendices

Appendix 1. Implementation

THE $m = 5$ regions for k^i are as follows: [1, 10], (10, 20], (20, 30], (30, 90], ($90 \leq k^i$). For the short-run simulations, we use 2,400-time series, with 50 observations in each one, starting the simulations between 100 and 50 times with initial conditions $k^i_{-1} \in [7.5{,}275]$. The number of observations in each of the m regions is between 16,000 and 33,000.

Following Krusell and Smith (KS), we approximate only the law of motion for the mean of the stock of capital, as indicated in equation (4.1). The length of the long-run simulation used to obtain the time series needed to run the OLS regression to update the parameters $\bar{\gamma}$ is 20,000 periods. We use as initial condition for these parameters the coefficients obtained from the OLS regression corresponding to the complete markets version of the economy. In the long-run simulation, we use 5,000 agents, reassigning in a random way the idiosyncratic state s^i (if needed), so that ϕ_t approximately satisfies the law of large numbers.

The computation of the transition matrices Π_t (and therefore ϕ_t) involves the Normal distribution (the cdf $G(\cdot)$ in the main text). Thanks to the symmetry of the Normal distribution, we approximate only the right tail of a standardized Normal distribution using a polynomial and transform the values in the usual way: for $x \geq 0$,

$$\Phi(x) = 1 - \frac{1}{2}(1 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4)^{-4},$$

with $a_1 = 0.196854$, $a_2 = 0.115194$, $a_3 = 0.000344$, $a_4 = 0.019527$. It is known that the error in the resulting approximation is smaller than 2.5×10^{-4} (see Johnson and Kotz, 1970: 55).

In the short-run simulations the aggregate states and the law of motion for the mean of Λ_t determine the equilibrium prices. Therefore, to pin down k^i_t, first we determine c^i_t assuming that the equation in (2.5) holds with equality (using the function $RHS^s(\Omega^i_t)$ to evaluate the conditional expectation), and then we find k^i_t as the residual of the budget constraint. If the resulting $k^i_t < B$, then we fix $k^i_t < B$, find c^i_t as the residual of the budget constraint, and continue to period $t + 1$. In the long-run simulations we follow the same strategy, the only differences here are that we need to compute capitals and consumptions in each period for all agents, and that equilibrium prices are computed using the actual average capital in the economy, not the one predicted by the function in (4.1).

In steps 3 and 6, at the end of an iteration *j*, we update the coefficients $\bar{\theta}$ and $\bar{\gamma}$ according to $\bar{\theta}_j = \kappa_1\, \bar{\theta}^* + (1 - \kappa_1)\, \bar{\theta}_{j-1}$ and $\bar{\gamma}_j = \kappa_2\, \bar{\gamma}^* + (1 - \kappa_2)\, \bar{\gamma}_{j-1}$, where $\bar{\theta}_{j-1}$ and $\bar{\gamma}_{j-1}$ are the vectors of coefficients in place in the previous iteration, $\bar{\theta}^*$ and $\bar{\gamma}^*$ are the vectors of coefficients obtained in the regressions corresponding to the *j* iteration, and $\kappa_{1,2} \in (0,1)$ are two constants governing the speed of convergence. We use $\kappa_1 = 0.6$ and $\kappa_2 = 0.1$. We stop iterating when the rate of change of the parameters is smaller than 45×10^{-4} (therefore, the average rate of change of a single parameter is about 0.01%). The choices for $\kappa_{1,2}$ may slow down a bit the algorithm, but they prevent erratic behavior and cycles between two sets of almost identical parameter without never converging. Nevertheless, computing costs are strikingly small: We programmed the algorithm in FORTRAN, and finding a solution took about one hour on a desktop at 2.00GHz [17].

The results about accuracy reported in the main text suggest that the above choices for *m* and for the length of each interval for k^i seem appropriate for the assumed calibration. Nevertheless, these choices are clearly problem-specific. Also, it is convenient to fix an upper limit for k^i in the long-run simulation (all grid methods, by construction, imply such an upper limit). This prevents agents from becoming too wealthy, i.e., to reach an asset level beyond the level for which expected marginal utility is accurately approximated. The upper limit for k^i should be chosen sufficiently large so that it is rarely attained and, thus, has a negligible impact on equilibrium prices and dynamics. In the examples reported in the main text, we fix the upper limit equal to 300 units. With this limit, in some periods there are about 0.2% constrained agents (this is why we start many short-run simulations with initial capitals as large as 200 or even 275 units).

Tables A.1.1 and A.1.2 report the coefficients obtained with $J = 6$ (for reasons of space, in table A.1.1, we only report five decimal points).

17. For comparison, Obiols-Homs (2003) solves with Parameterizing Expectations Approach (PEA) several versions of the economy in the main text and reports computing costs ranging between 24 hours and a week on a 866 MHz computer. Carroll (2000) solves a version of the previous economy using the KS code for MATHEMATICA, and reports a computing cost of 72 hours on a laptop at 333 MHz. Further in the text, we report that solving a similar economy using a version of the KS approach on a desktop at 2.00GHz took about 18 hours with a FORTRAN code.

TABLE A.1.1: Coefficients θ_j^s for $J = 6$

m	1	2	3	4	5
1	4.77852	3.01831	2.97960	3.79865	7.02985
k_{t-1}^i	–0.17922	–0.10298	–0.09588	–0.18233	–0.41915
z_t	–0.68778	–0.56472	–0.36441	–0.31303	–0.27321
ϕ_t	–0.04029	–0.01755	–0.02272	–0.02366	–0.03757
k_{t-1}	–0.58368	–0.47308	–0.47107	–0.45677	–0.33085
π_t^i	–1.62376	–0.30210	–0.11870	–0.09025	–0.12861
1	23.19105	3.56037	3.03930	3.86930	7.61193
k_{t-1}^i	–0.49746	–0.13698	–0.10656	–0.19114	–0.42634
z_t	1.48057	–0.54866	–0.42381	–0.31614	–0.25982
ϕ_t	1.18764	–0.06713	–0.05901	–0.04002	–0.01114
k_{t-1}	–0.81401	–0.48780	–0.46321	–0.44867	–0.33890
π_t^i	–0.90191	–0.07710	–0.01393	–0.01166	–0.01919

Note: Coefficients associated to the variables in the left colum for each *m*. The upper panel corresponds to $s = 1$, the lower panel corresponds to $s = 0$.

TABLE A.1.2: Coefficients γ for $J = 6$

$\gamma 1$	$\gamma 2$	$\gamma 3$	$\gamma 4$
0.1270167683	0.9655341564	0.0650106315	0.053370734

Note: Coefficients associated to equation 4.1 in the main text.

Appendix 2. More on Accuracy

WE performe the same test as in the main text but considering observations for both $s = 1$ and $s = 0$. The results with $J = 6$ are reported in table A.2.1.

TABLE A.2.1.: **Accuracy with Parameterizing Expectations Approach (PEA)-Krusell and Smith (KS) ($T = 3{,}000$)**

	h^2	h^3	h^4	h^5	h^6	h^7	h^8
1-5%	11.8	10	9.6	8.6	7	6.8	7.2
u-5%	0.8	3.2	4.2	3	4.8	5.4	7.5

Note: Test of accuracy using both $s = 1$ and $s = 0$.

The differences between table 4.2 and table A.2.1 suggest that the approximation of expected marginal utility in the *unproductive state* is not as accurate as in the *productive state.* Nevertheless, the results are still close to the theoretical 5%. As a comparison for the results on accuracy, below we report the results on three additional experiments. As a first example, we compute approximations to the stationary competitive equilibrium (SCE) using the same strategy as in the main text but assuming that agents predict the stock of capital in the next period using a linear function: $k_t = \gamma_1 + \gamma_2 k_{t-1} + \gamma_3 z_t + \gamma_4 \phi_t$. The R^2 of the corresponding regression is 0.99995, and the standard deviation of the regression error is about 0.52%. The goodness of fit is not as good as with equation (4.1), but the forecast errors are not terribly large. When we compute the test of accuracy under the previous specification, we find that the relative frequencies in the lower 5% tail are similar to the ones in table 4.2 (slightly lower), but the relative frequencies in the upper 5% tail are larger (about 9.8 or even 11.4%).

Applications of PEA to incomplete markets economies, as explained in section 3.2, are not as accurate as PEA-KS. For instance, Obiols-Homs (2003) solves several versions of the economy in the main text but with endogenous labor. The author reports that the solution passes the less stringent tests 1 and 2 in the working paper version of Den Haan and Marcet (1989). However the solution fails to pass the test in the main text (the statistic was never in the lower 5% tail, and that it was more than 30% of the times in the upper 5% tail of the corresponding X^2 distribution).

As a last example, we implement a version of the KS approach using a different grid method and study the accuracy of the approximation to the SCE for a simpler version of the economy. Specifically, in this version we assume that $z \in \{0.99, 1.01\}$ with probabilities of transition equal to 0.5, ϕ constant and equal to 92%, and probabilities of transition $\pi_{1|1} = 0.9305$ and $\pi_{1|0} = 0.8$ (same preferences and technologies as in the text). To further restrict the state space, we assume that, in the productive state, agents are able to provide 0.327 efficient units of labor. We solve for the equilibrium by approximating the derivative of the value function on a grid, as explained in detail in Huggett (1983) [18]. In the grid, we use 620 points in the k^i direction (with more points close to the borrowing limit), and 100 points in the k direction (these numbers of points are similar to those in KS). Decision rules associated to the derivatives of the value function are accurate up to the 10^{-5} decimal point. We stop iterating in the laws of motion for average capital when the rate of change in the parameters is smaller than 10^{-4}. Given the assumptions on shocks, the algorithm determines four policy functions corresponding to the agent's problem in each aggregate and idiosyncratic state s: $k\,(k^i, k, 1.01, 0.327)$, $k\,(k^i, k, 1.01, 0)$, $k\,(k^i, k, 0.99, 0.327)$ and $k\,(k^i, k, 0.99, 0)$. The computing cost of the solution was almost 18 hours on a desktop at 2.00GHz.

The estimated laws of motion for average capital are as follows:

$$\log(k') = 0.09297 + 0.9618 \log(k);\ R^2 = 0.92784,\ \hat{\sigma} = 0.0932\%,$$

in good times and

$$\log(k') = 0.09 + 0.96232 \log(k);\ R^2 = 0.93215,\ \hat{\sigma} = 0.0931\%,$$

in bad times ($\hat{\sigma}$ is the standard deviation of the regression errors). Clearly, the goodness of fit of these prediction rules is substantially smaller than the one obtained by KS or the one obtained with PEA-KS. This is surprising given the presumed high accuracy of decision rules, and it represents a serious indication of nonnegligible inaccuracies. The results of the test of accuracy for different elements of the vector $h(\omega_t^i) = [1, k_{t-1}, k_t, k_{t-1}^i]$ are

18. This method is less sophisticated than the KS method which approximates the value function, but still allows for choosing capital levels not in the grid by using linear interpolation, and in principle solutions can be refined up to arbitrary accuracy.

reported in table A.2.2 (500 realizations of the statistic, with 100 observations in each one).

TABLE A.2.2: Accuracy with a grid method ($T = 100$)

	h^2	h^3	h^4
Lower 5%	0.6	0.4	0
Upper 5%	10.6	16.4	27.4

Note: Test of accuracy using only observations with $s = 1$ (as in table 4.2).

We see again that even with a small sample size, the statistical test is able to detect severe inaccuracies. These results suggest that the high accuracy in KS may be very specific to their approximation method, and that it may not easily extend to other similar procedures.

References

AIYAGARI, S. R. (1994): "Uninsured Idiosyncratic Risk and Aggregate Saving", *The Quarterly Journal of Economics,* 109, 659-684.

BROWNING, M., L. P. HANSEN and J. J. HECKMAN (1999): "Micro Data and General Equilibrium Models", Handbook of Macroeconomics, in J. B. Taylor & M. Woodford (ed.), edition 1, 1, chapter 8, 543-633.

CARROLL, C. (2000): "Requiem for the Representative Consumer? Aggregate Implications of Microeconomic Consumption Behavior", *American Economic Review,* 90 (2), John Hopkins University, 110-115.

DÍAZ-GIMÉNEZ, J. (1997): "Uninsured Idiosyncratic Risk, Liquidity Constraints and Aggregate Fluctuations", *Economic Theory,* 10, 463-482.

HAAN, W. J. DEN (1996): "Heterogeneity, Aggregate Uncertainty and the Short-Term Interest rate", *Journal of Business and Economic Statistics,* 14, 399-411.

— and A. MARCET (1989): *Accuracy in Simulations,* Working Paper, Carnegie-Mellon University.

— and A. MARCET (1990): "Solving the Stochastic Growth Model by Parameterizing Expectations", *Journal of Business and Economic Statistics,* 8 (1), 31-34.

— and A. MARCET (1994): "Accuracy in Simulations", *Review of Economic Studies,* 61, 3-17.

HANSEN, G. D. (1985): "Indivisible Labor and the Business Cycle", *Journal of Monetary Economics,* 16 (3), 309-327.

HUGGETT, M. (1993): "The Risk-Free Rate in Heterogeneous-Agent Incomplete Insurance Economies", *Journal of Economic Dynamics and Control,* 17, 953-969.

— (1997): "The One-Sector Growth Model with Idiosyncratic Shocks: Steady States and Dynamics", *Journal of Monetary Economics,* 39, 385-403.

IMROHOROGLU, A. (1989): "Cost of Business Cycles with Indivisibilities and Liquidity Constraints", *Journal of Political Economy,* 97, 1364-1383.

JOHNSON, N. L. and S. KOTZ (1970): *Distributions in Statistics,* John Wiley & Sons.

KRUSELL, P. and A. A. SMITH Jr. (1998): "Income and Wealth Heterogeneity in the Macroeconomy", *Journal of Political Economy,* 106, 867-896.

LJUNGQVIST, L. and T. J. SARGENT (2007): "Two Questions about European Unemployment", *Econometrica* [forthcoming].

MARCET, A. (1988): *Solution of Nonlinear Models by Parameterizing Expectations,* Manuscript, GSIA, Carnegie Mellon University.

— and R. MARIMON (1992): "Communication, Commitment and Growth", *Journal of Economic Theory,* 58, 219-249.

OBIOLS-HOMS, F. (2003): "Incomplete Unemployment Insurance and Aggregate Fluctuations", *Review of Economic Dynamics,* 6, 602-636.

PRESCOTT, E. C. (2002): "Prosperity and Deression", Minneapolis Fed, Working Paper 618.

— (2004): "Why Do Americans Work so Much?", Minneapolis Fed, *Quarterly Review,* 28, 2-13.

ROGERSON, R. (1998): "Indivisible Labor, Lotteries and Equilibrium", *Journal of Monetary Economics,* 21 (1), 3-16.

*A B O U T T H E A U T H O R **

FRANCESC OBIOLS-HOMS holds a PhD and an MA in economics from the Pompeu Fabra University. He obtained his BS in economics from the Autonomous University of Barcelona where he is currently a professor of economic analysis. He was a visiting researcher at ECARE (Free University of Brussels), a visiting student at the University of Berkeley and at the University of Minnesota and a candidate-level member of the National Researcher System of Mexico. He has received scholarships from several institutions (Spanish Science and Technology Ministry [Ramon y Cajal Program], Spanish Science and Education Ministry, European Commission, etc.). His research fields are real economic cycles, heterogeneity, effects of incomplete insurance markets and simulation techniques. He has published several articles in Spanish and international journals such as *Journal of Monetary Economics, Review of Economic Dynamics, Journal of Money Credit and Banking, Economic Theory* and *Gaceta de Economía,* among others.
email: francesc.obiols@uab.es

Any comments on the contents of this paper can be addressed to Francesc Obiols-Homs at francesc.obiols@uab.es.

* I would like to thank Gary Hansen, Ken Judd, Todd Keister and Michael Reiter for helpful comments and suggestions. Financial support from the BBVA Foundation is also gratefully acknowledged.

Fundación BBVA

D O C U M E N T O S   D E   T R A B A J O

NÚMEROS PUBLICADOS

DT 01/04 *Métodos de solución de problemas de asignación de recursos sanitarios*
Helena Ramalhinho Dias Lourenço y Daniel Serra de la Figuera

DT 01/05 *Licensing of University Inventions: The Role of a Technology Transfer Office*
Inés Macho-Stadler, David Pérez-Castrillo y Reinhilde Veugelers

DT 02/05 *Estimating the Intensity of Price and Non-price Competition in Banking: An Application to the Spanish Case*
Santiago Carbó Valverde, Juan Fernández de Guevara Radoselovics, David Humphrey y Joaquín Maudos Villarroya

DT 03/05 *Sistemas de pensiones y fecundidad. Un enfoque de generaciones solapadas*
Gemma Abío Roig y Concepció Patxot Cardoner

DT 04/05 *Análisis de los factores de exclusión social*
Joan Subirats i Humet (Dir.), Ricard Gomà Carmona y Joaquim Brugué Torruella (Coords.)

DT 05/05 *Riesgos de exclusión social en las Comunidades Autónomas*
Joan Subirats i Humet (Dir.), Ricard Gomà Carmona y Joaquim Brugué Torruella (Coords.)

DT 06/05 *A Dynamic Stochastic Approach to Fisheries Management Assessment: An Application to some European Fisheries*
José M. Da-Rocha Álvarez y María-José Gutiérrez Huerta

DT 07/05 *The New Keynesian Monetary Model: Does it Show the Comovement between Output and Inflation in the U.S. and the Euro Area?*
Ramón María-Dolores Pedrero y Jesús Vázquez Pérez

DT 08/05 *The Relationship between Risk and Expected Return in Europe*
Ángel León Valle, Juan Nave Pineda y Gonzalo Rubio Irigoyen

DT 09/05 *License Allocation and Performance in Telecommunications Markets*
Roberto Burguet Verde

DT 10/05 *Procurement with Downward Sloping Demand: More Simple Economics*
Roberto Burguet Verde

DT 11/05 *Technological and Physical Obsolescence and the Timing of Adoption*
Ramón Caminal Echevarría

DT 01/06 *El efecto de la inmigración en las oportunidades de empleo de los trabajadores nacionales: Evidencia para España*
Raquel Carrasco Perea, Juan Francisco Jimeno Serrano y Ana Carolina Ortega Masagué

DT 02/06 *Inmigración y pensiones: ¿Qué sabemos?*
José Ignacio Conde-Ruiz, Juan Francisco Jimeno Serrano y Guadalupe Valera Blanes

DT 03/06 *A Survey Study of Factors Influencing Risk Taking Behavior in Real World Decisions under Uncertainty*
Manel Baucells Alibés y Cristina Rata

DT 04/06 *Measurement of Social Capital and Growth: An Economic Methodology*
Francisco Pérez García, Lorenzo Serrano Martínez, Vicente Montesinos Santalucía y Juan Fernández de Guevara Radoselovics

DT 05/06 *The Role of ICT in the Spanish Productivity Slowdown*
Matilde Mas Ivars y Javier Quesada Ibáñez

DT 06/06 *Cross-Country Comparisons of Competition and Pricing Power in European Banking*
David Humphrey, Santiago Carbó Valverde, Joaquín Maudos Villarroya y Philip Molyneux

DT 07/06 *The Design of Syndicates in Venture Capital*
Giacinta Cestone, Josh Lerner y Lucy White

DT 08/06 *Efectos de la confianza en la información contable sobre el coste de la deuda*
Belén Gill de Albornoz Noguer y Manuel Illueca Muñoz

DT 09/06 *Relaciones sociales y envejecimiento saludable*
Ángel Otero Puime, María Victoria Zunzunegui Pastor, François Béland, Ángel Rodríguez Laso y María Jesús García de Yébenes y Prous

DT 10/06 *Ciclo económico y convergencia real en la Unión Europea: Análisis de los PIB per cápita en la UE-15*
José Luis Cendejas Bueno, Juan Luis del Hoyo Bernat, Jesús Guillermo Llorente Álvarez, Manuel Monjas Barroso y Carlos Rivero Rodríguez

DT 11/06 *Esperanza de vida en España a lo largo del siglo xx: Las tablas de mortalidad del Instituto Nacional de Estadística*
Francisco José Goerlich Gisbert y Rafael Pinilla Pallejà

DT 12/06 *Convergencia y desigualdad en renta permanente y corriente: Factores determinantes*
Lorenzo Serrano Martínez

DT 13/06 *The Common Agricultural Policy and Farming in Protected Ecosystems: A Policy Analysis Matrix Approach*
Ernest Reig Martínez y Vicent Estruch Guitart

DT 14/06 *Infrastructures and New Technologies as Sources of Spanish Economic Growth*
Matilde Mas Ivars

DT 15/06 *Cumulative Dominance and Heuristic Performance in Binary Multi-Attribute Choice*
Manel Baucells Alibés, Juan Antonio Carrasco López y Robin M. Hogarth

DT 16/06 *Dynamic Mixed Duopoly: A Model Motivated by Linux versus Windows*
Ramon Casadesus-Masanell y Pankaj Ghemawat

DT 01/07 *Social Preferences, Skill Segregation and Wage Dynamics*
Antonio Cabrales Goitia, Antoni Calvó-Armengol y Nicola Pavoni

DT 02/07 *Stochastic Dominance and Cumulative Prospect Theory*
Manel Baucells Alibés y Franz H. Heukamp

DT 03/07 *Agency Revisited*
Ramon Casadesus-Masanell y Daniel F. Spulber

DT 04/07 *Social Capital and Bank Performance: An International Comparison for OECD Countries*
José Manuel Pastor Monsálvez y Emili Tortosa-Ausina

DT 05/07 *Cooperation and Cultural Transmission in a Coordination Game*
Gonzalo Olcina Vauteren y Vicente Calabuig Alcántara

DT 06/07 *The* Extended *Atkinson Family and Changes in Expenditure Distribution: Spain 1973/74 – 2003*
Francisco J. Goerlich Gisbert, María Casilda Lasso de la Vega Martínez y Ana Marta Urrutia Careaga

DT 07/07 *Análisis de la evolución de la dependencia en la tercera edad en España*
David Casado Marín

DT 08/07 *Designing Contracts for University Spin-offs*
Inés Macho-Stadler, David Pérez-Castrillo y Reinhilde Veugelers

DT 09/07 *Regional Differences in Socioeconomic Health Inequalities in Spain*
Pilar García Gómez y Ángel López Nicolás

DT 10/07 *The Evolution of Inequity in Access to Health Care in Spain: 1987-2001*
Pilar García Gómez y Ángel López Nicolás

DT 11/07 *The Economics of Credit Cards, Debit Cards and ATMs: A Survey and Some New Evidence*
Santiago Carbó-Valverde, Nadia Massoud, Francisco Rodríguez-Fernández, Anthony Saunders y Barry Scholnick

DT 12/07 *El impacto comercial de la integración europea, 1950-2000*
Luis Fernando Lanaspa Santolaria, Antonio Montañés Bernal, Marcos Sanso Frago y Fernando Sanz Gracia

DT 13/07 *Proyecciones de demanda de educación en España*
Andrés M. Alonso Fernández, Daniel Peña Sánchez de Rivera y Julio Rodríguez Puerta

DT 14/07 *Aversion to Inequality and Segregating Equilibria*
Antonio Cabrales Goitia y Antoni Calvó-Armengol

DT 15/07 *Corporate Downsizing to Rebuild Team Spirit*
Antonio Cabrales Goitia y Antoni Calvó-Armengol

DT 16/07 *Maternidad sin matrimonio: Nueva vía de formación de familias en España*
Teresa Castro Martín

DT 17/07 *Immigrant Mothers, Spanish Babies: Childbearing Patterns of Foreign Women in Spain*
Marta Roig Vila y Teresa Castro Martín

DT 18/07 *Los procesos de convergencia financiera en Europa y su relación con el ciclo económico*
José Luis Cendejas Bueno, Juan Luis del Hoyo Bernat, Jesús Guillermo Llorente Álvarez, Manuel Monjas Barroso y Carlos Rivero Rodríguez

DT 19/07 *On Capturing Rent from a Non-Renewable Resource International Monopoly: A Dynamic Game Approach*
Santiago J. Rubio Jorge

DT 20/07 *Simulación de políticas impositivas medioambientales: Un modelo de equilibrio general de la economía española*
Antonio Manresa Sánchez y Ferran Sancho Pifarré

DT 21/07 *Causas del crecimiento económico en Argentina (1990-2004): Otro caso de «tiranía de los números»*
Ariel Alberto Coremberg

DT 22/07 *Regional Financial Development and Bank Competition: Effects on Economic Growth*
Juan Fernández de Guevara Radoselovics y Joaquín Maudos Villarroya

DT 23/07 *Política fiscal e instituciones presupuestarias en los países de la reciente ampliación de la Unión Europea*
Carlos Mulas-Granados, Jorge Onrubia Fernández y Javier Salinas Jiménez

DT 24/07 *Measuring International Economic Integration: Theory and Evidence of Globalization*
Iván Arribas Fernández, Francisco Pérez García y Emili Tortosa-Ausina

DT 25/07 *Wage Inequality among Higher Education Graduates: Evidence from Europe*
José García Montalvo

DT 26/07 *Governance of the Knowledge-Intensive Firm*
Vicente Salas Fumás

DT 27/07 *Profit, Productivity and Distribution: Differences Across Organizational Form*
Emili Grifell-Tatjé y C. A. Knox Lovell

DT 28/07 *Identifying Human Capital Externalities: Theory with Applications*
Antonio Ciccone y Giovanni Peri

DT 01/08 *A Multiplicative Human Development Index*
Carmen Herrero Blanco, Ricardo Martínez Rico y Antonio Villar Notario

DT 02/08 *Real Exchange Rate Appreciation in Central and Eastern European Countries: Why the Balassa-Samuelson Effect Does Not Explain the Whole Story*
José García Solanes

DT 03/08 *Can International Environmental Cooperation Be Bought?*
Cristina Fuentes Albero y Santiago J. Rubio Jorge

DT 04/08 *On the Dynamics of Globalization*
Iván Arribas Fernández, Francisco Pérez García y Emili Tortosa-Ausina

DT 05/08 *Los motores de la aglomeración en España: Geografía versus historia*
Francisco J. Goerlich Gisbert y Matilde Mas Ivars

DT 06/08 *Sobre el tamaño de las ciudades en España: Dos reflexiones y una regularidad empírica*
Francisco J. Goerlich Gisbert y Matilde Mas Ivars

DT 07/08 *Managing Waiting Lists in a Fair Way*
Carmen Herrero

DT 08/08 *Estimación del capital social en España: Series temporales por territorios*
Francisco Pérez García, Lorenzo Serrano Martínez
y Juan Fernández de Guevara Radoselovics

DT 09/08 *Estimation of Social Capital in the World: Time Series by Country*
Francisco Pérez García, Lorenzo Serrano Martínez
y Juan Fernández de Guevara Radoselovics

DT 10/08 *Capabilities and Opportunities in Health*
Carmen Herrero y José Luis Pinto Prades

DT 11/08 *Cultural Transmission and the Evolution of Trust and Reciprocity in the Labor Market*
Gonzalo Olcina Vauteren y Vicente Calabuig Alcántara

DT 12/08 *Vertical and Horizontal Separation in the European Railway Sector: Effects on Productivity*
Pedro Cantos Sánchez, José Manuel Pastor Monsálvez y Lorenzo Serrano Martínez

DT 01/09 *How Effective Are Rewards Programs in Promoting Payment Card Usage?: Empirical Evidence*
Santiago Carbó-Valverde y José Manuel Liñares-Zegarra

DT 02/09 *Comparative Advantage Across Goods and Product Quality*
Francisco Alcalá

DT 03/09 *El transporte ferroviario de alta velocidad: Una visión económica*
Javier Campos Méndez, Ginés de Rus Mendoza e Ignacio Barrón de Angoiti

DT 04/09 *¿En qué circunstancias está justificado invertir en líneas de alta velocidad ferroviaria?*
Ginés de Rus Mendoza y Chris Nash

DT 05/09 *Openness and Geographic Neutrality: How Do They Contribute to International Banking Integration?*
Iván Arribas Fernández, Francisco Pérez García y Emili Tortosa-Ausina

DT 06/09 *Firms' Main Market, Human Capital and Wages*
Francisco Alcalá y Pedro J. Hernández

DT 07/09 *Tax Evasion, Technology Shocks and the Cyclicality of Government Revenues*
Judith Panadés Martí

Documentos de Trabajo

8

Fundación BBVA

Plaza de San Nicolás, 4
48005 Bilbao
España
Tel.: +34 94 487 52 52
Fax: +34 94 424 46 21

Paseo de Recoletos, 10
28001 Madrid
España
Tel.: +34 91 374 54 00
Fax: +34 91 374 85 22

publicaciones@fbbva.es
www.fbbva.es